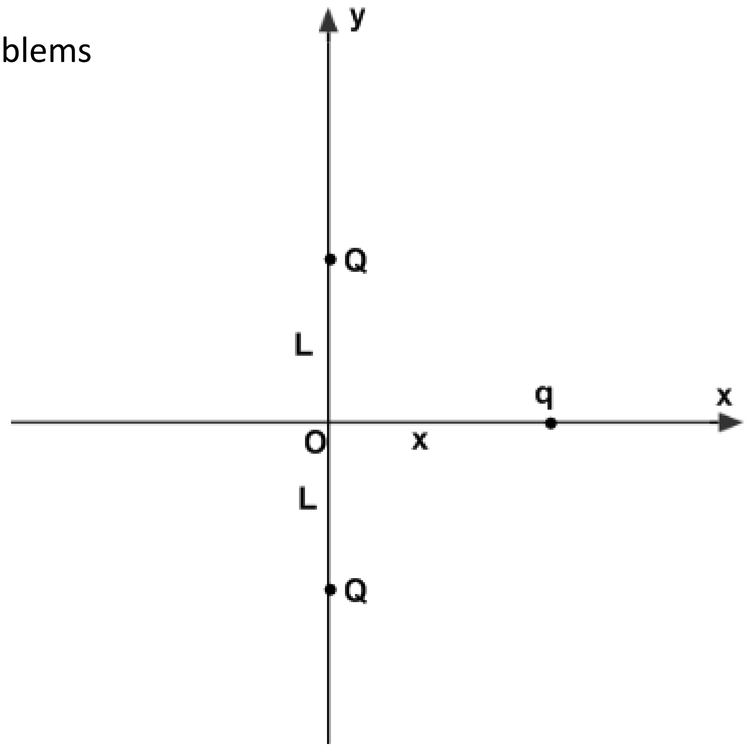
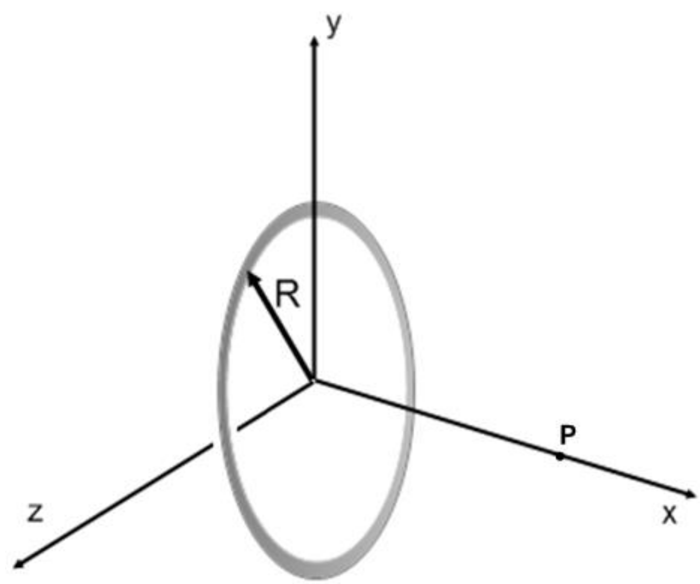
AP Physics C

Electric Potential and Capacitance

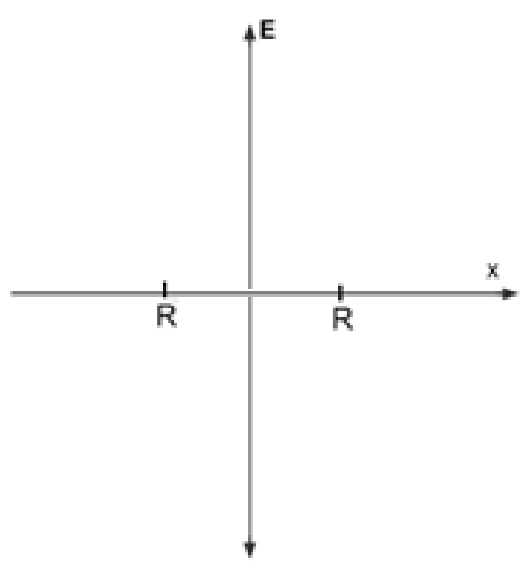
Free Response Problems

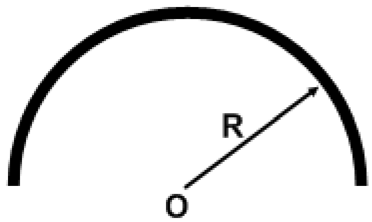


1. Two stationary point charges +Q are located on the y-axis at a distance L from the origin, as shown above. A third charge +q is brought in from infinity along the x-axis.
    a. Express the electric field E (magnitude and direction) due to two charges +q at a distance x = L from the origin on the x-axis.
    b. Determine the magnitude and direction of the force acting on the movable charge +q when it is located at the position x = L
    c. Determine the electric potential V at a distance x = L from the origin on the x-axis.
    d. Determine the electric potential energy of a movable charge +q when it is located at a distance x = L from the origin on the x-axis.
    e. Determine the work done by the electric field as the charge +q moves from infinity to the origin.

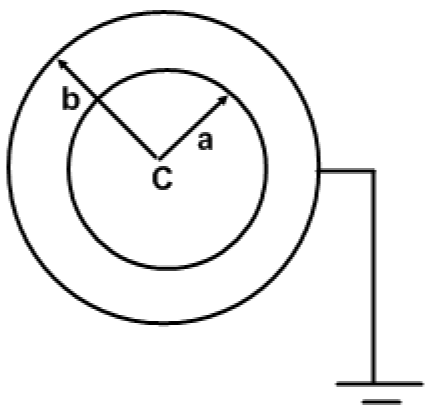


2. A nonconducting ring of radius R lies in yz – plane. The ring is charged with a positive charge Q that uniformly distributed on the on the ring.
    a. Find the electric potential at point P, which is a distance x from the center of the ring, located on the x-axis.
    b. Where along the x-axis is the electric potential the greatest?
    c. Find the x-component of the electric field at point P on the x-axis.
    d. What are the y- and z- components of the electric field at point P on the x-axis?
    e. Find the distance x for which the electric field is a maximum.
    f. Find the maximum value of the electric field.
    g. On the axes below, graph the x-component of the electric field as a function of distance x.

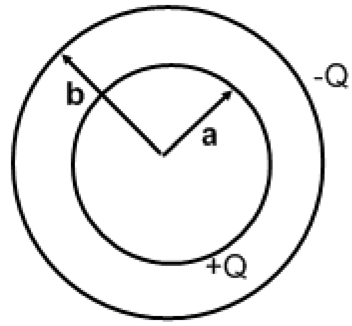




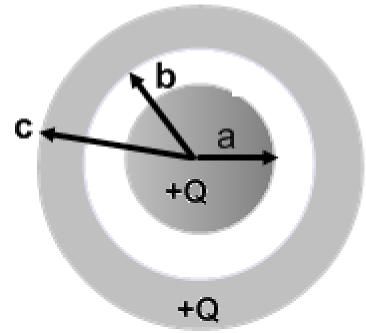
3. A thin plastic semi ring of radius R has a uniform linear positive charge density λ.
    a. Determine the electric potential V at point O, the center of the semi ring.
    b. What is the direction of the electric field at point O?
    c. Find the magnitude of the electric field E at point O.
    d. How much work must be done to bring a positive charge q from infinity to point O?



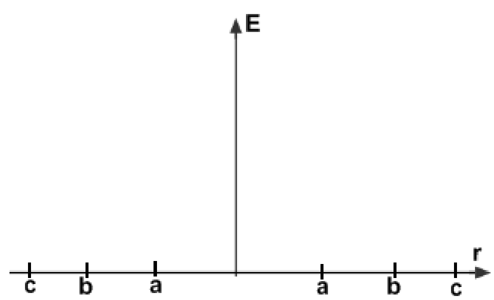
4. Two conducting spheres with a common center C have radii a and b ($a < b$). The outer shell is grounded. The inner shell is charged positively to a potential V.
    a. What is the net charge Q stored on the inner shell?
    b. Using Gauss's Law, determine the magnitude of the electric field in the region between the shells. Express your answer in terms of the net charge Q on the inner shell.
    c. Determine the potential difference between the spherical shells.
    d. Develop an expression for the capacitance of the system in terms of a, b, Q, and fundamental constants.



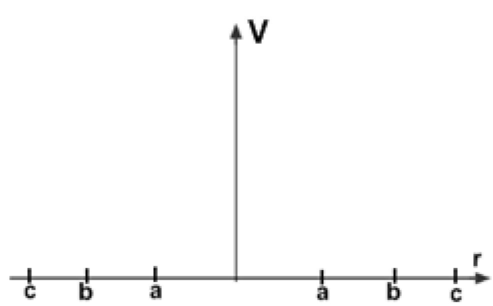
5. Two concentric spherical shells of radii a and b have equal but opposite charges. Just on the surface of the inner shell, the electric field radiates outward and has magnitude E_0.
    a. Find the magnitude of charge +Q on the inner shell as a function of E_0 and a.
    b. Find the magnitude of the electric field E between the shells as a function of E_0, a, and r.
    c. What is the potential difference V between the shells as a function of E_0, a, and b.
    d. Find the capacitance of the combination of two concentric spherical shells.
    e. Find the energy U stored in the capacitor in terms of E_0, a, and b.
    f. Determine the value of a that can be used to maximize the energy U stored in the capacitor. Assume that E_0 and b are fixed.

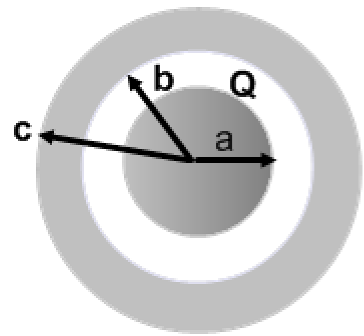


6. A solid conducting sphere of radius a is surrounded by a hollow conducting spherical shell of inner radius b and outer radius c. The sphere and the shell each have a charge +Q.
    a. Derive an expression for the magnitude of the electric field in the region when:
        i. r < a
        ii. a < r < b
        iii. b < r < c
        iv. r > c
    b. On the axes below, graph the electric field as a function of distance r from the center of the sphere.



    c. Derive an expression for the electric potential as a function of distance r from the center of the sphere for the following points: c, b, and a. Assume the potential is zero at r = ∞.
    d. On the axes below, graph the electric potential as a function of distance r from the center of the sphere.

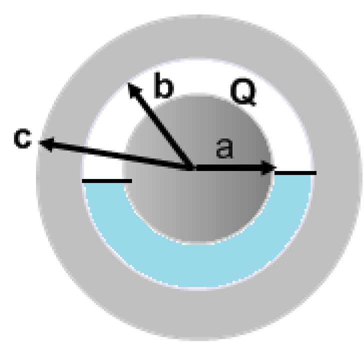




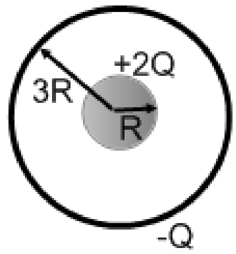
7. A conducting sphere of radius a and charge Q is surrounded by a concentric conducting shell of inner radius b and outer radius c. The outer shell is first grounded; then the grounding wire is removed.
   a. Using Gauss's law, determine the electric field in the region
      i. $r < a$,
      ii. $a < r < b$
      iii. $b < r < c$
      iv. $r > c$

      Where r is the distance from the center of the inner sphere.
   b. Develop an expression for the electric potential difference between the surface of the sphere and the inner surface of the shell.
   c. Develop an expression for the capacitance C of the system.

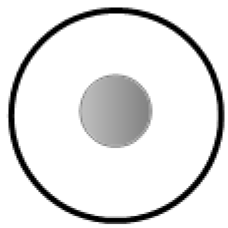
      A liquid dielectric with a dielectric constant of 7 is filled into the space between the sphere and the shell. The dielectric occupies a half of the space between the surfaces.



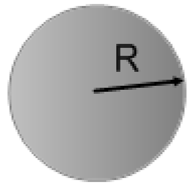
   d. Determine the new capacitance of the system in terms of C.



8. A solid metallic sphere of radius R has charge +2Q. A hollow spherical shell of radius 3R placed concentric with the sphere has net charge –Q.
   a. On the diagram below show the electric field lines in all regions inside and outside of the spheres.

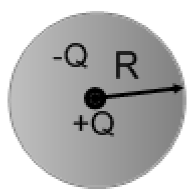


   b. Use Gauss's Law to determine the magnitude of the electric field in the region $R < r < 3R$.
   c. Find the potential difference between the sphere and shell.
   d. What would be the final distribution of the charge if the spheres were connected with a conducting wire?

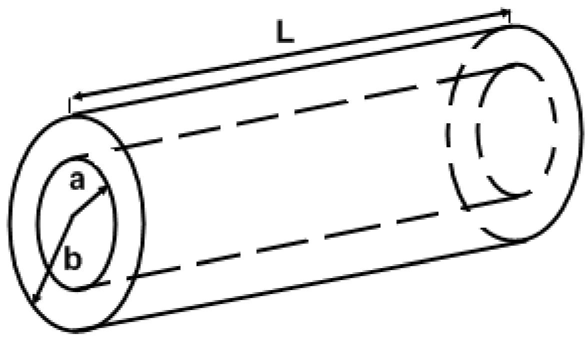


9. A total charge Q is distributed uniformly throughout a spherical volume of radius R. Let r denote the distance of a point inside the sphere to the center of the sphere.
   a. Use Gauss's law to derive an expression for the magnitude of the electric field at a point:
      i. outside the sphere, $r > R$.
      ii. inside the sphere, $r < R$.

      Assume the electrostatic potential to be zero at an infinite distance from the sphere.
   b. What is the potential at the surface of the sphere?
   c. Determine the potential at the center of the sphere.



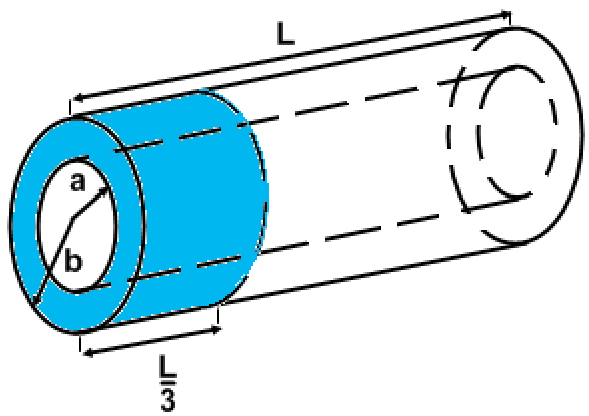
10. A negative charge $-Q$ is uniformly distributed throughout the spherical volume of radius R as shown above. A positive point charge $+Q$ is placed at the center of the sphere.
    a. Determine the electric field E outside the sphere at a distance $r > R$ from the center
    b. Determine the electric potential V outside the sphere at a distance $r > R$ from the center
    c. Determine the electric field inside the sphere at a distance $r < R$ from the center
    d. Determine the electric potential inside the sphere at a distance $r < R$ from the center



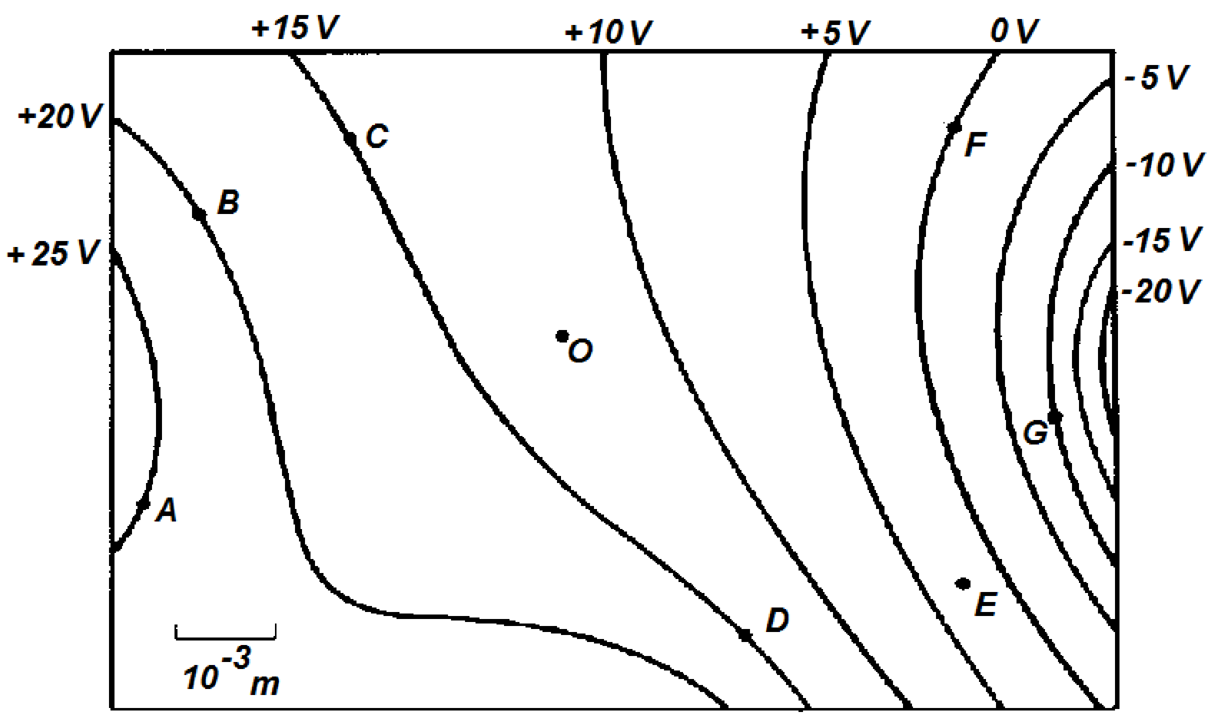
11. A capacitor consists of two conducting coaxial cylinders of radii a and b, and length L. When the capacitor is charged, the inner cylinder has a charge + Q and the outer cylinder has a charge -Q. Neglect end effects and assume that the region between the cylinders is filled with air.

    a. Determine an expression for the electric field at a distance r from the axis of the cylinder where $a < r < b$.
    b. Determine the potential difference between the cylinders.
    c. Determine the capacitance C of the capacitor.

    One third of the length of the capacitor is then filled with a dielectric of dielectric constant k = 6, as shown in the following diagram.



    d. Determine the new capacitance of the system of two cylinders in terms of C.



12. A non-uniform electric field is presented by a series of equipotential lines. The electric potential decreases from the left to the right.
    a. On the diagram above, draw arrows at the selected points to indicate the direction of the electric field vector.
    b. Which points represents the region where the electric field is the strongest. Explain.
    c. Estimate the magnitude of the electric field at point O.
    d. Find the potential difference between the following points: A and C, B and F, C and G, C and D.
    e. Determine the work done by the field to move an electric charge of magnitude 1 μC from point B to F.
    f. How much work would be done by the electric field to move a point charge along the following path: C→F→E→D→C?

Free Response Answers:

1. a. $E = \frac{kQ}{\sqrt{2}L^2}$ $in\ the\ positive\ x - direction$

   b. $F = \frac{kQq}{\sqrt{2}L^2}$ $in\ the\ positive\ x - direction$

   c. $V = \frac{2kQ}{\sqrt{2}L}$

   d. $U = \frac{2kQq}{\sqrt{2}L}$

   e. $W = \frac{2kQq}{\sqrt{2}L}$

2. a. $\frac{1}{4\pi\varepsilon_o\sqrt{R^2+x^2}}$

   b. $At\ x = 0$

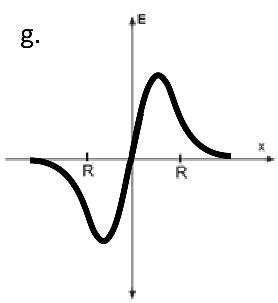
   c. $\frac{xQ}{4\pi\varepsilon_o(R^2+x^2)^{\frac{3}{2}}}$

   d. $0\ and\ 0$

   e. $\frac{R}{\sqrt{2}}$

   f. $\frac{Q}{6\sqrt{3}\pi\varepsilon_o R^2}$

   g.



3. a. $\frac{\lambda}{4\varepsilon_o}$

   b. $Down$

   c. $\frac{\lambda}{2\pi\varepsilon_o R}$

   d. $\frac{\lambda q}{4\pi}$

4. a. $4\pi\varepsilon_o Va$

   b. $\frac{Q_E}{4\pi\varepsilon_o r^2}$

   c. $\frac{Q}{4\pi\varepsilon_o}\left(\frac{1}{a}-\frac{1}{b}\right)$

   d. $4\pi\varepsilon_o\left(\frac{1}{a}-\frac{1}{b}\right)$

5. a. $4E_o\pi\varepsilon_o a^2$

   b. $\frac{E_o a^2}{r^2}$

   c. $E_o a^2\left(\frac{1}{a}-\frac{1}{b}\right)$

   d. $\frac{4\pi\varepsilon_o}{\frac{1}{a}-\frac{1}{b}}$

   e. $2E_o^2 a^3\pi\varepsilon_o\left(\frac{b-a}{b}\right)$

   f. $\frac{3}{4}$

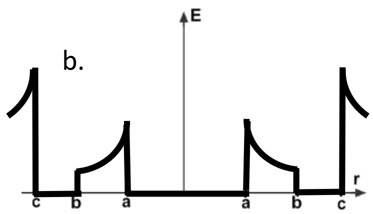
6. a. i. 0

ii. $\frac{Q}{4\pi r^2 \varepsilon_o}$

iii. 0

iv. $\frac{Q}{2\pi r^2 \varepsilon_o}$

b.

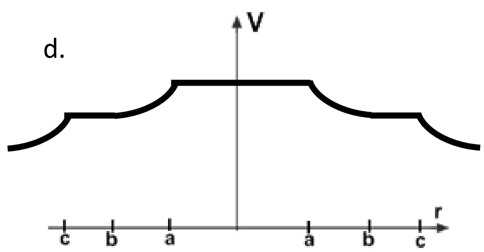


c. c: $\frac{Q}{2\pi\varepsilon_o c}$

b: $\frac{Q}{2\pi\varepsilon_o c}$

a: $\frac{Q}{2\pi\varepsilon_o}\left(\frac{1}{2a}+\frac{1}{2b}+\frac{1}{c}\right)$

d.



7. a. i. 0

ii. $\frac{Q}{4\pi r^2 \varepsilon_o}$

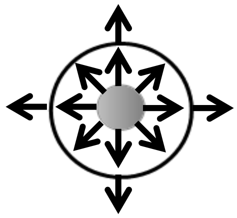
iii. 0

iv. $\frac{Q}{4\pi r^2 \varepsilon_o}$

b. $\frac{Q}{4\pi\varepsilon_o}(\frac{1}{a}-\frac{1}{b})$

c. $\frac{4\pi\varepsilon_o}{\frac{1}{a}-\frac{1}{b}}$

d. $4C_o$

8. a. 

b. $\frac{Q}{2\pi r^2 \varepsilon_o}$

c. $\frac{Q}{3\pi \varepsilon_o R}$

d. $+0.5Q\ on\ both\ spheres$

9. a. i. $\frac{Q}{4\pi r^2 \varepsilon_o}$

ii. $\frac{Qr}{4\pi R^3 \varepsilon_o}$

b. $\frac{Q}{4\pi \varepsilon_o R}$

c. $\frac{3Q}{8\pi \varepsilon_o R}$

10. a. 0

b. 0

c. $\frac{Q-\frac{Qr^3}{R^3}}{4\pi \varepsilon_o r^2}$

d. $\frac{Q}{4\pi \varepsilon_o}\left(\frac{1}{r}+\frac{r^2-2-R}{2R}\right)$

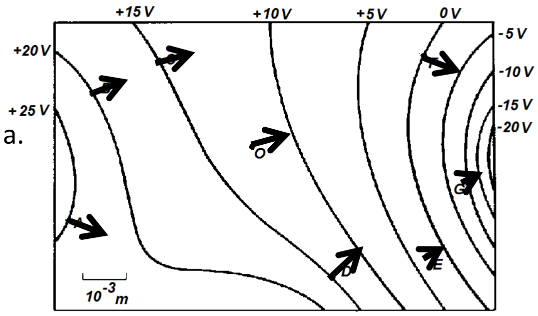
11. a. $\frac{Q}{2\pi r L \varepsilon_o}$

b. $\frac{Q}{2\pi L \varepsilon_o}\ln(\frac{b}{a})$

c. $\frac{2\pi \varepsilon_o L}{\ln\left(\frac{b}{a}\right)}$

d. $\frac{8C}{3}$

12. a.



b. $G. G\ is\ where\ two\ equipotential\ lines\ are\ the\ closest.$

c. $2500\frac{N}{C}$

d. $\boldsymbol{A} \rightarrow \boldsymbol{C}: 10V\ \ \boldsymbol{B} \rightarrow \boldsymbol{F}: 20V\ \ \boldsymbol{C} \rightarrow \boldsymbol{G}: 20V\ \ \boldsymbol{C} \rightarrow \boldsymbol{D}: 0V$

e. $0J$