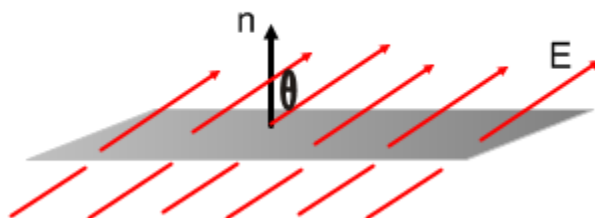


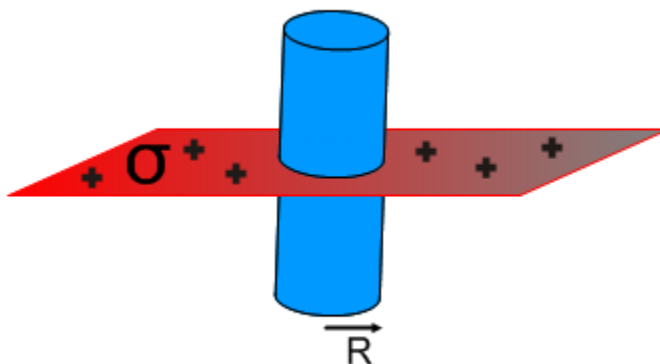
AP Physics C

Gauss's Law

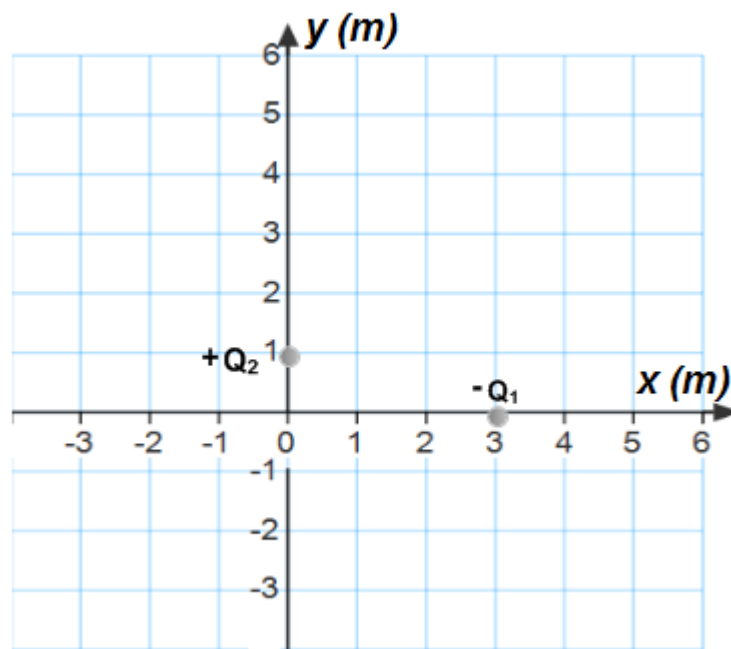
Free Response Problems



1. A flat sheet of glass of area 0.4 m^2 is placed in a uniform electric field $E = 500 \text{ N/C}$. The normal line to the sheet makes an angle $\theta = 60^\circ$ with the electric field.
 - a. Find the magnitude of the electric flux through the sheet.
 - b. Does the magnitude of the electric flux depend on the shape of the sheet?
 - c. For which angle θ between the normal and the field is the electric flux is the magnitude of the electric flux maximum?
 - d. For which angle θ between the normal and the field is the electric flux is the magnitude of the electric flux minimum?



2. A flat sheet has a uniform charge density $\sigma = 12 \text{ } \mu\text{C/m}^2$. A cylindrical surface crosses the sheet perpendicularly to the plane of the sheet. The cylindrical surface is divided by the sheet equally. The cylinder has a length of 0.8 m and radius 0.25 m .
 - a. Find the magnitude of the electric flux through the top surface of the cylinder?
 - b. Find the magnitude of the electric flux through the bottom surface of the cylinder?
 - c. Find the magnitude of the electric flux through the side surface of the cylinder.
 - d. What is the net flux through the cylindrical surface?
 - e. Does the length of the cylinder affect the magnitude of the flux?

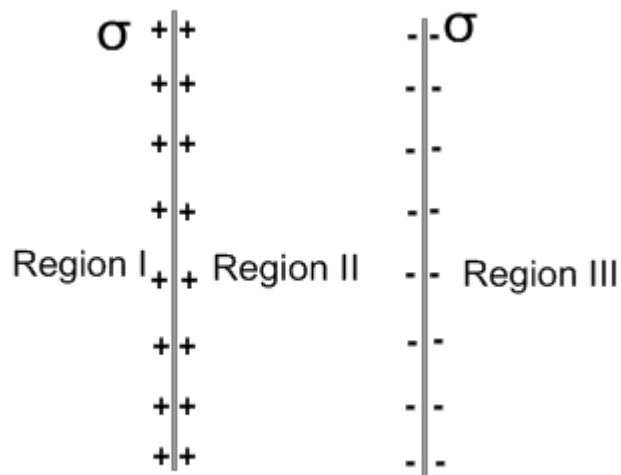


3. A point charge $Q_1 = -9 \text{ nC}$ is located on the x-axis at $x = 3 \text{ m}$, and a second point charge $Q_2 = 4 \text{ nC}$ is located on the y-axis at $y = 1 \text{ m}$.
 - a. What is the total electric flux due to two charges through a spherical surface of radius $R = 0.5 \text{ m}$ with its center at the origin?
 - b. What is the total electric flux due to two charges through a spherical surface of radius $R = 2 \text{ m}$ with its center at the origin?
 - c. What is the total electric flux due to two charges through a spherical surface of radius $R = 4 \text{ m}$ with its center at the origin?
 - d. Does the total flux change when we increase the radius of the spherical surface from 4
 - e. m to 6 m? Explain.

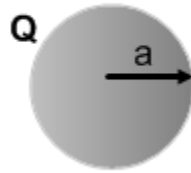


4. A very large conducting plate carries a positive charge with a constant surface charge density $\sigma = 6 \mu\text{C}/\text{m}^2$. Use Gauss's Law to answer following questions.
- Calculate the magnitude of the electric field in Region I.
 - Calculate the magnitude of the electric field in Region II.

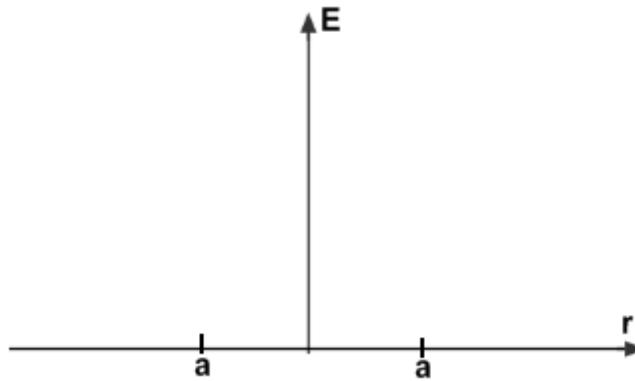
Another charged plate with the charge density $\sigma = -6 \mu\text{C}/\text{m}^2$ is placed in parallel to the first plate.

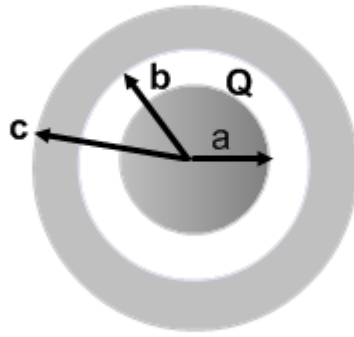


- Calculate the magnitude of the electric field in Regions I, II, and III.
- Calculate the electric force on an electron placed in Region II. Does this force depend on the electron's location?

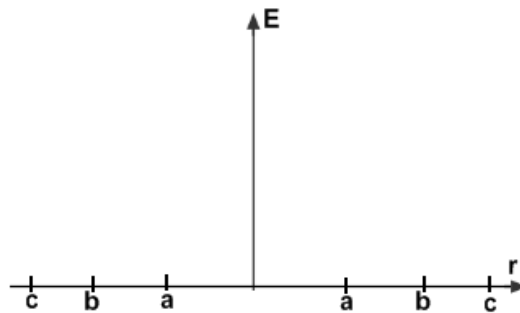


5. A solid conducting sphere carrying net charge Q has radius a .
- Derive an expression for the electric field at a distance $r < a$.
 - Derive an expression for the electric field at a distance $r > a$.
 - On the diagram below, sketch the electric field as a function of r .





6. A solid conducting sphere carrying a net charge Q has a radius a . The sphere is placed inside a conducting shell of inner radius b and outer radius c . The shell has no net charge.
- Derive an expression for the electric field in terms of distance r from the center of the sphere in the following regions:
 - $r < a$
 - $a < r < b$
 - $b < r < c$
 - $c < r$
 - On the axes below, graph the electric field as a function of r .

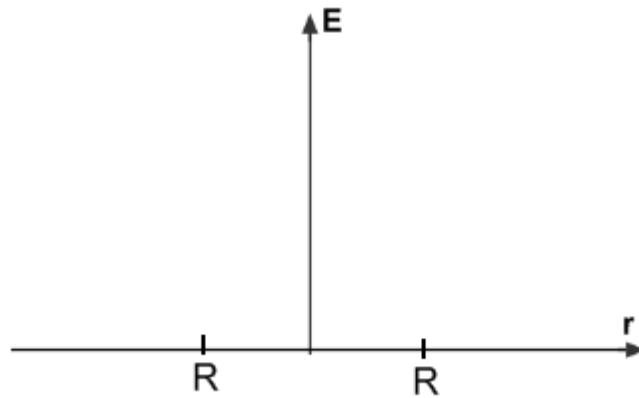


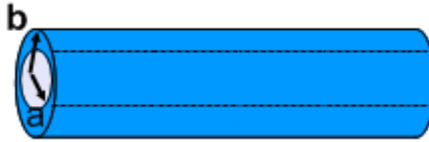
- What is the charge on the inner surface of the shell?
- What is the charge on the outer surface of the shell?
- On the diagram below, sketch the electric field lines within a spherical volume of radius $r > c$.





7. A very long conducting rod of radius R has a positive charge distributed uniformly throughout it. The charge per unit volume is ρ .
- Use Gauss's Law to derive an expression for the electric field inside the rod.
 - Use Gauss's Law to derive an expression for the electric field outside the rod in terms of the charge per unit length λ .
 - On the axes below, graph the electric field as a function of distance.





8. A very long conducting tube has an inner radius a and outer radius b . The tube carries charge per unit of length $-\lambda$. There is a line of charge that lies along the axis of the tube with a charge per unit of length λ .

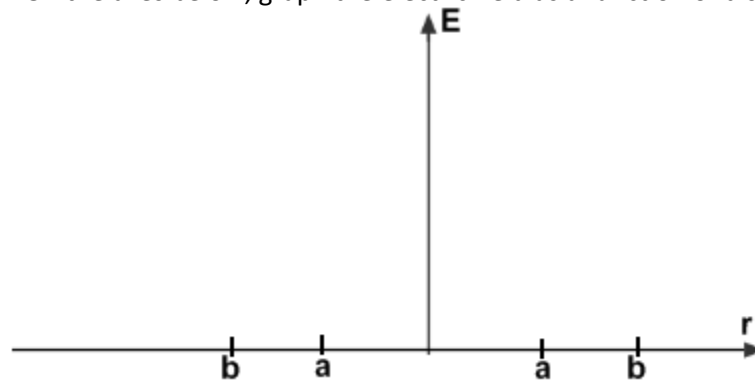
a. Derive an expression for the electric field as a function of distance r from the center of the tube for the following regions:

I. $r < a$

II. $a < r < b$

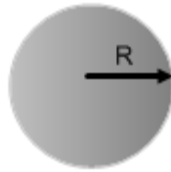
III. $r > b$

b. On the axes below, graph the electric field as a function of distance r .



c. What is the charge per unit of length on the inner surface of the tube?

d. What is the charge per unit of length on the outer surface of the tube?

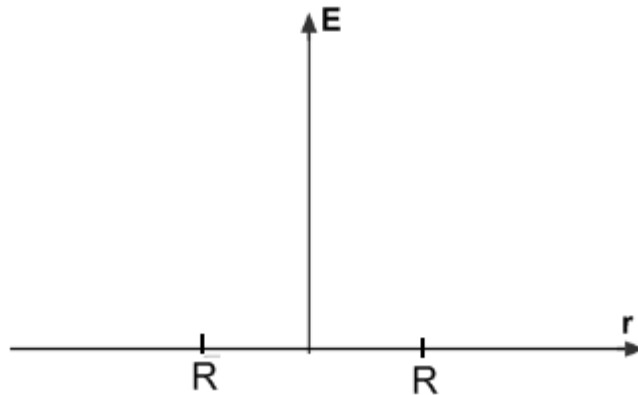


9. A non-conducting sphere with a radius R has a charge density distribution given by the formula:

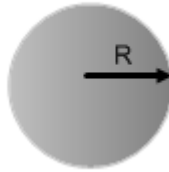
$$\rho = \rho_0 \left(1 - \frac{r}{R}\right), \text{ for } r \leq R$$

$$\rho = 0, \text{ for } r \geq R$$

- a. If $\rho_0 = \frac{3Q}{\pi R^3}$, show that the total charge of the sphere is Q .
- b. Derive an expression for the electric field as a function of distance for the region:
 - i. $r \geq R$
 - ii. $r \leq R$
- c. On the axes below, graph the electric field as a function of distance r .



- d. Find the distance r at which the electric field has a maximum value.
- e. Find the maximum value of the electric field.

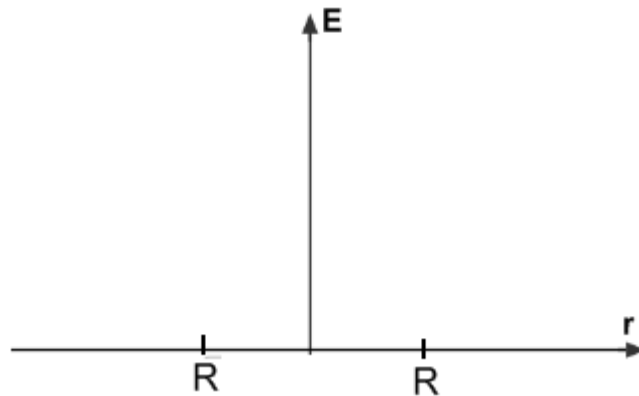


10. A non-conducting sphere with a radius R has a charge density distribution given by the formula:

$$\rho = \rho_0 \left(1 - \frac{4r}{3R}\right), \text{ for } r \leq R$$

$$\rho = 0, \text{ for } r \geq R, \text{ where } \rho_0 \text{ is a positive constant.}$$

- a. Determine the net charge distributed in the sphere.
- b. Derive an expression for the electric field as a function of distance for the region:
 - i. $r \geq R$
 - ii. $r \leq R$
- c. On the axes below, graph the electric field as a function of distance r .



- d. Find the distance r at which the electric field has a maximum value.
- e. Find the maximum value of the electric field.

Free Response Answers

1.
 - a. $173.21 \frac{Nm^2}{C}$
 - b. *No*
 - c. 0°
 - d. 90°

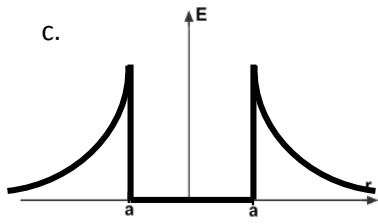
2.
 - a. $2.66 * 10^6 \frac{Nm^2}{C}$
 - b. $2.66 * 10^6 \frac{Nm^2}{C}$
 - c. 0
 - d. $5.32 * 10^6 \frac{Nm^2}{C}$

3.
 - a. $0 \frac{Nm^2}{C}$
 - b. $451.98 \frac{Nm^2}{C}$
 - c. $-564.97 \frac{Nm^2}{C}$
 - d. *No. $Q_{enclosed}$ does not change.*

4.
 - a. $3.68 * 10^5 \frac{N}{C}$
 - b. $3.68 * 10^5 \frac{N}{C}$
 - c. *I: $0 \frac{N}{C}$, II: $7.36 * 10^5 \frac{N}{C}$, III: $0 \frac{N}{C}$*
 - d. $1.17 * 10^{-13} N$. *No.*

5. a. 0

b. $E = \frac{Q}{4\pi r^2 \epsilon_0}$

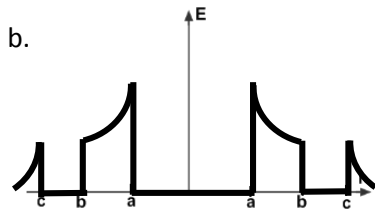


6. a. i. 0

ii. $\frac{Q}{4\pi r^2 \epsilon_0}$

iii. 0

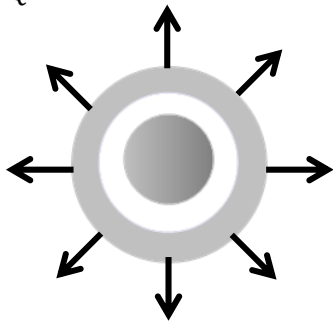
iv. $\frac{Q}{4\pi r^2 \epsilon_0}$



c. $-Q$

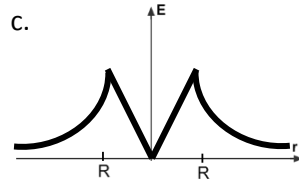
d. Q

e.



7. a. $E = \frac{\rho r}{2\epsilon_0}$

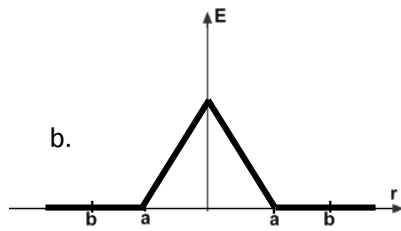
b. $E = \frac{\lambda}{2\pi r\epsilon_0}$



8. a. i. $E = \frac{\lambda}{2\pi r\epsilon_0}$

ii. $E = 0$

iii. $E = 0$



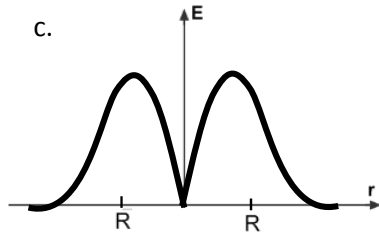
c. $-\lambda$

d. 0

9. a. After derivation, $Q = 4\pi\rho_o \left(\frac{R^3}{12}\right)$ when the distance is R , so $Q = 4\pi \left(\frac{3Q}{\pi R^3}\right) \left(\frac{R^3}{12}\right)$ when ρ_o is substituted. After simplification, the answer becomes Q .

b. i. $\frac{Q_E}{4\pi r^2 \epsilon_o}$

ii. $\frac{\rho_o}{\epsilon_o} \left(\frac{r}{3} - \frac{r^2}{4R}\right)$



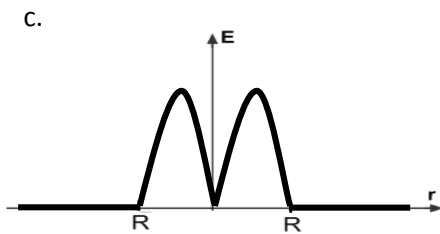
d. $\frac{2}{3}R$

e. $\frac{R\rho_o}{9\epsilon_o}$

10. a. 0

b. i. $E = \frac{Q}{4\pi r^2 \epsilon_o}$

ii. $E = \frac{\rho_o}{\epsilon_o} \left(\frac{r}{3} - \frac{r^2}{3R}\right)$



d. $\frac{R}{2} = r$

e. $\frac{R\rho_o}{12\epsilon_o}$