

The Special Theory of Relativity

$$E = mc^2$$

Inertial Reference Frames

Newton's laws are only valid in inertial reference frames:

An inertial reference frame is one which is not accelerating or rotating. It is an area in which every body remains in a state of rest unless acted on by an external unbalanced force.

Inertial Reference Frames

When your car accelerates, it is not an inertial reference frame.

This is why a drink on the dashboard of a car can suddenly seem to accelerate backwards without any force acting on it.

It's not accelerating, it's standing still. The reference frame, the car, is accelerating underneath it.



[Click here for a very famous video about frames of reference. watch the first 2:30 of the video](#)

Galilean-Newtonian Relativity

Relativity principle:

The basic laws of physics are the same in all inertial reference frames.

Think about that same drink in the reference frame of a stationary observer. To him, the car moves but the drink stays still.



1 You are riding in a spaceship that has no windows, radios, or other means for you to observe or measure what is outside. You wish to determine if the ship is stopped or moving at constant velocity. What should you do?

Move
For
Answer

- A You can determine if the ship is moving by determining the apparent velocity of light.**
- B You can determine if the ship is moving by checking your precision time piece. If it's running slow, the ship is moving.**
- C You can determine if the ship is moving either by determining the apparent velocity of light or by checking your precision time piece. If it's running slow, the ship is moving.**
- D You should give up because you have taken on an impossible task.**

Galilean-Newtonian Relativity

This principle works well for mechanical phenomena.

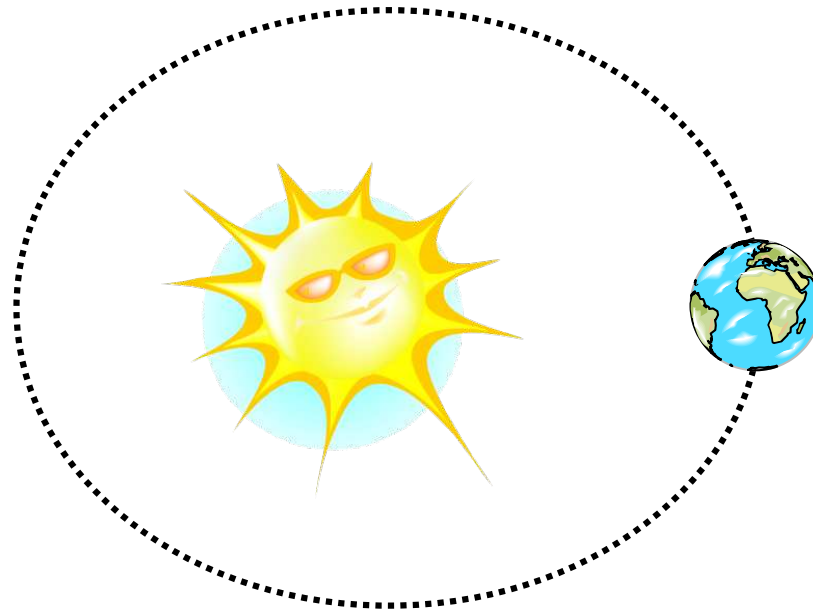
However, Maxwell's equations yield the velocity of light; it is 3.0×10^8 m/s.

So, which is the reference frame in which light travels at that speed?

Scientists searched for variations in the speed of light depending on the direction of the ray....

The Michelson - Morley Experiment

This experiment consisted of making very accurate measurements of the speed of light as Earth traveled around the sun over the course of the year. The idea was to determine the difference in the speed of light as Earth moved in different directions.

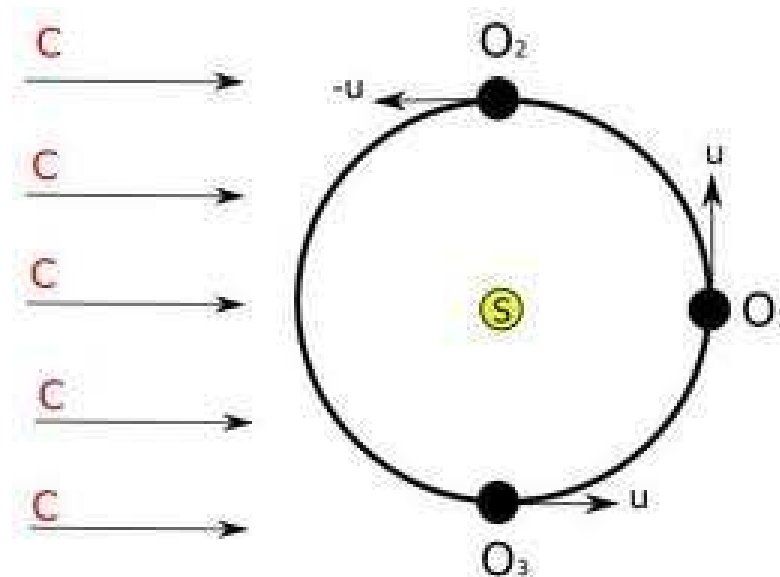


The Michelson - Morley Experiment

Michelson and Morley believed that light propagated through something called Ether. They believed that the ether moved through space and that its velocity as well as the velocity of the earth would effect the speed of light from the sun.

Two Swimmers Analogy

Michelson-Morley Experiment



The Michelson - Morley Experiment

They considered the experiment a failure as no difference was discovered.

This results was explained by Einstein's theory of special relativity. Some people believed that Einstein knew of the experiment to develop the theory, but that does not appear to be true.

2 The Michelson-Morley experiment was designed to measure

Move
For
Answer

- A the relativistic mass of the electron.
- B the relativistic energy of the electron.
- C the velocity of the Earth relative to the ether.
- D the acceleration of gravity on the Earth's surface.

3 **Michelson and Morley concluded from the results of their experiment that**

Move
For
Answer

- A the experiment was a failure since there was no detectable shift in the interference pattern.**
- B the experiment was successful in not detecting a shift in the interference pattern.**
- C the experiment was a failure since they detected a shift in the interference pattern.**
- D the experiment was successful in detecting a shift in the interference pattern.**

Postulates of the Special Theory of Relativity

- The laws of physics have the same form in all inertial reference frames.**
- Light propagates through empty space with speed c independent of the speed of source or observer.**

This solves the problem – the speed of light is in fact the same in all inertial reference frames.

4 One of Einstein's postulates in formulating the special theory of relativity was that the laws of physics are the same in reference frames that

- A accelerate.**
- B move at constant velocity with respect to an inertial frame.**
- C oscillate.**
- D are stationary, but not in moving frames.**

**Move
For
Answer**

Simultaneity

One of the implications of relativity theory is that time is not absolute. Observers do not necessarily agree on time intervals between events, or on whether they are simultaneous or not.

In relativity, an “event” is defined as occurring at a specific place and time....spacetime.

Simultaneity

Thought experiment: Lightning strikes at two separate places. One observer believes the events are simultaneous since the light has taken the same time to reach her.

Simultaneity

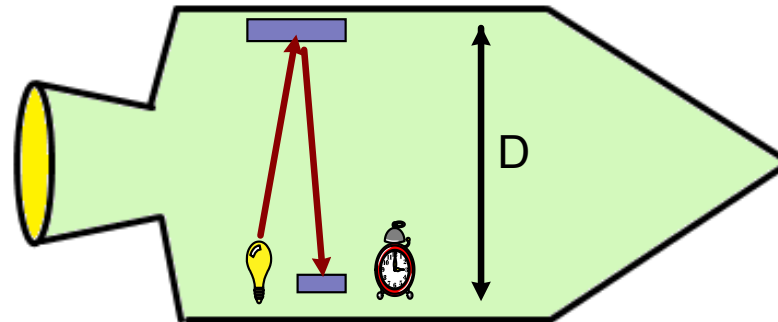
...but another, who is moving stationary to the observer does not agree that the two lightning strikes are simultaneous.



Time Dilation and the Twin Paradox

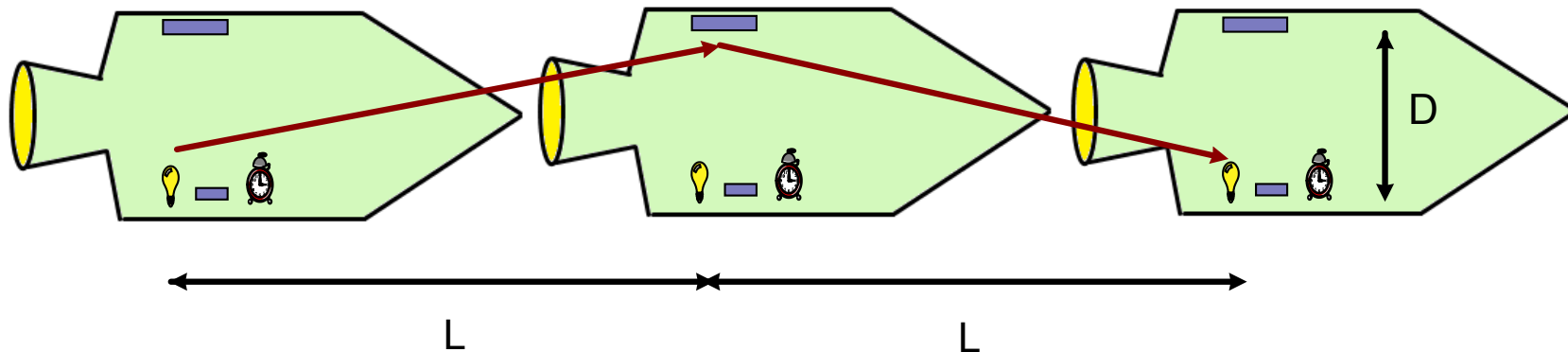
A different thought experiment, using a clock consisting of a light beam and mirrors in a space ship, shows that moving observers must disagree on the passage of time.

When the space ship is at rest, the light travels a distance of $2D$ in the time t .



Time Dilation and the Twin Paradox

But when the ship is moving, the light must travel a distance of $2\sqrt{D^2 + L^2}$ in the same time t .



Time Dilation Simulation

Time Dilation and the Twin Paradox

The time measured by the observer traveling with the clock is call proper time t_0 .

The outside observer sees the time traveled by the light as:

$$t = \frac{\sqrt{D^2 + L^2}}{c}$$

where

$$L = vt$$

and

$$D = ct_0$$

$$t = \frac{\sqrt{(ct_0)^2 + (vt)^2}}{c}$$

$$t^2 = \frac{c^2 t_0^2 + v^2 t^2}{c^2}$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

when simplified.

Time Dilation

Calculating the difference between clock “ticks,” we find that the interval in the moving frame is related to the interval in the clock’s rest frame:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}}$$

An outside observer will think that time is going slower for the traveling observer...and vice versa.

Time Dilation

The factor multiplying t_0 occurs so often in relativity that it is given its own symbol, γ .

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

We can then write:

$$\Delta t = \gamma \Delta t_0$$

Time Dilation and the Twin Paradox

Velocity, v	Gamma, γ
0	1
0.01c	1
0.10c	1.005
0.50c	1.15
0.90c	2.3
0.99c	7.1

Time Dilation

To clarify:

- The time interval in the frame where two events occur in the same place is t_0 . This is always the shortest measured time interval between events.
- The time interval in a frame moving with respect to the first one is Δt . That time interval is always larger than t_0 .

5 If you were to measure your pulse rate while in a spaceship moving away from the Sun at a speed close to the speed of light, you would find that it was

Move
For
Answer

- A much faster than normal.**
- B much slower than normal.**
- C the same as it was here on Earth.**

6 Relative to a stationary observer, a moving clock

- A always runs slower than normal.
- B always runs faster than normal.
- C keeps its normal time.
- D can do any of the above. It depends on the relative velocity between the observer and the clock.

Move
For
Answer

7 The gamma factor is defined as $\gamma \equiv 1 / \sqrt{1 - (v/c)^2}$, therefore gamma (γ)

Move
For
Answer

- A can be zero.**
- B can be any number less than or equal to one.**
- C can be any number greater than or equal to one.**
- D cannot equal one.**

8 A spaceship takes a nonstop journey to a planet and returns in 10 hours according to a clock on the spaceship. If the speed of the spaceship is $0.80c$, how much time has elapsed on the Earth?

- A 3.2 h**
- B 7.0 h**
- C 15 h**
- D 17 h**

Move
For
Answer

The Twin Paradox

It has been proposed that space travel could take advantage of time dilation – if an astronaut's speed is close enough to the speed of light, a trip of 100 light-years could appear to the astronaut as having been much shorter.

The astronaut would return to Earth after being away for a few years, and would find that hundreds of years had passed on Earth.

The Twin Paradox

This brings up the twin paradox – if any inertial frame is just as good as any other, why doesn't the astronaut age faster than the Earth traveling away from him?

The solution to the paradox is that the astronaut's reference frame has not been continuously inertial – he turns around at some point and comes back.

9 Suppose one twin takes a ride in a space ship traveling at a very high speed to a distant star and back again, while the other twin remains on Earth. The twin that remained on Earth predicts that the astronaut twin is

**Move
For
Answer**

- A younger.**
- B the same age.**
- C older.**
- D cannot be determined from the given information**

10 One 20-year-old twin brother takes a space trip with a speed of $0.80c$ for 30 years according to a clock on the spaceship. Upon returning to the Earth, what is his own age and the age of the Earth-based twin brother?

Move
For
Answer

- A 20; 30**
- B 30; 50**
- C 50; 70**
- D 70; 90**

Length Contraction

If time intervals are different in different reference frames, lengths must be different as well. Length contraction is given by:

$$L = L_0 \sqrt{1 - v^2/c^2}$$

or

$$L = \frac{L_0}{\gamma}$$

Length contraction occurs only along the direction of motion.

Length Contraction

$$L = L_0 \sqrt{1 - v^2/c^2}$$

or

$$L = \frac{L_0}{\gamma}$$

- The length of an object in a frame in which it has no velocity is called the object's proper length, L_0 ; L_0 is always the longest measurement of the length of an object.
- The length of an object in a frame moving with respect to the first one is L ; L is always less than L_0 .

Muon's and Length Contraction

11 Relative to a stationary observer, a moving object

- A appears shorter than normal.
- B appears longer than normal.
- C keeps its same length time.
- D can do any of the above. It depends on the relative velocity between the observer and the object.

Move
For
Answer

12 An object moves in a direction parallel to its length with a velocity that approaches the velocity of light. The width of this object, as measured by a stationary observer,

Move
For
Answer

- A approaches infinity.**
- B approaches zero.**
- C increases slightly.**
- D does not change.**

13 An object moves in a direction parallel to its length with a velocity that approaches the velocity of light. The length of this object, as measured by a stationary observer,

Move
For
Answer

- A approaches infinity.**
- B approaches zero.**
- C increases slightly.**
- D does not change.**

14 A meter stick is moving toward you with a speed of $0.80c$. What is its length?

- A zero**
- B 0.40 m**
- C 0.60 m**
- D 1 m**

**Move
For
Answer**

15 How fast would a rocket ship have to move to contract to half of its proper length (as observed by a stationary object)?

- A 0.50c**
- B 0.65c**
- C 0.78c**
- D 0.87c**

Move
For
Answer

16 The length of a spaceship is 10 m when it is at rest. If the spaceship travels by you with a speed of $0.70c$, what length does it appear to you?

- A 5.5 m**
- B 7.1 m**
- C 12 m**
- D 18 m**

**Move
For
Answer**

Four-Dimensional Space-Time

Space and time are even more intricately connected. Space has three dimensions, and time is a fourth. When viewed from different reference frames, the space and time coordinates can mix.

17 The theory of special relativity

Move
For
Answer

- A is based on a complex mathematical analysis.
- B has not been verified by experiment.
- C does not agree with Newtonian mechanics.
- D does not agree with electromagnetic theory.

Relativistic Momentum and Mass

Expressions for momentum and mass also change at relativistic speeds.

Momentum:

$$p = \frac{m_0 v}{\sqrt{1 - v^2/c^2}} = \gamma m_0 v$$

Gamma and the rest mass are sometimes combined to form the relativistic mass:

$$m_{\text{rel}} = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \gamma m_0$$

18 As the speed of a particle approaches the speed of light, the mass of the particle

- A increases.**
- B decreases.**
- C remains the same.**
- D approaches zero.**

**Move
For
Answer**

19 As the speed of a particle approaches the speed of light, the momentum of the particle

- A increases.**
- B decreases.**
- C remains the same.**
- D approaches zero.**

**Move
For
Answer**

20 An electron is traveling at $0.85c$. What is its mass? (The rest mass is 9.11×10^{-31} kg.)

- A 1.4×10^{-29} kg**
- B 7.2×10^{-30} kg**
- C 1.7×10^{-30} kg**
- D 2.4×10^{-30} kg**

**Move
For
Answer**

21 What is the momentum in kg·m/s of a proton when it is moving with a speed of $0.60c$?

- A 1.2×10^{-19} kg·m/s**
- B 1.5×10^{-19} kg·m/s**
- C 3.0×10^{-19} kg·m/s**
- D 3.8×10^{-19} kg·m/s**

**Move
For
Answer**

$E = mc^2$; Mass and Energy

At relativistic speeds, not only is the formula for momentum modified; that for energy is as well. The total energy can be written:

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

Where the particle is at rest,

$$E_0 = m_0 c^2$$

22 During a reaction, an element loses 4.8×10^{-28} kg of mass. How much energy (in Joules) is released?

- A 4.3×10^{-11} J
- B 1.4×10^{-19} J
- C 1.6×10^{-36} J
- D 5.3×10^{-45} J

Move
For
Answer

23 During a reaction, 1.7×10^{-4} J of energy is released. What change of mass would cause this?

- A 5.1×10^{-4} kg
- B 1.5×10^{-13} kg
- C 4.8×10^{-18} kg
- D 1.9×10^{-21} kg

Move
For
Answer

24 How much energy would be released if 2.0 kg of material was lost during a reaction?

- A** 1.8×10^{17} J
- B** 1.5×10^{16} J
- C** 6.0×10^8 J
- D** 4.7×10^{-8} J

Move
For
Answer

$E = mc^2$; Mass and Energy

Combining the relations for energy and momentum gives the relativistic relation between them:

$$E^2 = p^2c^2 + m_0^2c^4$$

Mass Dilation

$$m_{\text{rel}} = \frac{m_0}{\sqrt{1 - v^2/c^2}} = \gamma m_0$$

- The mass of an object in a frame in which it has no velocity is called the object's rest mass, m_0 ; m_0 is always the smallest measurement that can be made of that mass.
- The mass of an object in a frame moving with respect to the first one is m_{rel} ; m_{rel} is always greater than m_0 .

$E = mc^2$; Mass and Energy

All the formulas presented here become the usual Newtonian kinematic formulas when the speeds are much smaller than the speed of light.

There is no rule for when the speed is high enough that relativistic formulas must be used – it depends on the desired accuracy of the calculation.

The Ultimate Speed

A basic result of special relativity is that nothing can equal or exceed the speed of light. This would require infinite momentum – not possible for anything with mass.

(Sort of a circular argument since all of this was derived with the assumption that "c" could not be exceeded; the results of that cannot be used to prove it; the above statement is a tautology.)



25 A fast spaceship is traveling with a speed of $0.60c$ relative to Earth. It releases a rocket that is traveling at $0.60c$ relative to the spaceship. How fast is the rocket going relative to Earth?

- A $1.20c$
- B $1.00c$
- C $0.88c$
- D $0.33c$

Move
For
Answer

26 A fast spaceship is traveling with a speed of $0.80c$. How fast would light travel from the headlights of the ship, relative to a stationary observer?

- A $0.20c$**
- B $0.80c$**
- C $1.0c$**
- D $1.8c$**

**Move
For
Answer**

The Impact of Special Relativity

The predictions of special relativity have been tested thoroughly, and verified to great accuracy.

The correspondence principle says that a more general theory must agree with a more restricted theory where their realms of validity overlap.

This is why the effects of special relativity are not obvious in everyday life.

(There are everyday consequences of special relativity. For instance, it can be shown that magnetism is a result of length contraction plus electric force.)

27 Consider two spaceships, each traveling at $0.50c$ in a straight line. Ship A is moving directly away from the Sun and ship B is approaching the Sun. The science officers on each ship measure the velocity of light coming from the Sun. What do they measure for this velocity?

Move
For
Answer

- A Ship A measures it as less than c , and ship B measures it as greater than c .
- B Ship B measures it as less than c , and ship A measures it as greater than c .
- C On both ships it is measured to be less than c .
- D On both ships it is measured to be exactly c .

28 Which of the following depends on the observer's frame of reference?

- A the mass of the proton**
- B the length of a meter stick**
- C the half-life of radon**
- D all of the given answers**

Move
For
Answer

29 As the velocity of your spaceship increases, you would observe

- A that your precision clock runs slower than normal.**
- B that the length of your spaceship has decreased.**
- C that your mass has increased.**
- D all of the given answers**
- E none of the given answers**

**Move
For
Answer**

Summary

- **Inertial reference frame: one in which Newton's first law holds**
- **Principles of relativity: the laws of physics are the same in all inertial reference frames; the speed of light in vacuum is constant regardless of speed of source or observer**

- **Time dilation:**

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_0$$

- **Length contraction:**

$$L = L_0 \sqrt{1 - v^2/c^2} = \frac{L_0}{\gamma}$$

- **Gamma:**

$$\gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

Summary

- **Relativistic momentum:**

$$p = \gamma m_0 v = \frac{m_0 v}{\sqrt{1 - v^2/c^2}}$$

- **Relativistic mass:**

$$m_{\text{rel}} = \gamma m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

- **Mass-energy relation:**

$$E_0 = m_0 c^2$$

- **Kinetic energy:**

$$\text{KE} = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}} - m_0 c^2 = (\gamma - 1) m_0 c^2$$

- **Relationship between energy and momentum:**

$$E^2 = p^2 c^2 + m_0^2 c^4$$