How to Use this File

- Each topic is composed of brief direct instruction
- There are formative assessment questions after every topic denoted by black text and a number in the upper left.
  > Students work in groups to solve these problems but use student responders to enter their own answers.
  > Designed for SMART Response PE student response systems.
  > Use only as many questions as necessary for a sufficient number of students to learn a topic.
- Full information on how to teach with NJCTL courses can be found at njctl.org/courses/teaching methods
Newton's Law of Universal Gravitation

The law of gravitation states that every mass in the universe exerts an attractive force on every other mass directly proportional to the product of the two masses and inversely proportional to the square of the distance between their centers of mass. Sir Isaac Newton published this law in 1687 in *Principia*.

\[
\vec{F} = G \frac{m_1 m_2}{r^2} \hat{r}
\]

The force is along a line connecting the two masses.
Newton's Law of Universal Gravitation

Since the gravitational force is attractive, we define $\vec{F}_{12}$ as the force that $m_1$ exerts on $m_2$, $r$ is the magnitude of the distance between the two masses and $\hat{r}_{12}$ is a unit vector directed from $m_1$ to $m_2$.

$$\vec{F}_{12} = -G \frac{m_1 m_2}{r^2} \hat{r}_{12}$$

The negative sign shows that each mass exerts an attractive force on the other. The second equation is for the force that $m_2$ exerts on $m_1$.

The universal gravitational constant, $G$, was calculated in a 1798 experiment by Henry Cavendish, using a torsion balance.

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

Newton's Law of Universal Gravitation

The origin of the Law of Universal Gravitation, like most great scientific advances, is properly shared by multiple people, and is also shrouded in the confusion of history. Brahe's and Kepler's work led to it. Robert Hooke corresponded with Newton about it. Edmond Halley questioned Newton about it.

Sir Isaac Newton put it in its most complete form and published it in his *Principia*, one of the most famous mathematics and physics books of all time.

Newton connected the idea that objects, like apples, fall towards the center of Earth, with the idea that the moon orbits around the Earth while also falling towards the center of the Earth.

The moon remains in a circular orbit because it has a velocity perpendicular to its acceleration.
Newton's Law of Universal Gravitation

The apple part of this story was first related to Newton's friend William Stukeley, and separately to John Conduitt, the husband of his niece - but fifty years after he wrote the law! So, it's unclear what role the apple played. Probably, Brahe, Kepler, Hooke and Halley had a greater influence.

1. Two objects of mass \( m \) are originally separated by a distance \( r \) and the force acting on each one is \( F \). What is the new force if the distance between them is cut in half?

   - A \( F/4 \)
   - B \( F/2 \)
   - C \( F \)
   - D \( 2F \)
   - E \( 4F \)

2. Two objects, one of mass \( m \), and the other of mass \( 2m \) are separated by a distance \( r \). Which is true about the force between the two objects?

   - A Mass \( m \) experiences a greater force.
   - B Mass \( 2m \) experiences a greater force.
   - C Both experience the same attractive force.
   - D Both experience the same repulsive force.
   - E The force increases as they get farther apart.
Here's where history and physics are out of synch.

Newton created his theory of gravitation in 1687, along with the definition of the gravitational force. It clearly works, in that the motions of the celestial objects are described by it as well as earthly phenomena.

But, there is a problem - it implies "action at a distance," where objects interact with each other without contact. This was hard to accept.

This was solved in 1832 by Michael Faraday when he came up with the theory of the electric field.

Field theory relies on an object creating a "field" which exists everywhere in space. Other objects interact with the field and not the object that generated it.

Faraday was working with electricity and magnetism, and then field theory was applied to Gravity - most notably by Albert Einstein in General Relativity.

Start with the gravitational force equation, and let $M$ be the object that is generating the gravitational field, and $m$ is a small "test particle." Divide the force by this test particle, and the result is the gravitational field, $\mathbf{g}$.

$$\mathbf{g} = \frac{F}{m} = \frac{GM}{r^2} \hat{r}$$
Gravitational Field

By now, you should be familiar with g; you’ve been taught it is the gravitational acceleration due to a mass. And when that mass is the planet Earth, \( g = 9.8 \text{ m/s}^2 \).

\( g \) is now being defined more generally as the gravitational field - it is a vector field and is illustrated below. A similar diagram will be seen again in the electricity and magnetism chapters of this course.

The lines are a mathematical abstraction - the arrows point in the direction of the field (towards the mass generating it), and the closer the lines are to each other imply a stronger field.

3 An object of mass 10.0 kg is sitting on the surface of the earth. What is its weight?

- A 10 N
- B 49 N
- C 98 N
- D 116 N
- E 980 N
4 An object of mass 10 kg is sitting on the surface of Mars. What is the value of g for Mars? What is the weight of the object?

\[ M_{\text{Mars}} = 6.4 \times 10^{23} \text{ kg} \quad R_{\text{Mars}} = 3.4 \times 10^6 \text{ m} \quad G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \]

A \( g = 3.4 \text{ m/s}^2 \quad W = 34 \text{ N} \)
B \( g = 3.4 \text{ m/s}^2 \quad W = 340 \text{ N} \)
C \( g = 6.8 \text{ m/s}^2 \quad W = 68 \text{ N} \)
D \( g = 6.8 \text{ m/s}^2 \quad W = 680 \text{ N} \)
E \( g = 9.8 \text{ m/s}^2 \quad W = 98 \text{ N} \)

---

**Gravitational Field of a Sphere**

The gravitational field outside a solid sphere is measured from the center of mass of the object - it is the same as if all of the object’s mass was concentrated at its center (assuming a uniform mass density within the sphere, or if it changes only with the distance from the center).

Newton wanted to prove why this was true - and this was one of the drivers for his invention of calculus.

![First, lets start with the gravitational field due a uniform thin spherical shell (it's hollow).](image)

**Gravitational Field of a Shell**

Using integral calculus, the gravitational field due to a uniform thin spherical shell is found to be (the derivation can be found in more advanced college physics texts):

\[ \ddot{g} = -\frac{GM}{r^2} \quad \text{for } r > R \]

\[ \ddot{g} = 0 \quad \text{for } r < R \]

Let's now show a geometric explanation of why \( g = 0 \) within the shell.

![Image of a shell with a gravitational field](image)
Consider a point mass, \( m \), somewhere within a uniform thin spherical shell.

Extend two cones, of lengths \( r_a \) and \( r_b \), from the point mass that end up on the surface of the shell, subtending surface areas of \( A_a \) and \( A_b \).

The masses of the surface segments are proportional to their surface areas, which are proportional to the squares of the radii:

\[
\frac{m_a}{m} = \frac{A_a}{A} = \frac{r_a^2}{r^2}.
\]

Rearranging the equation:

\[
\frac{m_a}{r_a^2} = \frac{m}{r^2}.
\]

The gravitational forces between \( m \) and the two shell segments decrease as the square of the radius vectors.

Therefore, the force from segment \( A_a \) on the point mass \( m \) is equal to the force on the point mass from \( A_b \), resulting in zero net force on mass \( m \).

\[ Since \ there \ is \ no \ force \ on \ the \ point \ mass, \ there \ is \ no \ gravitational \ field \ within \ the \ shell. \]

This result - gained both by calculus and illustrated by geometry is identical to a result obtained by Gauss using vector calculus. **Gauss’s Law for Gravity** makes the same statement that Newton’s Universal Law of Gravitation does! And as we’ll see later, this law is mathematically similar to describing the behavior of electric and magnetic fields.
Gravitational Field of a Solid Sphere

Time to calculate the gravitational field of a uniform solid sphere. This is more appropriate for planetary and cosmological applications - planets and stars for the most part are not hollow.

By building up an infinite number of infinitely thin concentric uniform spherical shells, each of which’s gravitational field is the same as if its mass was concentrated at its center, we arrive at a solid sphere where:

\[ g = \frac{GM}{r^2} \quad \text{for} \quad r > R \]

But what about the gravitational field within the sphere? It is NOT equal to zero as in the case of the uniform thin spherical shell.

Gravitational Field Inside a Solid Sphere

The gravitational field within a uniform thin spherical shell is zero. It feels no force from any mass outside the shell. If a sphere with radius \( r \) is inscribed within the sphere of radius \( R \), only the mass within the smaller sphere contributes to its gravitational field.

The gravitational field for \( r \leq R \) is then:

\[ g_r = \frac{Gm}{r^2} \quad \text{and} \quad g_r = \frac{GM}{R^2} \]

The mass within the sphere of radius \( r \) is defined as \( m \). The mass of the sphere of radius \( R \) is defined as \( M \). The gravitational field at \( r \) is due solely to the mass within the smaller sphere.
Gravitational Field of a Solid Sphere

We can now show the gravitational fields inside and outside a solid sphere. The field increases linearly as the radius increases, and the field outside decreases as the square of the distance from the center of mass.

\[
g_r = \frac{GM}{R^2} \quad \text{for } r \leq R
\]

\[
g_r = \frac{GM}{r^2} \quad \text{for } r > R
\]
Gravitational Potential Energy

Gravitational Potential Energy (GPE or $U_g$) was defined in an earlier unit as $U_g = mg\Delta h$ and it was restricted to objects close to the surface of the planet (normally earth), with the promise that this restriction would be lifted. We are now going to work on that.

Newton's Law of Universal Gravitation states that the gravitational force between two objects is between their centers of mass. The gravitational field was then described as:

$$ \ddot{r} = \frac{F}{m} = \frac{GM}{r^2} \hat{r} $$

By choosing an appropriate reference point, we can talk about an absolute gravitational potential energy, $U$. It is conventional to state that $U(r = \infty) = 0$. Assume $m_2$ started its journey at $r_0 = \infty$ and finished at $r_f = r$.

The gravitational potential energy is negative. This reflects the attractive nature of the force. How does this give a positive number for $U_g = mgh$, in the limiting case where $m_2$ is an object near the surface of a planet, $m_1$?
Gravitational Potential Energy

Here's what $U_g$ looks like as a function of $r$, the distance between the center of masses of two objects:

Next consider the case of a small object that is a distance $h$ above the surface of the earth.

Near the surface of the earth, $U(r)$ can be approximated by the red line - it is nearly linear, so $mg\Delta h$ is an accurate expression for the change in gravitational potential energy.

5 Why can't the expression $U = mg\Delta h$ be used to determine the potential energy of an earth-weather satellite system in geosynchronous orbit 35.8 km above the earth?

- A The satellites orbital period is twice that of the earth's rotational period.
- B The satellite is moving at a great speed.
- C The value of $g$ is different at the earth's surface and at the satellite's location.
- D The value of $g$ is the same at the earth's surface and at the satellite's location.
- E The value of $g$ is always positive.
6 Describe how the negative sign in the generalized potential energy equation was determined.

7 How much work is required by an external force on an object of mass \( m \) to bring it from the surface of the earth (mass \( M \) and radius \( r \)) to infinity? Assume the object starts at rest and arrives at infinity with zero kinetic energy.

- A \(-\frac{GMm}{r} \)
- B \(-\frac{GMm}{r^2} \)
- C Zero
- D \(\frac{GMm}{r} \)
- E \(\frac{GMm}{r^2} \)

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**A trip through the center of the earth**

Suppose you could drill a tunnel through the earth that starts in Madrid, Spain, passes through the center of the earth, and comes out the other side in Weber, New Zealand?

And then have a vehicle that could convey you safely through the tunnel. Ignoring the impossibility of such a task and the obvious risks, what kind of motion would you experience?

Start with the magnitude of the force acting on this vehicle of mass \( m \), using the value of \( g \), previously derived for the inside of the earth:

\[ F = mg, \quad -\frac{GMm}{R^2} \]
A trip through the center of the earth

The vehicle is moved over the hole, and it starts falling through the tunnel with an acceleration of 9.8 m/s². It continues to increase its speed, but the magnitude of the acceleration continually decreases until it equals zero at the center of the earth. At that point, the velocity is at a maximum.

\[ F = mg_r = -\frac{GMmr}{R^2} = 0 \quad \text{at the center of the earth} \]

where \( r = 0 \)

Once through the earth’s center, the magnitude of the acceleration starts increasing, and since it is in the opposite direction of the motion, the vehicle slows down.

Until it reaches New Zealand, where it pops above the surface.... And then reverses its journey. What does this motion remind you of?

A trip through the center of the earth

Simple Harmonic Motion. Take the force equation, and replace all the constants with one constant; k.

\[ F = mg_r = \frac{GMmr}{R^2} = -kr \]

This is the equation of simple harmonic motion where there is a restorative force that is proportional to the displacement of the object.

A trip through the center of the earth

Calculate the speed of the vehicle as it passes the center of the earth.

Since the force is constantly changing, and there are no external non-conservative forces (we're ignoring friction and other forces that would really appear if this problem was not so unrealistic), conservation of total mechanical energy is the right approach.

The kinetic energy when the trip starts is zero. The potential energy is known. But what is the potential energy at the center of the earth? You can’t say the potential energy at the center of the earth is zero. We've already said the potential energy is zero at infinity.

The change in potential energy from the surface to the center needs to be calculated.
8 An object of mass 110 kg is dropped above the surface of a tunnel that goes through and comes out the other side of the earth. What is the velocity of the object at the center of the earth? What is its velocity when it reaches the other side of the earth?

\( M_{\text{earth}} = 5.97 \times 10^{24} \text{ kg}, r_e = 6.38 \times 10^6 \text{ m}, G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2 \)

9 Assuming the gravitational potential energy at infinity is 0, what is the gravitational potential energy at the center of the earth?

- A \ GMm/R
- B \ GMm/2R
- C \ 0
- D \ -GMm/2R
- E \ -GMm/R
Circular Orbits

Here is Newton's own drawing of a thought experiment where a cannon on a very high mountain (above the atmosphere) shoots a shell with increasing speed, shown by trajectories for the shell of D, E, F, and G. The shell finally moves so fast that it never falls to earth, but goes into an elliptical orbit.

(From M. Fowler, http://galileoandeinstein.physics.virginia.edu/lectures/newton.html)

Circular Orbits

This drawing is deceptive - and there is debate over what Newton meant by points A and B.

Once an object has sufficient tangential velocity to reach point G, it goes into orbit - it will not land past point G.

If the object exceeds the escape velocity, then it could enter a parabolic or hyperbolic orbit.

(From M. Fowler, http://galileoandeinstein.physics.virginia.edu)
Circular Orbits

Assume the simplest type of an elliptical orbit - a circle. This means that the object is always the same distance from the surface, has a centripetal acceleration (due to the gravitational force), and a tangential velocity.

This enables us to find the velocity that an object needs to remain in a stable orbit at a distance \( r \) from the center of the earth:

\[
F = \frac{GMm}{r^2} = m\frac{v^2}{r}
\]

\[
v_{\text{orb}} = \sqrt{\frac{GM}{r}}
\]

This is a very interesting result - the orbital velocity for a stable circular orbit at the earth's surface \( (r = R_E) \) is equal to the maximum velocity achieved by our imaginary vehicle as it passed through the earth's center in the "A trip through the center of the earth" from the last chapter!

\[
v_{\text{orb}} = \sqrt{\frac{GM}{R_E}}
\]

Another example of the relationship of simple harmonic motion to rotational motion.

10 What velocity must a satellite maintain in order to stay in a circular orbit one earth radius above the surface of the earth?

\( M_{\text{earth}} = 5.97\times10^{24} \text{ kg}, \ r_e = 6.38\times10^6 \text{ m}, \ G = 6.67\times10^{-11} \text{ Nm}^2/\text{kg}^2 \)

- A 2540 m/s
- B 3700 m/s
- C 5600 m/s
- D 6800 m/s
- E 7900 m/s
Circular Orbits

By establishing the relation between the force due to gravity and uniform circular motion we can also solve for the Period, $T$, the time it takes to complete one orbit. The orbital velocity is the distance traveled in one orbit ($2\pi r$) divided by the time it takes for one orbit - $T$.

$$v = \frac{2\pi r}{T}$$

Substitute this value for $v$ into the following:

$$\frac{GMm}{r^2} = ma = m\frac{v^2}{r} = m\frac{4\pi^2r^3}{rT^2}$$

Solve for $T$:

$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

Orbital Period for an orbit radius of $r$

This works for any object orbiting another object. In the example of the earth orbiting the sun, $T$ represents the period of the orbit, $r$ represents the distance from the earth to the sun, and $M$ is the mass of the sun. The same equation works for the moon orbiting the earth, and other moons orbiting their planets.

Very few orbits are perfectly circular, so instead of using the radius of a circular orbit, the semi major axis, $a$, of the elliptical orbit is used for $r$.

$$T = 2\pi \sqrt{\frac{a^3}{GM}}$$

11 What is the orbital period of the earth orbiting the sun?

$$(M_{\text{sun}} = 1.99\times10^{30} \text{ kg}, r_{\text{earth orbit}} = 1.50\times10^{11} \text{ m}, G = 6.67\times10^{-11} \text{ Nm}^2/\text{kg}^2)$$

- A 1.56x10^7 s
- B 3.17x10^7 s
- C 5.28x10^7 s
- D 1.06x10^8 s
- E 9.08x10^4 s
Total Energy and Escape Velocity

Total Energy

An object of relatively small mass, m, is orbiting around a much larger mass, M (the earth around the sun, the moon around the earth), at a distance r between their centers of mass, and a velocity v, and the larger mass is at rest in an inertial reference frame.

The total energy of the two body system is:

\[ E_T = KE + U_T \]
\[ E_T = \frac{1}{2}m v^2 - \frac{GMm}{r} \]

In the previous chapter, we found: \[ v_{esc} = \sqrt{\frac{GM}{r}} \]

Substituting \( v_{esc} = \sqrt{\frac{GM}{r}} \) into \( E_T = \frac{1}{2}mv^2 - \frac{GMm}{r} \):

\[ E_T = \frac{1}{2}m \left( \frac{GM}{r} \right)^2 - \frac{GMm}{r} = \frac{GMm}{2r} - \frac{GMm}{r} \]

**The total energy of an object in a bound circular orbit is negative.** If it were an elliptical orbit, r would be replaced by the semi-major axis, a.

The total energy of this object is conserved.
12 A satellite is in a stable orbit of $3R_e$ above the surface of the earth. What is the total amount of energy of the satellite?

- $A -\frac{GMm}{3R_e}$
- $B -\frac{GMm}{4R_e}$
- $C -\frac{GMm}{6R_e}$
- $D -\frac{GMm}{8R_e}$
- $E -\frac{GMm}{16R_e}$

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**Escape Velocity**

Apply the conservation of total energy to escape velocity.

Escape velocity is the minimum velocity an object (such as a spaceship) needs to attain to "escape" the earth's gravitational pull. Actually, since the force of gravity has an infinite range, this is never really possible, but the force will be extremely tiny at great distances.

We're going to use conservation of energy, and our system is going to be the earth-spaceship system. If we're looking for the minimum velocity, what will the magnitude of the potential energy and kinetic energy of the spaceship when it is an infinite distance from the earth?

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**Escape Velocity**

Since we're looking for the minimum required velocity to escape the earth, we want the object to get to infinity with zero kinetic energy. If it has a positive kinetic energy, then its velocity leaving the earth was too great.

Potential energy was already defined to be zero at infinity. Using the Conservation of Energy:

\[
\frac{1}{2}mv_{\text{escape}}^2 - \frac{GMm}{R_e} = 0
\]

\[
v_{\text{escape}}^2 = \frac{2GM}{R_e}
\]

\[
v_{\text{escape}} = \sqrt{\frac{2GM}{R_e}}
\]
Escape Velocity

\[ v_{\text{escape}} = \sqrt{\frac{2GM}{R_e}} \]

Note that escape velocity is independent of the mass of the escaping object. It only depends on the mass and radius of the object being escaped from.

Using this fact, can you explain why despite the fact that Hydrogen and Helium are the most abundant elements in the universe, there are only trace amounts of each element in our atmosphere?

When the earth was first formed, the atmosphere contained significant amounts of both elements.

However - Hydrogen and Helium are the lightest elements, so for a given atmospheric temperature, they would move faster than the heavier elements.

Over time, their average speeds exceeded the escape velocity of earth, thus they left the planet.

13 What is the escape velocity on the planet Earth?
\( (M_{\text{earth}} = 5.97 \times 10^{24} \text{ kg}, R_e = 6.38 \times 10^6 \text{ m}, G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2) \)
Kepler's Laws

Tyco Brahe and Johannes Kepler

Kepler's Laws start with the magnificent observational work done in last thirty years of the 1500s by Tyco Brahe, using a sextant and compass. He recorded the positions of the planets and the stars visible with the eye, constantly improving the quality of his observation instruments - but never using a telescope (it hadn't been invented yet)!

Kepler assisted Brahe in the last years of Brahe's life, then took 16 years to fit a math model to the work.

Kepler's laws tie together multiple concepts to explain the orbits of celestial bodies.

Kepler's First Law

The planets of the solar system move in elliptical orbits with the sun at one of the foci.

This is a consequence of the inverse square nature of the gravitational force - it results in elliptical orbits for objects in a bound orbit such as planets, asteroids and comets.

The planetary orbits look very much like circles - the limiting case of an ellipse where the two axes (minor and major) are equal. Even Mercury's orbit, which has the most eccentricity (least like a circle), looks very much like a circle.
Kepler's First Law

A highly eccentric ellipse - and what you may have thought planetary orbits looked like from your first science classes:

What planetary orbits really look like - which makes Kepler's statement that orbits are really elliptical, a great tribute to Brahe's exacting and comprehensive measurements and Kepler's mathematics:

Kepler's Second Law

A radius vector drawn from the sun to the orbital path sweeps out equal areas in equal times, which shows that planets speed up as they get closer to the sun in their elliptical orbits. The figure below is greatly exaggerated to make the effect more noticeable.

This is a result of applying the law of the conservation of angular momentum and was shown in the Rotational Motion unit of this course. This conservation law will be used to prove the swept out area dependence on time.

Kepler's Second Law

The system is defined as the sun and one of its planets. Assume no external forces and that the sun is stationary (and defines the reference frame).

The gravitational force is a central force and is directed from the planet to the sun. Note that it is NOT perpendicular to the planet's velocity - the work done on the planet by the sun is not zero - its kinetic energy (and speed) will change.
Kepler's Second Law

But the force will always be parallel to a line connecting the earth and the sun, therefore the torque done by the force will be zero, and the conservation of angular momentum can be used.

\[ \sum \vec{F} = \frac{d\vec{L}}{dt} = 0 \] which implies that: \[ \vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} = \text{const} \]

Kepler's Second Law

As the planet moves from point 1 in the orbit to point 2, it travels a tangential distance of \( dr = vt \). Construct a parallelogram that shows this:

The area subtended by the two radii vectors is \( dA \) which is half the area of the parallelogram:

\[ dA = \frac{1}{2} |\vec{r} \times d\vec{r}| = \frac{1}{2} |\vec{r} \times \vec{v} dt| \]

Kepler's Second Law

We have two equations:

\[ \vec{L} = \vec{r} \times \vec{p} = m\vec{r} \times \vec{v} = \text{const} \]

\[ dA = \frac{1}{2} |\vec{r} \times d\vec{r}| = \frac{1}{2} |\vec{r} \times \vec{v} dt| \]

Rearranging the first equation:

\[ \vec{r} \times \vec{v} = \frac{\vec{L}}{m} \]

Substitute into second equation:

\[ dA - \frac{1}{2} |\vec{r} \times \vec{v} dt| = \frac{1}{2} \left| \frac{\vec{L}}{m} \right| dt - \frac{L dt}{2m} \]

\[ \frac{dA}{dt} = \frac{\vec{L}}{2m} \]
Kepler's Second Law

Since $L$ and $m$ are constants, the time rate of change of the area subtended by the two radii vectors is constant. Hence, Kepler's Second Law!

\[
\frac{d\vec{A}}{dt} = \frac{\vec{L}}{2m}
\]

Kepler's Third Law

The square of the period of the orbit is proportional to the cube of the orbital radius.

In the Orbital Motion chapter of this unit, Newton's Law of Universal Gravitation and the definition of centripetal acceleration was used to derive the following property of elliptical orbits ($a$ is the length of the semi major axis):

\[
T = 2\pi \sqrt{\frac{a^3}{GM}}
\]

Square both sides, and the Third Law is found (although the Third Law chronologically preceded Newton's Law - it's nice that it all works out):

\[
T^2 = \frac{4\pi^2}{GM} a^3 \quad \frac{T^2}{a^3} = \text{const}
\]

Kepler's Third Law

Since the ratio of the square of the period and the cube of the radius give us a constant, we can determine the value of another object's orbital period or distance from the planet's center based on an orbit we already know, if given one of the two quantities for the new object.

\[
\frac{T^2}{a^3} = \text{const}
\]

\[
\frac{T_1^2}{r_1^3} = \frac{T_2^2}{r_2^3}
\]
**Kepler's Laws**

**First Law:** The planets of the solar system move in elliptical orbits with the sun at one of the foci.

**Second Law:** A radius vector drawn from the sun to the orbital path sweeps out equal areas in equal times.

**Third Law:** The square of the period of the orbit is proportional to the cube of the orbital radius.

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14 What is the ratio of $T^2/a^3$ for the orbit of Mercury about the Sun?

$T_{\text{Mercury}} = 7.60 \times 10^6 \text{ s} \quad a_{\text{Mercury}} = 5.79 \times 10^{10} \text{ m}$

- A $2.97 \times 10^{-19} \text{ s}^2/\text{m}^3$
- B $2.97 \times 10^{-8} \text{ s}^2/\text{m}^3$
- C $1.78 \times 10^{-19} \text{ s}^2/\text{m}^3$
- D $1.78 \times 10^{-9} \text{ s}^2/\text{m}^3$
- E $3.36 \times 10^{-19} \text{ s}^2/\text{m}^3$

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15 What is the ratio of $T^2/a^3$ for the orbit of the Earth about the Sun?

$T_{\text{Earth}} = 3.156 \times 10^7 \text{ s} \quad a_{\text{Earth}} = 1.496 \times 10^{11} \text{ m}$

- A $2.97 \times 10^{-19} \text{ s}^2/\text{m}^3$
- B $2.97 \times 10^{-8} \text{ s}^2/\text{m}^3$
- C $1.78 \times 10^{-19} \text{ s}^2/\text{m}^3$
- D $1.78 \times 10^{-9} \text{ s}^2/\text{m}^3$
- E $3.36 \times 10^{-19} \text{ s}^2/\text{m}^3$
16 Use the following measurements to calculate the mass of the Sun.

\[ T_{\text{Earth}} = 3.156 \times 10^7 \text{ s} \quad a_{\text{Earth}} = 1.496 \times 10^{11} \text{ m} \]

- A 2.01 x 10^{29} kg
- B 6.33 x 10^{29} kg
- C 1.99 x 10^{30} kg
- D 6.25 x 10^{30} kg
- E 1.25 x 10^{31} kg