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# Electric Field, Potential Energy, & Voltage

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## Work


The force changes as charges move towards each other since the force depends on the distance between the charges.

As these two charges get closer together, it takes more force to keep them from flying apart since they're repelling each other.

It takes work to move the charges.

W = Force x $distance_{parallel}$

$W = \frac{kQq}{r}$

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## Electric Potential Energy

$W = \frac{kQq}{r}$


*If we have two charges initially at rest, and infinitely far apart:*

$E_0 + W = E_f$

$0 + \frac{kQq}{r} = E_f$

$E_f = U_E = \frac{kQq}{r}$

$U_E = \frac{kQq}{r}$

Potential Energy due to two point charges

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## Electric Potential Energy

This is the equation for the potential energy due to two point charges being near each other. $U_E = \frac{kQq}{r}$

Energy is NOT a vector, it is a scalar. There is no direction, but the sign matters.

If you have two positive charges or two negative charges, there will be a positive potential energy. This means it is taking energy to keep them from flying apart.

If you have a positive charge and a negative charge near each other, you will have a negative potential energy. This means that it takes energy to keep them from getting closer together.

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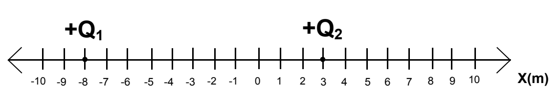


A positive charge $Q_1$ = 5 mC is located at $x_1$ = -8 m and a charge $Q_2$ = 2.5 mC is located at $x_2$ = 3 m. Compute the potential energy of the two charges.

$U_E = \frac{kQq}{r} = \frac{(9\times10^9\ Nm^2/C^2)(5\times10^{-3}\ C)(2.5\times10^{-3}\ C)}{11m}$

$U_E$ = 10227 J

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## Energy of Multiple Charges

To get the total energy for multiple charges, you must first find the energy due to each pair of charges.

Then, you can add these energies together. Since energy is a scalar, there is no direction involved.

$U_{total} = U_1 + U_2 + U_3 + ...$

## Electric Potential or Voltage

We know that:

Just as we can break electrical force into two parts:

$F = qE$ and $E = \frac{kQ}{r^2}$,

we could separate potential energy into two parts:

$U_E = qV$ and $V = \frac{kQ}{r}$

where V is called the ***voltage***.



## Electric Potential or Voltage

Voltage is also called electric potential (NOT to be confused with electric potential energy).

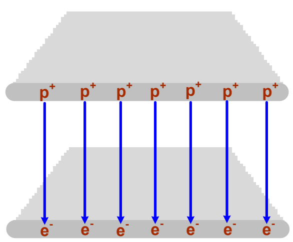
Voltage is measured in Volts (V)

where one $V = \frac{J}{C}$

Voltage is NOT a vector, so multiple voltages can be added directly (sign is important!).



## Uniform Electric Field

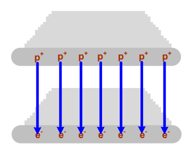



The field cancels outside the plates and adds together between the plates for a strong electric field.

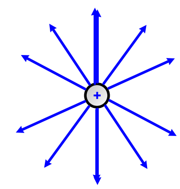
Uniform means that the strength of the field is the same everywhere (between the plates).



## Uniform Electric Field



Only some equations we have learned apply to uniform electric fields.




Point charges have a non-uniform field strength since the field weakens with distance.



$F = \frac{kQq}{r^2}$

$E = \frac{kQ}{r^2}$

$U_E = \frac{kQq}{r}$

$V = \frac{kQ}{r}$

Use ONLY with point charges.

Equations with the "k" are point charges ONLY.

$F = qE$

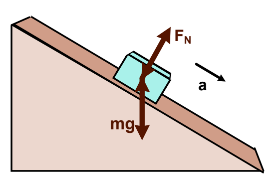
$U_E = qV$

Use in ANY situation.

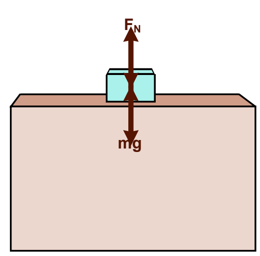
For point charges AND uniform electric fields



## Electric Field & Voltage




The slope of the plane determines the acceleration and the net force on the object.




Slope = 0

$F_{net} = 0$

no acceleration!

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## Electric Field & Voltage

If we look at the energy of the block on the inclined plane...

$E_0 + W = E_f$ where $W = 0$

$mg\Delta h = \frac{1}{2}mv_f^2$

If $v_f^2 = v_0^2 + 2a\Delta x$ and $v_0 = 0$ then $v_f^2 = 2a\Delta x$

$mg\Delta h = \frac{1}{2}m(2a\Delta x)$

$mg\Delta h = ma\Delta x$

$g\Delta h = a\Delta x$

$$a = \frac{g\Delta h}{\Delta x}$$

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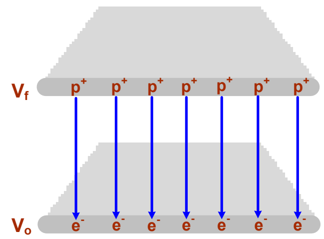
## Electric Field & Voltage

A similar relationship exists with uniform electric fields and voltage.

With the inclined plane, a difference in height was responsible for acceleration. Here, a difference in electric potential (voltage) is responsible for the electric field.

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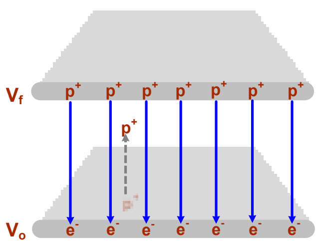
## Electric Field & Voltage

The change in voltage is defined as the work done per unit charge against the electric field.

Therefore energy is being put into the system when a positive charge moves in the opposite direction of the electric field (or when a negative charge moves in the same direction of the electric field).

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## Electric Field & Voltage

To see the exact relationship, look at the energy of the system.

$E_0 + W = E_f$ where $W = 0$

$qV_0 = qV_f + \frac{1}{2}mv_f^2$

$qV_0 - qV_f = \frac{1}{2}mv_f^2$

$-q\Delta V = \frac{1}{2}mv_f^2$ where $\Delta V = V_f - V_0$

If $v_f^2 = v_0^2 + 2a\Delta x$ and $v_0 = 0$ then $v_f^2 = 2a\Delta x$

$-q\Delta V = \frac{1}{2}m(2a\Delta x)$

$-q\Delta V = ma\Delta x$

If $F = ma$, and $F = qE$, then we can substitute $ma = qE$

$-q\Delta V = qE\Delta x$

$-\Delta V = E\Delta x$

$$E = \frac{-\Delta V}{\Delta x} = \frac{-\Delta V}{d}$$

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## Electric Field & Voltage

The equation only applies to uniform electric fields.

$$E = -\frac{\Delta V}{\Delta x} = -\frac{\Delta V}{d}$$

It follows that the electric field can also be shown in terms of volts per meter (V/m) in addition to Newtons per Coulomb (N/C). This can be shown:

$1\frac{N}{C} = \frac{V}{m}$ and a $V = \frac{J}{C}$

$1\frac{N}{C} = \frac{(J/C)}{m}$ and a $J = N \cdot m$

$1\frac{N}{C} = \frac{(N \cdot m/C)}{m}$

$1\frac{N}{C} = 1\frac{N}{C}$ The units are equivalent.

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## Electric Field & Voltage

A more intuitive way to understand the negative sign in the relationship

$$E = -\frac{\Delta V}{\Delta x}$$

is to consider that just like a mass falls down, from higher gravitational potential energy to lower, a positive charge "falls down" from higher electric potential (V) to lower.

Since the electric field points in the direction of the force on a hypothetical positive test charge, it must also point from higher to lower potential.

The negative sign just means that objects feel a force from locations with greater potential energy to locations with lower potential energy. This applies to all forms of potential energy.

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1 In order for a charged object to experience an electrostatic force, there must be a:

- A a large electric potential
- B a small electric potential
- C the same electric potential everywhere
- D a difference in electric potential

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2 How strong (in V/m) is the electric field between two metal plates 25 cm apart if the potential difference between them is 100 V?

- A 400 V/m
- B 600 V/m
- C 800 V/m
- D 1000 V/m
- E 1200 V/m

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3 An electric field of 3500 N/C is desired between two plates which are 4.0 mm apart; what voltage should be applied?

- A 10 V
- B 12 V
- C 14 V
- D 16 V
- E 18 V

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## Combining Two Ideas...

$E = -\frac{\Delta V}{d}$

$U_E = q\Delta V$ ← $\Delta V = -Ed$

$U_E = -qEd$

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4 How much work (in Joules)is done by a uniform 300 N/C electric field on a charge of 6.1 mC in accelerating it through a distance of 20 cm?

- A $4.23 \times 10^{-2}$ J
- B $3.66 \times 10^{-2}$ J
- C $3.81 \times 10^{-2}$ J
- D $3.12 \times 10^{-2}$ J
- E $4.93 \times 10^{-2}$ J

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$F = \frac{kQq}{r^2}$

$E = \frac{kQ}{r^2}$

$U_E = \frac{kQq}{r}$

$V = \frac{kQ}{r}$

Use ONLY with point charges.

Equations with the "k" are point charges ONLY.

$F = qE$

$U_E = qV$

$E = -\frac{\Delta V}{d}$

Use in ANY situation.

For point charges AND uniform electric fields

$U_E = -qEd$

ONLY for uniform electric fields

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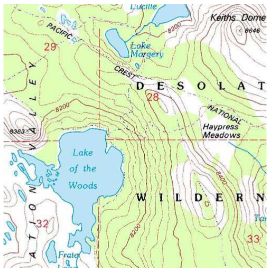
## Topographic Maps




Each line represents the same height value.

The area between lines represents the change between lines.

A big space between lines, there is a slow change in height.

Little space between lines means there is a very quick change in height.

***Where in this picture is the steepest incline?***

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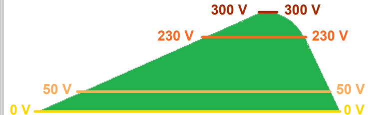
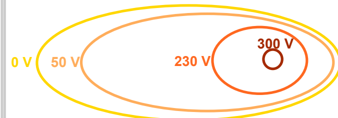
## Equipotential Lines




These "topography" lines are called "equipotential lines" when we use them to represent the electric potential.




The closer the lines, the faster the change in voltage.... the bigger the change in voltage, the larger the electric field.

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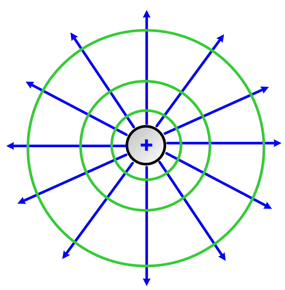
## Equipotential Lines

1. The direction of the electric field and force are always perpendicular to the lines.

2. The electric field lines are farther apart when the equipotentials are farther apart.

3. The electric field goes from high to low potential (just like a positive charge).

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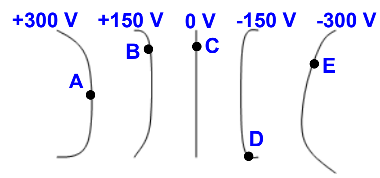
5 At point A in the diagram, what is the direction of the electric field?

- A Up
- B Down
- C Left
- D Right

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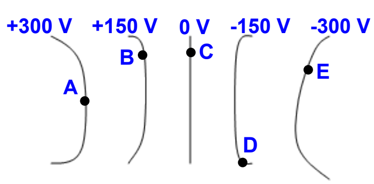
6 How much work is done to a +10μC charge that moves from point C to B?

- A 1 x 10 J
- B 1.3 x 10 J
- C 2 x10 J
- D 3.5 x 10 J
- E 1.5 x 10 J

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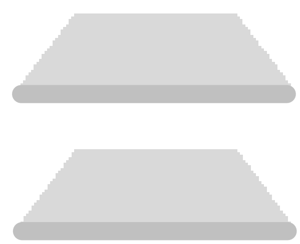
## Parallel Plate Capacitors

The simplest version of a capacitor is the

parallel plate capacitor

which consists of two metal plates that are parallel to one another and located a distance apart.



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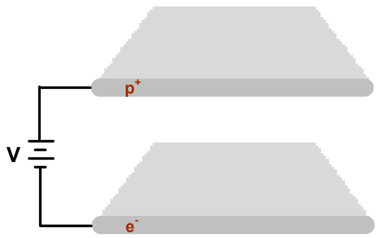
## Parallel Plate Capacitors

When a battery is connected to the plates, charge moves between them. Every electron that moves to the negative plate leaves a positive nucleus behind.

The plates have equal magnitudes of charges, but one is positive, the other negative.

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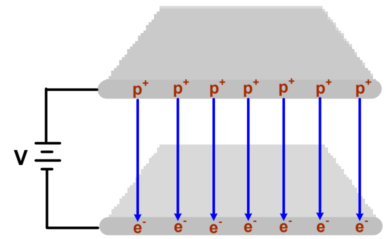
## Parallel Plate Capacitors

Only unpaired protons and electrons are represented here.

Most of the atoms are neutral since they have equal numbers of protons and electrons.

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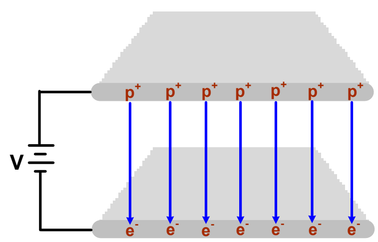
## Parallel Plate Capacitors

Drawing the Electric Field from the positive to negative charges reveals that the Electric Field is uniform everywhere in a capacitor's gap.

Also, there is no field outside the gap.

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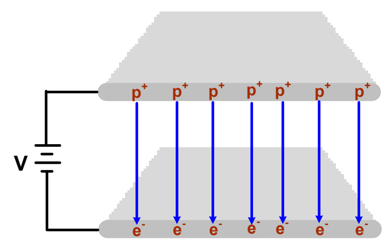
## Capacitors

ANY capacitor can store a certain amount of charge for a given voltage. That is called its capacitance, C.

$$C = \frac{Q}{V}$$

This is just a DEFINITION and is true of all capacitors, not just parallel plate capacitors.

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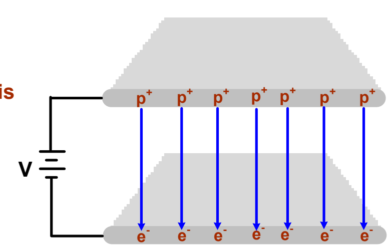
## Units of Capacitance

$$C = \frac{Q}{V}$$

The unit of capacitance is the farad (F). A farad is a Coulomb per Volt.

A farad is huge; so capacitance is given as

picofarad (1pf = $10^{-12}$ F),
nanofarad (1nf = $10^{-9}$ F),
microfarad (1μf = $10^{-6}$ F),
millifarads (1mf = $10^{-3}$ F)

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7 What is the capacitance of a fully charged parallel plate capacitor that has a charge of 25 μC and a difference in potential of 50 V?

- A $5 \times 10^{-7}$ F
- B $2 \times 10^{-7}$ F
- C $4 \times 10^{-7}$ F
- D $8 \times 10^{-7}$ F
- E $1 \times 10^{-7}$ F

8 A fully charged 50μF parallel place capacitor has a potential of 100V across it's plates. How much charge is stored in the capacitor?

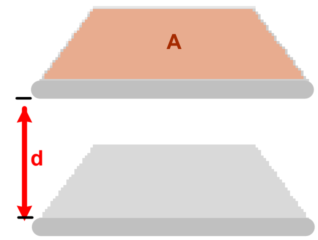
- A $6 \times 10^{-3}$ F
- B $4 \times 10^{-3}$ F
- C $5 \times 10^{-3}$ F
- D $9 \times 10^{-3}$ F
- E $10 \times 10^{-3}$ F



## Parallel Plate Capacitors

The Area of the capacitor is just the surface area of ONE PLATE, and is represented by the letter A.

The distance between the plates is represented by the letter d.



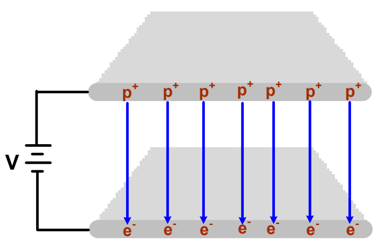



## Parallel Plate Capacitors

For PARALLEL PLATE CAPACITORS, the capacity to store charge increases with the area of the plates and decreases as the plates get farther apart.

C # A

C # 1/d





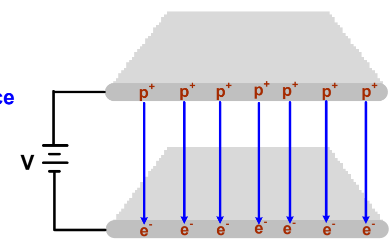

## Parallel Plate Capacitors

The constant of proportionality is called the

Permitivity of Free Space

and has the symbol $\#_o$.

$\#_o = 8.85 \times 10^{-12}\ C^2/N\text{-}m^2$





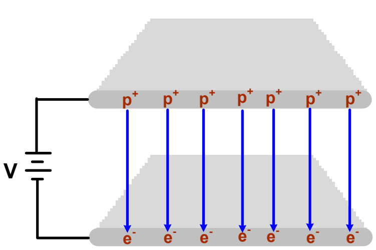

## Parallel Plate Capacitors

So for PARALLEL PLATE CAPACITORS:

$$C = \frac{\#_o A}{d}$$

The larger the Area, A, the higher the capacitance.

The closer together the plates get, the higher the capacitance.



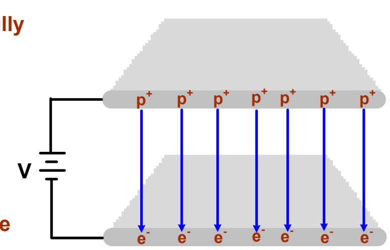



## Parallel Plate Capacitors

Imagine you have a fully charged capacitor.

If you disconnect the battery and change either the area or distance between the plates, what do you know about the charge on the capacitor?

The charge remains the same.

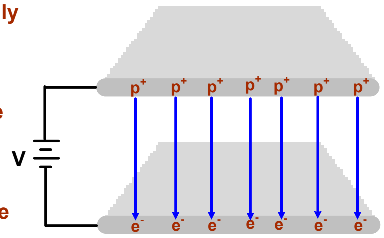


## Parallel Plate Capacitors

Imagine you have a fully charged capacitor.

If you keep the battery connected and change either the area or distance between the plates, what do you know about the voltage across the plates?

The voltage remains the same.






9 A parallel plate capacitor has a capacitance $C_0$. If the area on the plates is doubled and the distance between the plates drops by one half, what will be the new capacitance?

- A $C_0/4$
- B $C_0/2$
- C $C_0$
- D $2C_0$
- E $4C_0$



10 A parallel-plate capacitor is charged by connection to a battery and remains connected. What will happen to the charge on the capacitor and the voltage across it if the area of the plates increases and the distance between them decreases?

- A Both increase
- B Both decrease
- C Both remain the same
- D The voltage remains the same and charge increases
- E The voltage remains the same and the charge decreases



11 A parallel-plate capacitor is charged by connection to a battery and the battery is disconnected. What will happen to the charge on the capacitor and the voltage across it if the area of the plates decreases and the distance between them increases?

- A Both increase
- B Both decrease
- C Both remain the same
- D The charge remains the same and voltage increases
- E The charge remains the same and the voltage decreases

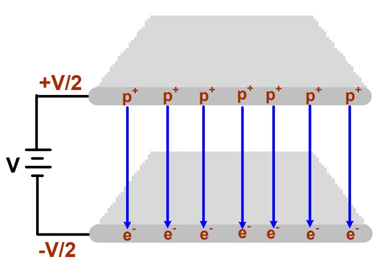


## E-field and Voltage in P.P. Capacitors

After being charged the plates have equal and opposite voltage, V.

There is a uniform electric field, E, between the plates.

We learned earlier that with a UNIFORM E-FIELD that #V = -Ed; this is true in the case of the parallel plate capacitor.



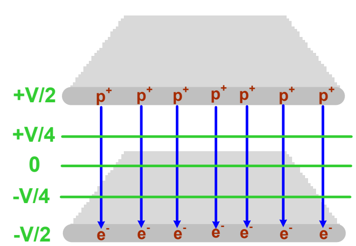



## E-field and Voltage in PP Capacitors

The Electric Field is constant everywhere in the gap.

The Voltage (also know as the Electric Potential) declines uniformly from +V to -V within the gap; it is zero at the location midway between the plates.

It is always perpendicular to the E-Field.



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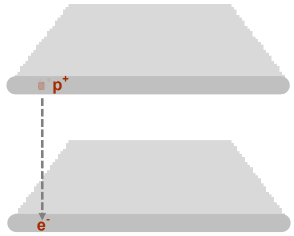
## Energy Storage in Capacitors

The energy stored in ANY capacitor is given by formulas most easily derived from the parallel plate capacitor.

Consider how much work it would take to move a single electron between two initially uncharged plates.

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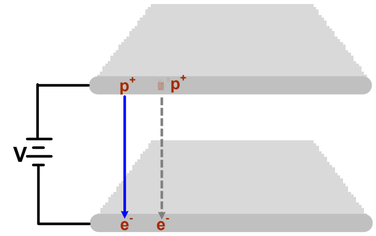
## Energy Storage in Capacitors

That takes ZERO work since there is no difference in voltage.

However, to move a second electron to the negative plate requires work to overcome the repulsion from the first one...and to overcome the attraction of the positive plate.

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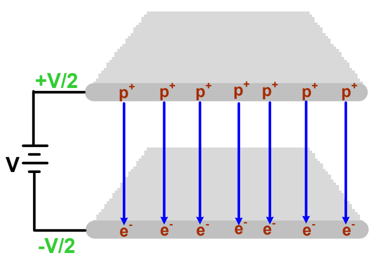
## Energy Storage in Capacitors

To move the last electron from the positive plate to the negative plate requires carrying it through a voltage difference of V.

The work required to do that is q#V...




Here the charge of a electron is -e, and the difference in potential is -V (-V/2 - V/2)... the work = eV.

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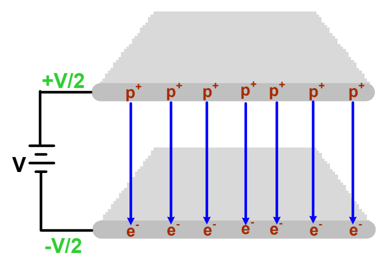
## Energy Storage in Capacitors

If the work to move the first electron is zero.

And the work to move the last electron is eV.

The the AVERAGE work for ALL electrons is eV/2.

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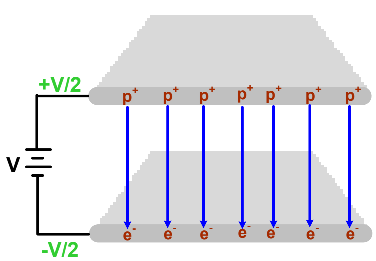
## Energy Storage in Capacitors

Then, the work needed to move a total charge Q from one plate to the other is given by

W = QV/2

That energy is stored in the electric field within the capacitor.

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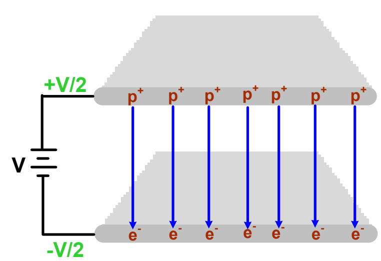
## Energy Storage in Capacitors

So the energy stored in a capacitor is given by:

$$U_C = \frac{QV}{2}$$

Where Q is the charge on one plate and V is the voltage difference between the plates.



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## Energy Storage in Capacitors

Using our equation for capacitance (C=Q/V) and our equation for electric potential energy in a capacitor, we can derive three different results.

$C = \frac{Q}{V}$ → solve for V → $V = \frac{Q}{C}$ → solve for Q → $Q = CV$

$U_C = \frac{QV}{2}$

substitute: $U_C = \frac{QV}{2}$, $U_C = \frac{Q}{2}\frac{Q}{C}$

substitute: $U_C = \frac{QV}{2}$, $U_C = \frac{(CV)V}{2}$

$U_C = \frac{Q^2}{2C}$

$U_C = 1/2\ CV^2$

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12 How much energy is stored in a fully charged parallel plate capacitor that is storing 15 nC with 3 V across its plates?

- A $1.3 \times 10^{-8}$ J
- B $1.9 \times 10^{-8}$ J
- C $2.3 \times 10^{-8}$ J
- D $2.7 \times 10^{-8}$ J
- E $3.0 \times 10^{-8}$ J

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13 How much energy is stored in a fully charged 3 mF parallel plate capacitor with 2 V across its plates?

- A $6 \times 10^{-3}$ J
- B $3 \times 10^{-3}$ J
- C $5 \times 10^{-3}$ J
- D $8 \times 10^{-3}$ J
- E $12 \times 10^{-3}$ J

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14 How much energy is stored in a fully charged 16 mF parallel plate capacitor with 220 V across its plates?

- A 234.6 J
- B 294.9 J
- C 372.8 J
- D 387.2 J
- E 408.4 J

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15 A parallel plate capacitor is connected to a battery. The capacitor becomes fully charged and stays connected to the battery. What will happen to the energy held in a parallel plate capacitor if the area of the plates increases?

- A Remains the same
- B Increased
- C Decreased
- D Zero
- E More information is required

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16 A parallel plate capacitor is connected to a battery. The capacitor becomes fully charged and is then disconnected from the battery. What will happen to the energy held in a parallel plate capacitor if the area of the plates increases?

- A Remains the same
- B Increased
- C Decreased
- D Zero
- E More information is required

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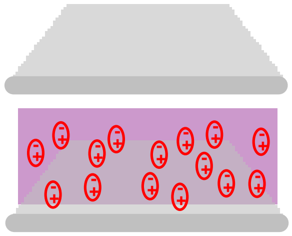
## Dielectrics and Capacitors

Capacitance can be increased by inserting a dielectric (an insulator) into the gap.

Before the plates are charges the atoms are unpolarized, the electron is bound to the nucleus and not oriented in any direction.



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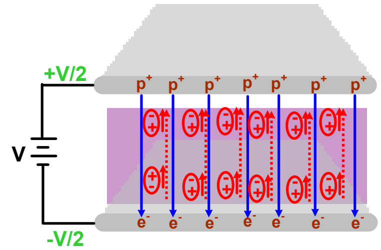
## Dielectrics and Capacitors

When the plates charge, the atoms are polarized and line up to reduce the external E-field.

This reduces the electric field, which lowers the voltage for a given charge (since V = Ed).

Since C = Q/V, this increases the capacitance.

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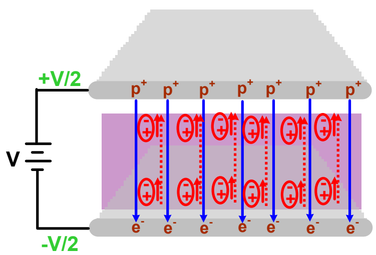
## Dielectrics and Capacitors

Every material has a dielectric constant, # (kappa), which is given in a table.

For a vacuum, = 1; air is about 1. If a dielectric is present, then:

$$C = \frac{\#\#_o A}{d}$$

The larger #, the larger C.

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## Capacitor Problems

These two equations are true for all capacitors.

$$C = \frac{Q}{V} \qquad U_C = \frac{QV}{2}$$

This equation is true for Parallel Plate Capacitors. Unless indicated otherwise, # = 1.

$$C = \frac{\#\#_o A}{d}$$

Some combination of these can solve all problems related to the Voltage, Charge, Electric Field and Voltage of a Capacitor.

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## Capacitor Problems

A capacitor is fully charged by a battery. While the battery is connected the distance between the plates is doubled. By what factor is the energy stored in the capacitor changed?

$$C = \frac{Q}{V} \qquad C = \frac{\#\#_o A}{d} \qquad U_C = \frac{QV}{2}$$

Since the battery is connected, V is constant...but Q will change.

So we need an equation for energy which doesn't rely on Q since it may change. Combine $C = \frac{Q}{V}$ and $U_C = \frac{QV}{2}$

to get: $U_C = 1/2\ CV^2$

Combine this with $C = \frac{\#\#_o A}{d}$ to get a relationship between energy and distance for constant voltage: $U_C = 1/2\ \frac{\#\#_o A}{d} V^2$

From this we see if we double the distance we halve the energy.

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