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Kinematics in Two-Dimensions

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How to Use this File

- Each topic is composed of brief direct instruction
- There are formative assessment questions after every topic denoted by black text and a number in the upper left.
  - > Students work in groups to solve these problems but use student responders to enter their own answers.
  - > Designed for SMART Response PE student response systems.
  - > Use only as many questions as necessary for a sufficient number of students to learn a topic.
- Full information on how to teach with NJCTL courses can be found at njctl.org/courses/teaching methods

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Vector Notation

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Position and Velocity Vectors

Motion problems in one dimension are interesting, but frequently, objects are moving in two, and even three dimensions (four, when you count time as a dimension in special and general relativity).

This is where the vector notation learned earlier comes in very handy, and we will start by defining a position vector, $\vec{r}$.

Average Velocity

As an object moves from one point in space to another, the average velocity of its motion can be described as the displacement of the object divided by the time it takes to move.

$$\vec{v}_{av} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$ (average velocity vector)

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

Instantaneous Velocity

To find the instantaneous velocity (the velocity at a specific point in time) requires the time interval to be so small that it can effectively be reduced to 0, which is represented as a limit. **#**

$$\vec{v}(t) = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$ (instantaneous velocity vector)

$$\vec{r} = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$$

Notation note: it will be assumed that all the motion vectors are time dependent, so after this slide x(t), y(t) and z(t) will be shown as x, y and z (same convention for velocity and acceleration).

Instantaneous Velocity Components

The instantaneous velocity has three different components: v_x, v_y, and v_z (any of which can equal zero).

Each component is shown below:

$$v_x = \frac{dx}{dt} \qquad v_y = \frac{dy}{dt} \qquad v_z = \frac{dz}{dt}$$

Vector representation:

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Average Acceleration

Acceleration is the rate at which the velocity is changing, and the average acceleration can be found by taking the difference of the final and initial velocity and dividing it by the time it takes for that event to occur.

$$\vec{a}_{av} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta\vec{v}}{\Delta t}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Instantaneous Acceleration

Just as we can find the velocity at a specific point in time, we can also find the instantaneous acceleration using a limit.

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

Instantaneous Acceleration

The instantaneous acceleration has three different components: a_x, a_y, and a_z (any of which can equal zero).

Each component is shown below:

$$a_x = \frac{dv_x}{dt} \qquad a_y = \frac{dv_y}{dt} \qquad a_z = \frac{dv_z}{dt}$$

Vector representation:

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$

1 The vector, $\vec{r} = (5t)\hat{\imath} + (3 - 2t^2)\hat{\jmath} + (10 + 2t)\hat{k}$, describes the position of a particle as a function of time. Find the expression for the velocity and acceleration vectors expressed as a function of time.

2 The vector, $\vec{r} = (6t^2 + 3)\hat{i} + (7t^3 + 4t^2)\hat{j} - 9\hat{k}$, describes the position of a particle as a function of time. Find the expression for the velocity and acceleration vectors expressed as a function of time.

Integration

The unit on One Dimension Kinematics showed how to obtain position from velocity, and velocity from acceleration through integration techniques. The same method works for two and three dimensions.

Each component is shown below, and since we are only looking for instantaneous values, we will leave out the limits of integration:

$$v_x = \int a_x(t)dt \qquad x = \int v_x(t)dt$$

$$v_y = \int a_y(t)dt \qquad y = \int v_y(t)dt$$

$$v_z = \int a_z(t)dt \qquad z = \int v_z(t)dt$$

Integration

Here is it what it looks like from a vector point of view, where we start with acceleration and integrate twice to get to position:

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{v} = \left(\int a_x(t)dt\right)\hat{i} + \left(\int a_y(t)dt\right)\hat{j} + \left(\int a_z(t)dt\right)\hat{k}$$

$$\vec{r} = \left(\int v_x(t)dt\right)\hat{i} + \left(\int v_y(t)dt\right)\hat{j} + \left(\int v_z(t)dt\right)\hat{k}$$

3 The vector, $\vec{a} = (6 - 3t^2)\hat{\imath} - 9\hat{\jmath} - (8t)\hat{k}$, describes the acceleration of a particle as a function of time. Find the expression for the velocity and position vectors expressed as a function of time.

Instantaneous values

Once the vector for position, velocity or acceleration is found, either by differentiation or integration, the instantaneous value can be found by substituting the value of time in for t.

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$\vec{v} = \left(\int a_x(t)dt\right)\hat{i} + \left(\int a_y(t)dt\right)\hat{j} + \left(\int a_z(t)dt\right)\hat{k}$$

$$\vec{r} = \left(\int v_x(t)dt\right)\hat{i} + \left(\int v_y(t)dt\right)\hat{j} + \left(\int v_z(t)dt\right)\hat{k}$$

Notation note: When you find the value of the position, velocity or vector, just leave it in vector notation - don't worry about the units - at this point in your physics education, its assumed you know them!

6 What is the velocity of an object at t = 3 s if its acceleration is described by $\vec{a} = (6t^2)\hat{i} - (6t + 7)\hat{j} + 9\hat{k}$?

Projectile Motion

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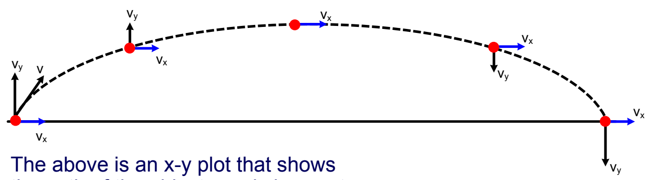
Projectile Motion

Have you ever thrown an object in the air or kicked a soccer ball to a friend and watched the path in space it followed?

The path is described by mathematics and physics - it is a parabolic path - another reason why you studied parabolas in mathematics.

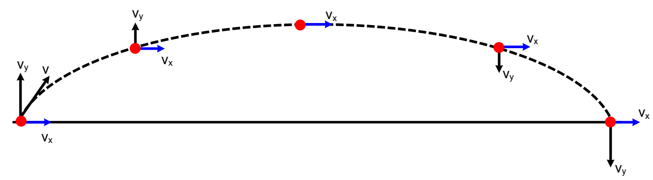


The above is an x-y plot that shows the path of the object - and shows at various points, the velocity vectors.

Take a minute and discuss the behavior of the v_y vectors.

Projectile Motion

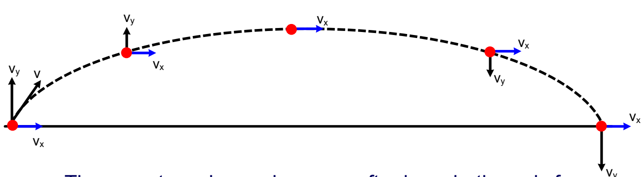
The v_y vectors are acting as studied earlier - v_y is maximum at the launch point, decreases under the influence of the gravitational field, reaches zero at the apex, and then increases until it reaches the negative of the initial velocity right before it strikes the ground.



Now that the v_y behavior has been reviewed, what else do you notice about this picture?

Projectile Motion

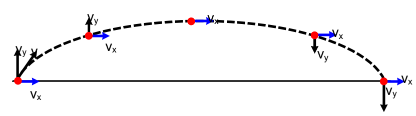
Just as in mathematics where a vector is resolved into two perpendicular vectors (x and y), in real life, the x motion is independent of the y motion and can be dealt with separately.



The v_y vectors change because after launch, the only force acting on the ball in the y direction is gravity. But, neglecting friction, there are NO forces acting in the x direction.

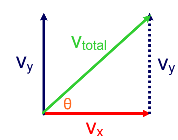
So v_x is constant throughout the motion.

Projectile Velocity



Vector analysis for the velocity gives us:

$$v_{total} = \sqrt{v_x^2 + v_y^2}$$

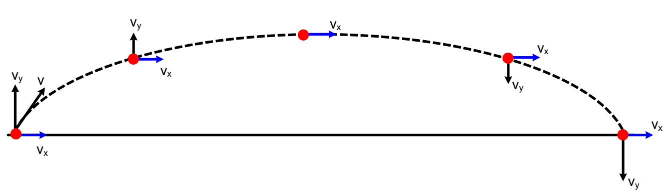


$$v_y = v\sin\theta$$

$$v_x = v\cos\theta$$

$$\theta = \tan^{-1}\frac{v_y}{v_x}$$

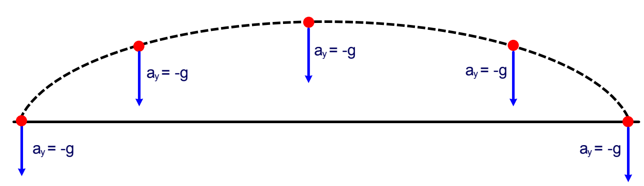
Velocity of a Projectile



In 1D Kinematics, you are used to the velocity of the object at its apex being zero. For 2D Kinematics, the y velocity is zero, but it has a total velocity because it still has a velocity component in the x direction.

What is the direction of the acceleration vector at each point?

Acceleration of a Projectile



Near the surface of the planet Earth, there is zero acceleration in the x direction, and a constant acceleration, with magnitude, g, in the negative y direction. This is true, regardless of the direction of the velocity or displacement of the projectile.

$a_x = 0$ $a_y = -g$

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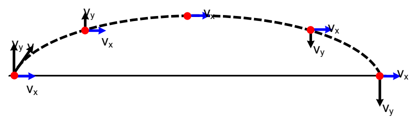
Motion of a Projectile



You know from experience that this motion is a parabola. Let's see if this can be derived mathematically, by examining the position equations in the x and y direction.

$$x = x_0 + v_{0x}t + 1/2\,a_x t^2$$
$$x = v_{0x}t$$

$$y = y_0 + v_{0y}t + 1/2\,a_y t^2$$
$$y = v_{0y}t - 1/2\,gt^2$$

In the absence of a given initial point, we are free to set $x_0 = y_0 = 0$. The acceleration in the x direction is zero, and the acceleration in the y direction is "-g."

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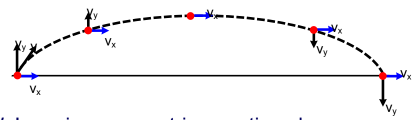
Motion of a Projectile

$$x = v_{0x}t$$
$$t = \frac{x}{v_{0x}}$$
$$y = v_{0y}t - 1/2\,gt^2$$
$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{gx^2}{2v_{0x}^2}$$
$$y = Ax + Bx^2$$



We're using parametric equations here, where t is the parameter, and we're free to manipulate the x and y equations simultaneously, since they both are true for any given t. For more info - see your math teacher!

The constants are combined, represented by A and B, and y is now expressed in terms of x. Plug A = 2.8 and B = 0.18 and see what your graphing calculator or other electronic device plots.

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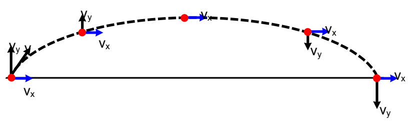
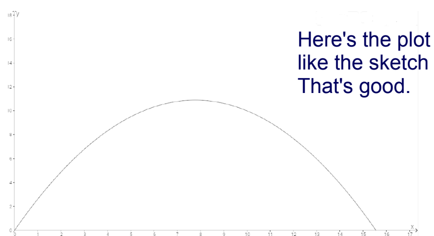
Motion of a Projectile





Here's the plot - looks like the sketch above. That's good.

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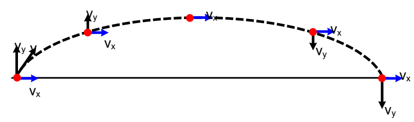
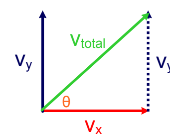
Projectile Displacement



We now have a group of equations and vectors to work with to predict the future motion of a projectile, given initial conditions.

$$v_{total} = \sqrt{v_x^2 + v_y^2}$$



$$v_y = v\sin\theta$$
$$v_x = v\cos\theta$$
$$\theta = \tan^{-1}\frac{v_y}{v_x}$$

$$x = x_0 + v_{0x}t \qquad a_x = 0$$
$$y = y_0 + v_{0y}t - 1/2\,gt^2 \qquad a_y = -g$$

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7 Which of the following statements are true regarding projectile motion?

- A $\sqrt{v_x^2 + v_y^2}$ is constant.
- B Total acceleration is +g when the object is rising and -g when it is falling.
- C In the absence of friction, the trajectory depends upon the mass of the object.
- D The velocity of the object is zero at the apex of the motion.
- E The horizontal motion is independent of the vertical motion.

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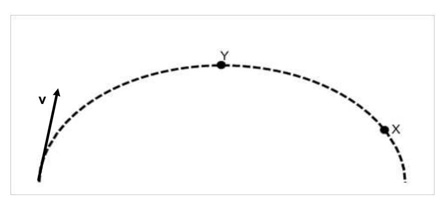
8 A marble is shot by a marble launcher and follows a parabolic path as shown. Air resistance is negligible. Which of the following shows the direction of the velocity at point Y, the highest point on the path?

- A None.
- B 
- C
- D
- E



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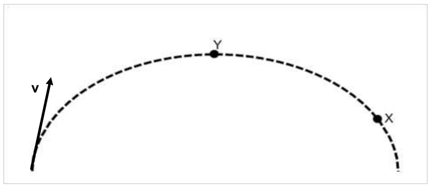
9 A marble is shot by a marble launcher and follows a parabolic path as shown. Air resistance is negligible. Which of the following shows the direction of the net force at point X?



- A None.
- B ↗
- C →
- D ↓
- E ←

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Solving Projectile Motion Problems

Several different problems will now be solved.

The critical point in solving these is realizing that in the absence of friction, the only force acting on the projectile in flight is the gravitational force.

Without this force, the projectile would move in a straight line and never hit the ground - and leave the planet. With gravity, the projectile gets pulled to the ground (unless it reaches a sufficient velocity to orbit the planet - but this will be covered in the Universal Gravitation unit of this course).

Based on this, in most cases, do you begin with the kinematics equations in the y or in the x direction?

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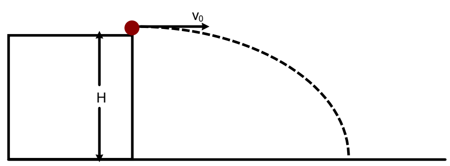
Time to fall

You solve for the time to fall - using the kinematics equations for the y axis. Nothing very interesting is going on in the x direction - the velocity stays constant (no acceleration).

But, the x displacement depends on how long the ball stays in the air.

Start with this problem - a ball is pushed off a height, H, above the ground, with an initial velocity v_0 in the horizontal direction.



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10 Two cannon balls, with different masses and initial velocities, are launched horizontally off a cliff at the same time. Which will strike the ground first?

- A The cannon ball with the greatest mass.
- B The cannon ball with the smallest mass.
- C The cannon ball with the greatest initial velocity.
- D The cannon ball with the smallest initial velocity.
- E They both strike the ground at the same time.

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11 Two cannon balls, with different masses and initial velocities, are launched horizontally off a cliff at the same time. Which will go further in the x direction?

- A The cannon ball with the greatest mass.
- B The cannon ball with the smallest mass.
- C The cannon ball with the greatest initial velocity.
- D The cannon ball with the smallest initial velocity.
- E They both strike the ground at the same time.

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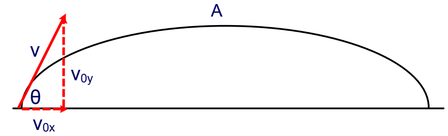
Finding Maximum Height (Apex) and Time to Apex

An object is propelled with an initial velocity, v_0, at an angle θ with the ground. Find the time it takes to reach point A (apex) and its maximum height.



There are two approaches to finding the maximum height. We'll take the more complicated one first (which involves finding the time to apex)!

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Finding Time to Apex

The initial velocity in the y direction is $v_0 \sin\theta$ and at the apex, the velocity in the y direction is zero.

$$v_y = v_{oy} - gt$$

$$v_y = v_o \sin\theta - gt = 0$$

$$v_o \sin\theta = gt$$

$$t = \frac{v_o \sin\theta}{g}$$

This is the time to apex.

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Finding Maximum Height (Apex)

Did you notice how we used the first two Kinematics equations to find the maximum height?

If the problem did not ask for the time to apex, but just for the height, the third Kinematics equation could have been used, as it was derived from combining the first two Kinematics equations:

$$v_y^2 = v_y^2 + 2a\Delta y$$

$$h = \frac{v_y^2 - v_{0y}^2}{2a} = \frac{0 - (v_0 \sin\theta)^2}{-2g}$$

Δy = h and v_y = 0 for maximum height

$$h = \frac{v_0^2 \sin^2\theta}{2g}$$

Matches the answer from using the first two Kinematics equations.

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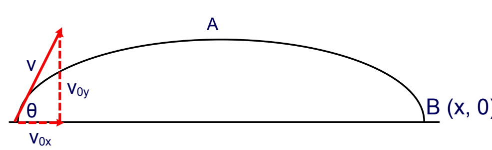
Finding Maximum Displacement

The last problem to be worked is to find the maximum displacement in the x direction - point B (x, 0).

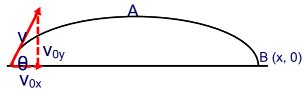


The first step is to find out how long the projectile is in the air. Since the projectile takes as long to reach point A from the ground as it does to go from point A to B, we can just double the apex time. But that's using a symmetry argument, and we can be more formal.

How?

Finding Maximum Displacement

Use the second kinematics equation, realizing that $y = y_0 = 0$.



$$y = y_o + v_{oy}t - \frac{1}{2}gt^2$$

$$v_{oy}t - \frac{1}{2}gt^2 = 0$$

$$t\left(v_o \sin\theta - \frac{1}{2}gt\right) = 0$$

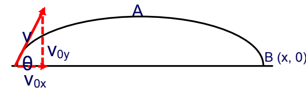
$$t = 0, \frac{2v_o \sin(\theta)}{g}$$

There are two solutions for t. Many times, the math gives two solutions and only one is physically relevant. In this case, both solutions are valid - they show the initial and final times at which the projectile is on the ground.

One more step......

Finding Maximum Displacement

The maximum displacement is frequently labeled R (range).



Time in the air

$$x = x_0 + v_{0x}t$$

$$R = v_o \cos(\theta)\left(\frac{2v_o \sin\theta}{g}\right)$$

$$R = \frac{v_o^{\,2}\, 2\sin\theta\cos\theta}{g} = \frac{v_o^{\,2}\sin 2\theta}{g}$$

Maximum displacement (range)

Double Angle formula (trigonometry)

12 At what angle will a projectile have the greatest vertical displacement?

- A 0^0
- B 30^0
- C 45^0
- D 60^0
- E 90^0

13 At what angle will a projectile have the greatest horizontal displacement?

- A 0^0
- B 30^0
- C 45^0
- D 60^0
- E 90^0

14 A marble launcher fires two marbles with the same initial velocity but at different angles to the horizontal. Which pair of angles will result in the same maximum displacement in the x direction?

- A 0^0 and 30^0
- B 30^0 and 60^0
- C 0^0 and 45^0
- D 20^0 and 80^0
- E None of the above.

Uniform Circular Motion

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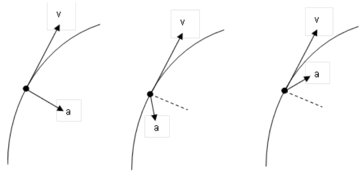
Uniform Circular Motion



constant speeddecreasing speedincreasing speed

Uniform Circular Motion occurs when an object moves in a circle with constant speed, and its acceleration is perpendicular to its velocity. The velocity and displacement vectors are tangent to the circle.

In the above picture, UCM is represented on the left. *Why is it necessary for the acceleration to be perpendicular to the velocity for UCM?*

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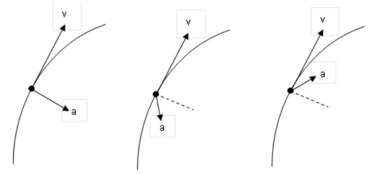
Uniform Circular Motion



constant speeddecreasing speedincreasing speed

This perpendicular acceleration is called centripetal (center-seeking) acceleration. The acceleration is caused by the centripetal force - and when force is perpendicular to an object's displacement, then no Work is done on the particle.

With zero work, the Work-Energy equation states that $KE_f = KE_0$, so the speed remains constant.

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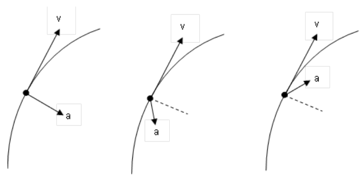
Uniform Circular Motion



constant speeddecreasing speedincreasing speed

Use the Work-Energy equation to explain what is going on in the middle and right diagrams.

These two diagrams will be explained in more detail in the Dynamics unit of this course as they involve free body diagrams.

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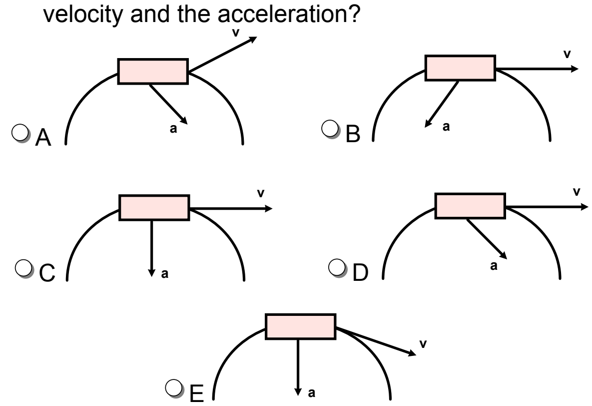
15 A car is driving with decreasing speed on a curved path. Which diagram shows the correct direction for the velocity and the acceleration?



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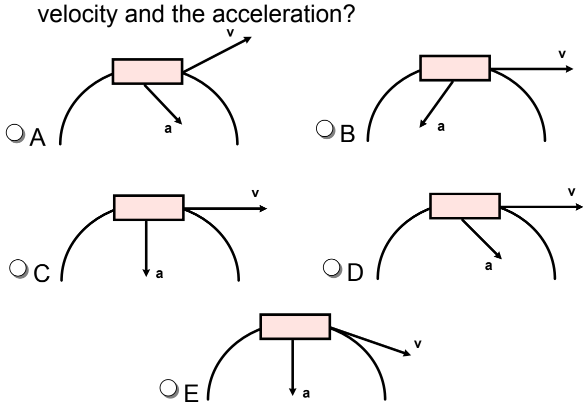
16 A car is driving with constant speed on a curved path. Which diagram shows the correct direction for the velocity and the acceleration?



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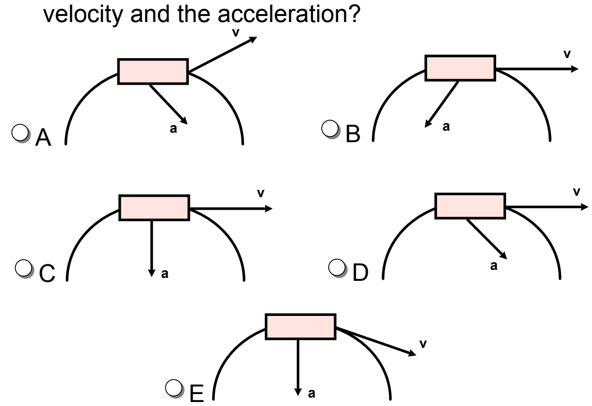
17 A car is driving with increasing speed on a curved path. Which diagram shows the correct direction for the velocity and the acceleration?

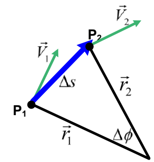


18 Is the velocity for an object in Uniform Circular Motion a constant? Explain your answer.

Students type their answers here

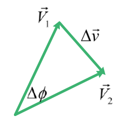
Centripetal Acceleration Derivation

You've already learned that $a_c = \frac{v^2}{r}$ for objects in uniform circular motion (the speed is constant); now it will be derived.



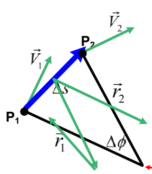
Compare these two triangles.

This is a position sketch of UCM. As an object moves from point P_1 to P_2, it undergoes a linear displacement of **Δs** and sweeps out an angle Φ.



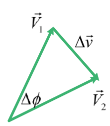
This is a sketch of the velocities at the two points. The magnitudes are equal, but their differences in direction give rise to **Δv**.

Centripetal Acceleration Derivation



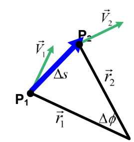
Since $\mathbf{v_1}$ is perpendicular to $\mathbf{r_1}$ and $\mathbf{v_2}$ is perpendicular to $\mathbf{r_2}$, the position and the velocity triangles both subtend the same angle, Δθ.

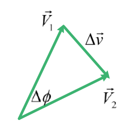
The velocity triangle has been rotated and superimposed on the position triangle so you can how the angles are the same.



Both of these are isosceles triangles, and have the same vertex angle. A geometric proof shows that these are similar triangles.

Centripetal Acceleration Derivation





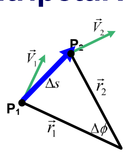
Knowing that the triangles are similar, the ratios of their corresponding sides are equal (geometry):

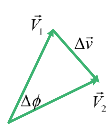
$$\frac{|\Delta\vec{v}|}{v_1} = \frac{\Delta s}{r_1}$$

And since $v_1 = v_2 = v$, and $r_1 = r_2 = r$, this simplifies as:

$$\frac{|\Delta\vec{v}|}{v} = \frac{\Delta s}{r} \quad \text{or} \quad |\Delta\vec{v}| = \frac{v\Delta s}{r}$$

Centripetal Acceleration Derivation





To find the instantaneous acceleration, we first have to come up with a representation for the average acceleration as before:

$$a_{av} = \frac{|\Delta\vec{v}|}{\Delta t} = \frac{v\Delta s}{r}\frac{1}{\Delta t} = \frac{v}{r}\frac{\Delta s}{\Delta t}$$

Centripetal Acceleration Derivation

$$a_{av} = \frac{v}{r}\frac{\Delta s}{\Delta t}$$

$$a_c = \lim_{\Delta t \to 0} a_{av} = \lim_{\Delta t \to 0} \frac{v}{r}\frac{\Delta s}{\Delta t} = \frac{v}{r}\lim_{\Delta t \to 0}\frac{\Delta s}{\Delta t}$$

$$a_c = \frac{v}{r}v = \frac{v^2}{r}$$

The instantaneous acceleration is the value of the average acceleration as Δt⇒ 0.

The instantaneous acceleration at every point on the circle is the same - and is called centripetal acceleration, a_c.

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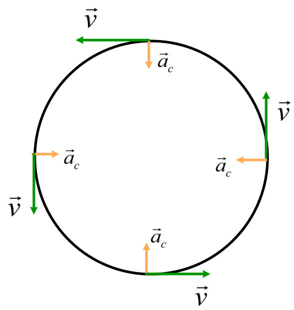
Centripetal Acceleration



One more handy expression:

$$a_c = \frac{v^2}{r} \quad \leftarrow \quad v = \frac{2\pi r}{T}$$

$$a_c = \frac{4\pi^2 r}{T^2}$$

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19 What is the centripetal acceleration of a ball that is swung in a circle of radius, 1.0 m, with a velocity of 5.0 m/s?

- A 0.040 m/s^2
- B 0.20 m/s^2
- C 5.0 m/s^2
- D 10 m/s^2
- E 25 m/s^2

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20 A ball is swung in a circle of 9.0 m and its centripetal acceleration is 1 m/s^2. What is its velocity?

- A √3.0 m/s
- B 3.0 m/s
- C 9 m/s
- D 18 m/s
- E 81 m/s

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21 What is the radius of the orbit of an object moving in a circle with a speed of 15 m/s and a centripetal acceleration of 45 m/s^2?

- A 0.33 m
- B 3.0 m
- C 5.0 m
- D 10 m
- E 15 m

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22 A ball spins in a horizontal orbit with a period of 5.0 s. The orbit's radius is 0.98 m. What is the centripetal acceleration of the ball?

- A 1.3 m/s^2
- B 2.5 m/s^2
- C 3.8 m/s^2
- D 7.7 m/s^2
- E 15 m/s^2

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Relative Motion

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Reference Frames

In order to describe where something is, you need to relate it to something else.

That's the purpose of a reference frame - you choose a coordinate system in space and make measurements relative to the system.

There is no one preferred reference frame - and frequently, there is a need to translate measurements made in one frame to another.

One reference frame can even be moving compared to the other (or is the other way around?).

As long as the frames move with a constant velocity relative to each other, we can use a Galilean transformation. Accelerating reference frames are the province of Special Relativity and won't be discussed now.

Reference Frames

Before the math is done, think about different reference frames.

If you're sitting in a bus that is going 25 m/s (56 mph), the person sitting next to you, relative to the reference frame that has you at the origin, is not changing his position, and his speed relative to you is 0 m/s.

If someone is standing at a bus stop, relative to a reference system that has her at the origin, the person in the bus is moving at 25 m/s, and is getting further away from her.

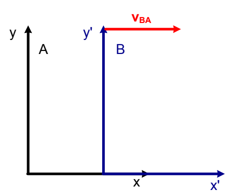
And if a bus is moving parallel to your bus with the same velocity, a person in that bus would agree with you. Your seat mate looks stationary.

Who is correct? Why, you all are!

Galilean Transformation

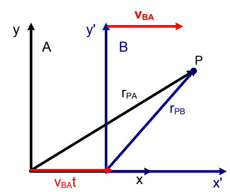
Choose a reference frame based on Cartesian coordinates - and for clarity, we'll just work in two dimensions - the same principles apply in three dimensions. This will be the A frame.

Then, choose the B system and put it in motion with a constant velocity, $\mathbf{v_{BA}}$ (velocity of the B system with respect to the A system), in the x direction.



In the bus example, reference frame A would be the woman at the bus stop, and you and your seat mate would be using the B system.

Galilean Transformation



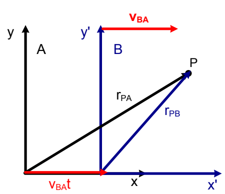
The seat mate is at point P. You are at the origin of the B coordinate system. r_{PB} is the position of the seat mate relative to you.

The woman at the bus stop is at the origin of the A system. r_{PA} is the position of the seat mate relative to her.

Start the problem with the two reference systems coincident with each other at t = 0. After time t, the distance traveled by the bus is $x = x_0 + v_{BA}t$ (assuming a = 0).

Using vector analysis, derive the equation for the relationship of the positions measured by both observers.

Galilean Transformation



The vector analysis gives us the **First Galilean transformation**:

$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{v}_{BA}t$$

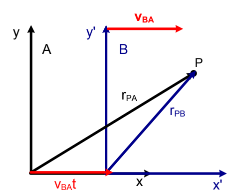
Take the time derivative of both sides:

$$\frac{d\vec{r}_{PA}}{dt} = \frac{d\vec{r}_{PB}}{dt} + \vec{v}_{BA}$$ ← $\mathbf{V_{BA}}$ is constant

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

This is the **Second Galilean transformation**.

Galilean Transformation



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{v}_{BA}t$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

There is no transformation in the direction perpendicular to the motion. The y component of point P stays constant (as does the z); only the x component changes.

A handy way to keep track of the subscripts is to note that the "internal" subscript (B) on the right side of the equations are the same. The "external" subscripts on the right side match the subscripts on the left side of the equation (PA).

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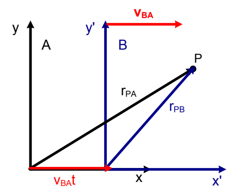
Galilean Transformation - acceleration



$$\vec{r}_{PA} = \vec{r}_{PB} + \vec{v}_{BA}t$$

$$\vec{v}_{PA} = \vec{v}_{PB} + \vec{v}_{BA}$$

Take the time derivative of the second transformation:

$$\frac{dv_{PA}}{dt} = \frac{d\vec{v}_{PB}}{dt} + \frac{d\vec{v}_{BA}}{dt} = \frac{d\vec{v}_{PB}}{dt} + 0$$

$$\vec{a}_{PA} = \vec{a}_{PB}$$

V_{BA} is constant

The acceleration of an object measured in two reference frames moving with a constant velocity relative to each other is the same.

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23 An airplane is flying south (assume south is in the negative x direction) at a speed of 500 km/h at a constant altitude (positive y direction). You are at rest on the ground. What is the velocity of a plane passenger in the x and y directions relative to your position?

- A v_x = 500 km/h v_y = -500 km/h
- B v_x = -500 km/h v_y = 500 km/h
- C v_x = 500 km/h v_y = 0 km/h
- D v_x = -500 km/h v_y = -500 km/h
- E v_x = -500 km/h v_y = 0 km/h

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24 You are in a bus, moving at a constant speed of 30 m/s. The person next to you gets up and starts walking up the aisle with an acceleration of 0.42 m/s^2 from your vantage point. A person on the side of the road simultaneously measures the the person's acceleration. What measurement does she obtain for his acceleration?

- A 0.21 m/s^2
- B 0.42 m/s^2
- C 0.63 m/s^2
- D 0.82 m/s^2
- E 9.8 m/s^2

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25 Which of the following is required in order to use a Galilean transformation between two reference frames?

- A Both frames must be moving in the same direction.
- B The frames must be moving in opposite directions.
- C The frames must be moving perpendicular to each other.
- D The frames must move with a constant velocity relative to each other.
- E The frames must move with a constant acceleration relative to each other.

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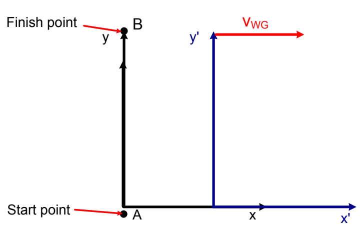
Boat problem



Two kids are on a boat capable of a maximum speed of 4.2 m/s, and wish to cross a river 1200 m to a point directly across from their starting point. If the speed of the water in the river is 2.8 m/s, how much time is required for the crossing (assume their engine is operating at maximum speed)?

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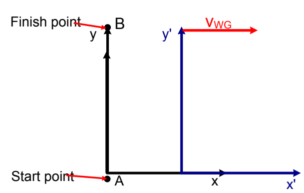
Boat problem



This is a classic Galilean transformation problem. The , xy axes, represent a coordinate system fixed to the earth. The boat will start at point A and go to point B.

The x'y' axes represent the flowing water reference frame - it is moving at a velocity v_{WG} = 2.8 m/s to the right (v_{WG} is the notation for the velocity of the water, relative to the fixed ground).

From your own experience, if you were driving the boat, would you point the bow at point B and just start the engine?

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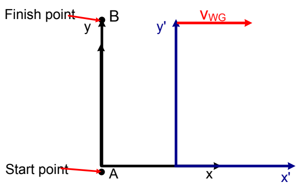
Boat problem



The water current will push the boat to the right. So, if the boat is headed directly for point B, it will wind up somewhere to the right of it on the opposite shore.

You want to aim the boat to the left of point B. The boat's maximum speed is 4.2 m/s. Try drawing a vector diagram that shows the boat's velocity, and what velocity the boat has because of the current pushing it to the right. Add the v_{WG} to the two vectors and make a triangle.

Hint: you are free to move v_{WG} from where it is shown above.

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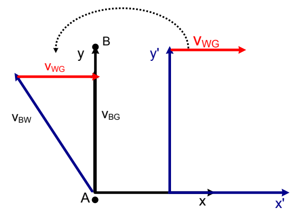
Boat problem



The current (v_{WG}) pushes the boat to the right - so although it is pointing to the left, the boat is actually going straight across the river - v_{BG}, the velocity of the boat in the ground fixed xy system.

The maximum speed of the boat is v_{BW} - the velocity of the boat over the water reference frame (x'y'). It represents the direction that the boat's bow is pointing. A fixed observer in the x'y' frame (a duck floating with the current) would see the boat under power move opposite the current flow (to the left) and away from the duck.

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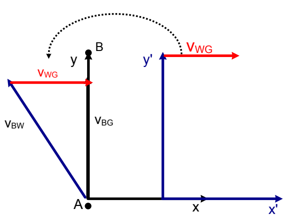
Boat problem



We're now ready to solve the problem by using vector analysis.

Note how the inner subscripts "cancel" and only the outer subscripts remain on the right.

$$\vec{v}_{BG} = \vec{v}_{BW} + \vec{v}_{WG}$$

The three velocity vectors form a right triangle, so we can use Pythagoras to find the magnitude of v_{BG}:

$$|\vec{v}_{BG}| = \sqrt{|\vec{v}_{BW}|^2 - |\vec{v}_{WG}|^2}$$

$$|\vec{v}_{BG}| = \sqrt{(4.2\,m/s)^2 - (2.8\,m/s)^2}$$

$$|\vec{v}_{BG}| = 3.13\,m/s$$

How long does it take the boat to reach the other side?

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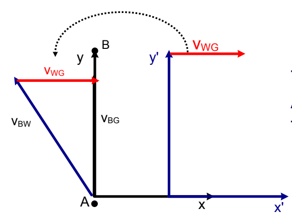
Boat problem



The distance to be traveled from point A to B is 1200 m, so the time it takes to cross, given v_{BG} = 3.13 m/s is:

$$t = \frac{\Delta y}{v_{BG}} = \frac{1200m}{3.13\,m/s} = 383s$$

What angle, θ, with the vertical, should the boat be pointing at to get to point B?

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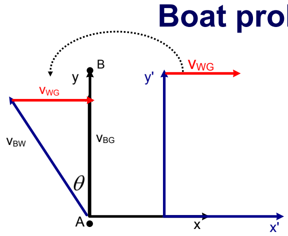
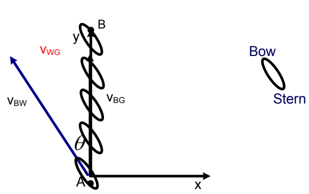
Boat problem



$$\theta = \tan^{-1}\frac{v_{WG}}{v_{BG}}$$

$$\theta = \tan^{-1}\frac{2.8}{3.13}$$

$$\theta = 42^0$$



The oval to the left represents the boat. A flying duck looking down on the boat would see it move from point A to B, but it would be pointing to the left as it worked against the current - as if it was sliding across the river.

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26 Two sailors are on a motor whaleboat traveling at a speed of 10.0 km/h, and wish to cross a river 2.0 km wide to a point directly across from their starting point. If the speed of the river is 9.0 km/h parallel to the shore, how much time is required for the crossing?

- A 0.45 h
- B 0.90 h
- C 1.0 h
- D 10.0 h
- E The boat will never reach the other shore.

27 Two sailors are on a motor whaleboat that travels at a speed of 10.0 km/h and wish to cross a river 2.0 km wide in the quickest time. What direction should they point the bow? Draw and label the velocity vector diagram. The speed of the river is 9.0 km/h parallel to the shore. How far down river do they travel before they reach the other shore?

28 An airplane is flying with a constant speed of 1200 km/h through the air, while experiencing a cross wind with a speed of 500 km/h relative to the ground. What is the airplane's speed relative to ground?

- A 700 km/h
- B 1300 km/h
- C 1600 km/h
- D 1700 km/h
- E 2500 km/h