Midpoint and Distance Formula – Class Work

M is the midpoint of A and B. Use the given information to find the missing point.

1. A(4, 2) and B(3, -8), find M

$(3.5, 3)$

2. A(5, 7) and B(-2, -9), find M

$(1.5, -1)$

3. A(2,0) and B(6, -2), find M

$(4, -1)$

4. A(3, 7) and M(4,-3), find B

$(5, -13)$

5. M(4, -9) and B(-10, 11) find A

$(18, -29)$

6. B(4, 8) and M(-2, 5), find A

$(-8, 2)$

7. Find the distance from A(4, 2) to B(3, -8).

$\sqrt{101}$

8. Find the distance from A(5, 7) to B(-2, -9).

$\sqrt{305}$

9. Find the distance from A(2,0) to B(6, -2).

$2\sqrt{5}$

10. The distance from A(2, 3) to B(-6, y) is 10, find y.

$y = -3 \text{ or } 9$

11. The distance from A(-4, 7) to B(x, 9) is 7, find x.

$x = -4 \pm 3\sqrt{5} \approx 2.71 \text{ or } -10.71$

Midpoint and Distance Formula – Homework

M is the midpoint of A and B. Use the given information to find the missing point.

12. A(4, -2) and B(5, 6), find M

$(4.5, 2)$

13. A(9, 4) and B(-3, -7), find M

$(3, -1.5)$

14. A(1, 10) and B(6, -2), find M

$(3.5, 4)$

15. A(4, 8) and M(4,-3), find B

$(4, -14)$

16. M(8, 7) and B(-10, 11) find A

$(26, 3)$

17. B(-5, 10) and M(-2, 5), find A

$(1, 0)$

18. Find the distance from A(-3, 9) to B(3, -8).

$5\sqrt{13}$

19. Find the distance from A(5, -9) to B(-2, -9).

7

20. Find the distance from A(-2,10) to B(-6, 0).

$2\sqrt{29}$

21. The distance from A(2, -3) to B(5, y) is 10, find y.

$y = -3 \pm \sqrt{91} \approx 6.54 \text{ or } -12.54$

22. The distance from A(4, 6) to B(2x, 9) is 7, find x.

$x = 2 \pm \sqrt{10} \approx 5.16 \text{ or } -1.16$

Parabolas – Class Work

What is the vertex of the parabola?

23. $(x-2)^2=(y-4)$ **$(2,4)$**

24. $(x+5)^2=-\frac{1}{3}(y-5)$ **$(-5,5)$**

25. $(y-7)^2=\frac{1}{5}(x+6)$ **$(-6,7)$**

Write the following equations in standard form. State the direction of the opening. Identify vertex and the focus and give the equations of the directrix and axis of symmetry.

26. $x^2+4x-y=0$

$$(x+2)^2=(y+4)$$
$$\uparrow \quad V:(-2,-4) \quad F:(-2,-3.75)$$
$$D:y=-4.25 \quad A:x=-2$$

27. $y^2-8y-x=0$

$$(y-4)^2=(x+16)$$
$$\rightarrow \quad V:(-16,4) \quad F:(-15.75,4)$$
$$D:x=-16.25 \quad A:y=4$$

28. $x^2-6x-y+8=0$

$$(x-3)^2=(y+1)$$
$$\uparrow \quad V:(3,-1) \quad F:(3,-0.75)$$
$$D:y=-1.25 \quad A:x=3$$

29. $y^2+2y-x+10=0$

$$(y+1)^2=(x-9)$$
$$\rightarrow \quad V:(9,-1) \quad F:(9.25,-1)$$
$$D:x=8.75 \quad A:y=-1$$

30. $x^2+10x-y-12=0$

$$(x+5)^2=(y+37)$$
$$\uparrow \quad V:(-5,-37) \quad F:(-5,-36.75)$$
$$D:y=-37.25 \quad A:x=-5$$

31. $y^2-8y-x+16=0$

$$(y-4)^2=x$$
$$\rightarrow \quad V:(0,4) \quad F:(0.25,4)$$
$$D:x=-0.25 \quad A:y=4$$

32. $2x^2+12x-10y-2=0$

$$(x+3)^2=5(y+2)$$
$$\uparrow \quad V:(-3,-2) \quad F:(-3,-0.8)$$
$$D:y=-3.2 \quad A:x=-3$$

33. $3y^2-6y-12x+39=0$

$$(y-1)^2=4(x-3)$$
$$\rightarrow \quad V:(3,1) \quad F:(4,1)$$
$$D:x=2 \quad A:y=1$$

34. $-4x^2+8x-48y-100=0$

$$(x-1)^2=-12(y+2)$$
$$\downarrow \quad V:(1,-2) \quad F:(1,-5)$$
$$D:y=1 \quad A:x=1$$

35. $-6y^2+48y-36x+48=0$

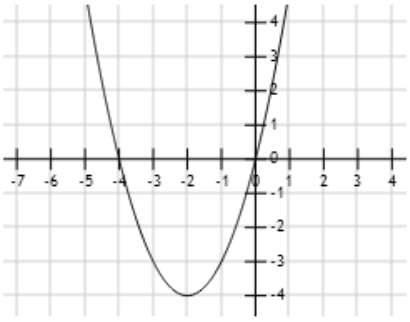
$$(y-4)^2=-6(x-4)$$
$$\leftarrow \quad V:(4,4) \quad F:(2.5,4)$$
$$D:x=5.5 \quad A:y=4$$

Graph each of the following. State the direction of the opening. Identify vertex and the focus and give the equations of the directrix and axis of symmetry.

36. $(x+6)^2 = 2(y-5)$

↑ $\boldsymbol{V}: (-6, 5)$ $\boldsymbol{F}: (-6, 5.5)$

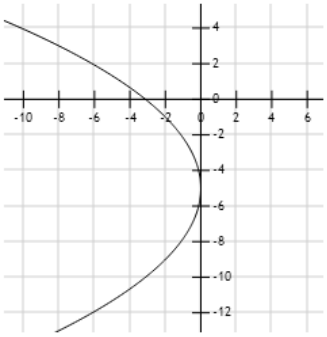
$\boldsymbol{D}: y = 4.5$ $\boldsymbol{A}: x = -6$



37. $(y+5)^2 = -8x$

← $\boldsymbol{V}: (0, -5)$ $\boldsymbol{F}: (-2, -5)$

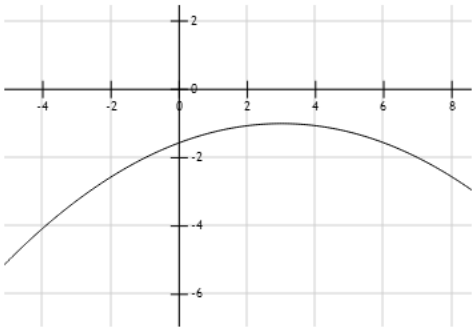
$\boldsymbol{D}: x = 2$ $\boldsymbol{A}: y = -5$



38. $(x-3)^2 = -16(y+1)$

↓ $\boldsymbol{V}: (3, -1)$ $\boldsymbol{F}: (3, -5)$

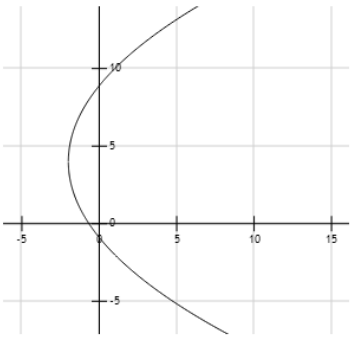
$\boldsymbol{D}: y = 3$ $\boldsymbol{A}: x = 3$



39. $(y-4)^2 = 12(x+2)$

→ $\boldsymbol{V}: (-2, 4)$ $\boldsymbol{F}: (1, 4)$

$\boldsymbol{D}: x = -5$ $\boldsymbol{A}: y = 4$

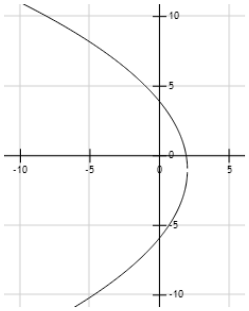


40. $y^2 + 2y + 12x - 23 = 0$

$\boldsymbol{(y+1)^2 = 12(x-2)}$

← $\boldsymbol{V}: (2, -1)$ $\boldsymbol{F}: (-1, -1)$

$\boldsymbol{D}: x = 5$ $\boldsymbol{A}: y = -1$

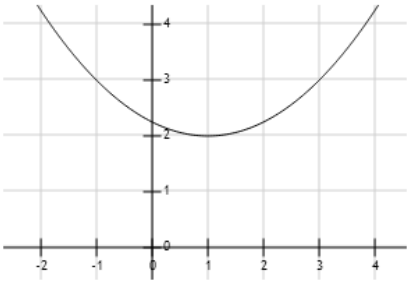


41. $x^2 - 2x - 4y + 9 = 0$

$\boldsymbol{(x-1)^2 = 4(y-2)}$

↑ $\boldsymbol{V}: (1, 2)$ $\boldsymbol{F}: (1, 3)$

$\boldsymbol{D}: y = 1$ $\boldsymbol{A}: x = 1$



Parabolas – Homework

What is the vertex of the parabola?

42. $(x+3)^2 = (y-7)$

$\boldsymbol{(2,4)}$

43. $(x+4)^2 = -\frac{1}{2}(y-8)$

$\boldsymbol{(-4,8)}$

44. $(y-3)^2 = \frac{1}{6}(x+5)$

$\boldsymbol{(-5,3)}$

Write the following equations in standard form. State the direction of the opening. Identify vertex and the focus and give the equations of the directrix and axis of symmetry.

45. $x^2 + 6x - y = 0$

$$\boldsymbol{(x+3)^2 = (y+9)}$$
$$\boldsymbol{\uparrow \quad V:(-3,-9) \quad F:(-3,-8.75)}$$
$$\boldsymbol{D: y = -9.25 \quad A: x = -3}$$

46. $y^2 - 10y - x = 0$

$$\boldsymbol{(y-5)^2 = (x+25)}$$
$$\boldsymbol{\rightarrow \quad V:(-25,5) \quad F:(-24.75,5)}$$
$$\boldsymbol{D: x = -25.25 \quad A: y = 5}$$

47. $x^2 - 4x - y + 11 = 0$

$$\boldsymbol{(x-2)^2 = (y-7)}$$
$$\boldsymbol{\uparrow \quad V:(2,7) \quad F:(2,7.25)}$$
$$\boldsymbol{D: y = 6.75 \quad A: x = 2}$$

48. $y^2 + 8y - x + 12 = 0$

$$\boldsymbol{(y+4)^2 = (x+4)}$$
$$\boldsymbol{\rightarrow \quad V:(-4,-4) \quad F:(-3.75,-4)}$$
$$\boldsymbol{D: x = -4.25 \quad A: y = -4}$$

49. $x^2 + 16x - y + 49 = 0$

$$\boldsymbol{(x+8)^2 = (y+15)}$$
$$\boldsymbol{\uparrow \quad V:(-8,-15) \quad F:(-8,-14.75)}$$
$$\boldsymbol{D: y = -15.25 \quad A: x = -8}$$

50. $-y^2 - 8y - x + 8 = 0$

$$\boldsymbol{(y+4)^2 = -(x-24)}$$
$$\boldsymbol{\leftarrow \quad V:(24,-4) \quad F:(23.75,-4)}$$
$$\boldsymbol{D: x = 24.25 \quad A: y = -4}$$

51. $2x^2 + 8x - 4y = 0$

$$\boldsymbol{(x+2)^2 = 2(y+2)}$$
$$\boldsymbol{\uparrow \quad V:(-2,-2) \quad F:(-2,-1.5)}$$
$$\boldsymbol{D: y = -2.5 \quad A: x = -2}$$

52. $3y^2 - 18y - 24x - 45 = 0$

$$\boldsymbol{(y-3)^2 = 8(x+3)}$$
$$\boldsymbol{\rightarrow \quad V:(-3,3) \quad F:(-1,3)}$$
$$\boldsymbol{D: x = -5 \quad A: y = 3}$$

53. $-5x^2 + 10x - 20y + 15 = 0$

$$\boldsymbol{(x-1)^2 = -4(y-1)}$$
$$\boldsymbol{\downarrow \quad V:(1,1) \quad F:(1,0)}$$
$$\boldsymbol{D: y = 2 \quad A: x = 1}$$

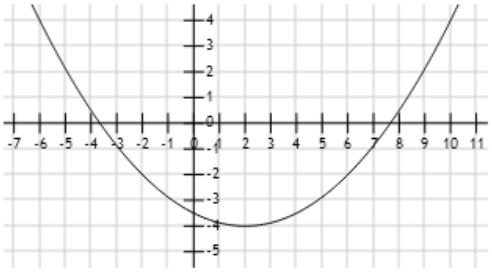
54. $-2y^2 - 12y - 20x - 78 = 0$

$$\boldsymbol{(y+3)^2 = -10(x+3)}$$
$$\boldsymbol{\leftarrow \quad V:(-3,-3) \quad F:(-5.5,-3)}$$
$$\boldsymbol{D: x = -0.5 \quad A: y = -3}$$

Graph each of the following. State the direction of the opening. Identify vertex and the focus and give the equations of the directrix and axis of symmetry.

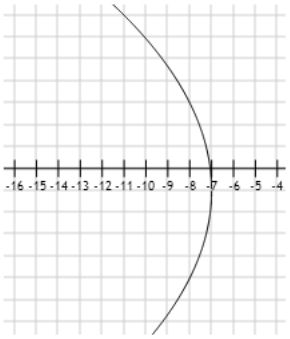
55. $(x-2)^2 = 8(y+4)$

$\uparrow$ $\boldsymbol{V}: (2,-4)$ $\boldsymbol{F}: (2,-2)$
$\boldsymbol{D}: y = -6$ $\boldsymbol{A}: x = 2$



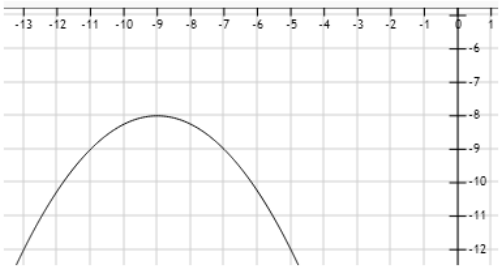
56. $(y+1)^2 = -16(x+7)$

$\leftarrow$ $\boldsymbol{V}: (-7,-1)$ $\boldsymbol{F}: (-11,-1)$
$\boldsymbol{D}: x = -3$ $\boldsymbol{A}: y = -1$



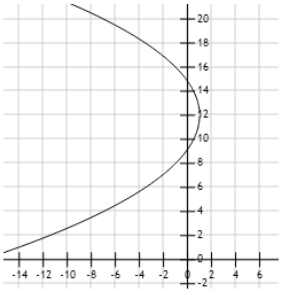
57. $(x+9)^2 = -4(y+8)$

$\downarrow$ $\boldsymbol{V}: (-9,-8)$ $\boldsymbol{F}: (-9,-9)$
$\boldsymbol{D}: y = -7$ $\boldsymbol{A}: x = -9$



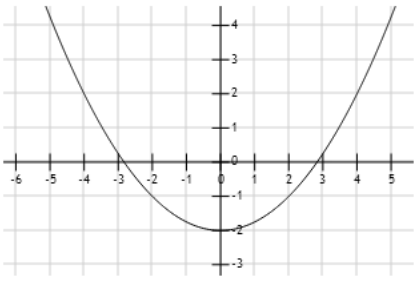
58. $y^2 - 24y + 8x + 136 = 0$

$(y-12)^2 = -8(x-1)$
$\leftarrow$ $\boldsymbol{V}: (1,12)$ $\boldsymbol{F}: (-1,12)$
$\boldsymbol{D}: x = 3$ $\boldsymbol{A}: y = 12$



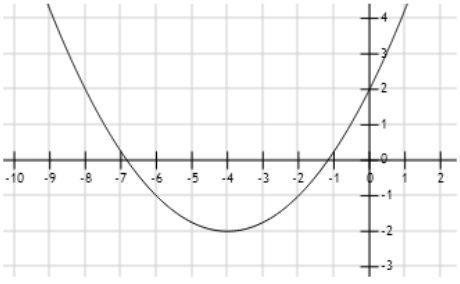
59. $x^2 - 4y - 8 = 0$

$x^2 = 4(y+2)$
$\uparrow$ $\boldsymbol{V}: (0,-2)$ $\boldsymbol{F}: (0,-1)$
$\boldsymbol{D}: y = -3$ $\boldsymbol{A}: x = 0$



60. $3x^2 + 24x - 12y + 24 = 0$

$(x+4)^2 = 4(y+2)$
$\uparrow$ $\boldsymbol{V}: (-4,-2)$ $\boldsymbol{F}: (-4,-1)$
$\boldsymbol{D}: y = -3$ $\boldsymbol{A}: x = -4$



Circles – Class Work

What are the center and the radius of the following circles?

61. $(x+2)^2+(y-4)^2=16$

$\boldsymbol{C:(-2,4)}$ $\boldsymbol{r:4}$

62. $(x-3)^2+(y-7)^2=25$

$\boldsymbol{C:(3,7)}$ $\boldsymbol{r:5}$

63. $(x)^2+(y+8)^2=1$

$\boldsymbol{C:(0,-8)}$ $\boldsymbol{r:1}$

64. $(x-7)^2+(y+1)^2=17$

$\boldsymbol{C:(7,-1)}$ $\boldsymbol{r:\sqrt{17}}$

65. $(x+6)^2+(y)^2=32$

$\boldsymbol{C:(-6,0)}$ $\boldsymbol{r:4\sqrt{2}}$

Write the standard form of the equation for the given information.

66. center (3,2) radius 6

$\boldsymbol{(x-3)^2+(y-2)^2=36}$

67. center (-4, -7) radius 8

$\boldsymbol{(x+4)^2+(y+7)^2=64}$

68. center (5, -9) radius 10

$\boldsymbol{(x-5)^2+(y+9)^2=100}$

69. center (-8, 0) diameter 14

$\boldsymbol{(x+8)^2+y^2=49}$

70. center (4,5) and point on the circle (3, -7)

$\boldsymbol{(x-4)^2+(y-5)^2=145}$

71. diameter with endpoints (6, 4) and (10, -8)

$\boldsymbol{(x-8)^2+(y+2)^2=40}$

72. center (4, 9) and tangent to the x-axis

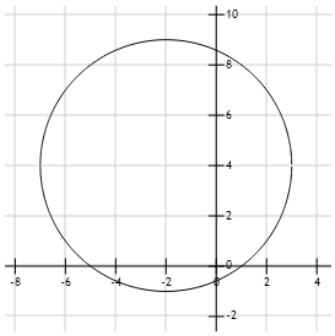
$\boldsymbol{(x-4)^2+(y-9)^2=81}$

Write the standard form of the equation, identify the Center and Radius, then graph.

73. $x^2+4x+y^2-8y=5$

$\boldsymbol{(x+2)^2+(y-4)^2=25}$

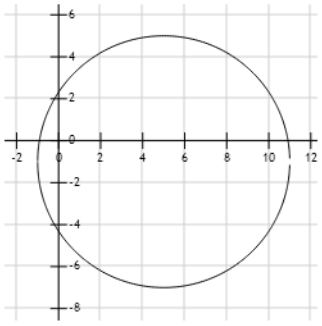
$\boldsymbol{C:(-2,4)}$ $\boldsymbol{r:5}$



74. $x^2-10x+y^2+2y=10$

$\boldsymbol{(x-5)^2+(y+1)^2=36}$

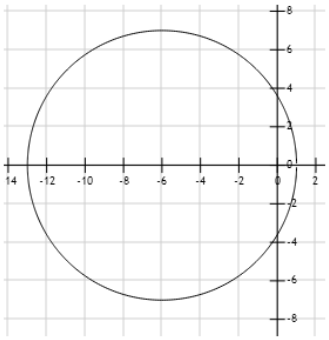
$\boldsymbol{C:(5,-1)}$ $\boldsymbol{r:6}$



75. $x^2+12x+y^2=13$

$\boldsymbol{(x+6)^2+y^2=49}$

$\boldsymbol{C:(-6,0)}$ $\boldsymbol{r:7}$



Circles – Homework

What are the center and the radius of the following circles?

76. $(x-9)^2+(y+5)^2=9$

$\boldsymbol{C:(9,-5)}$ $\boldsymbol{r:3}$

77. $(x+11)^2+(y-8)^2=64$

$\boldsymbol{C:(-11,8)}$ $\boldsymbol{r:8}$

78. $(x+13)^2+(y-3)^2=144$

$\boldsymbol{C:(-13,3)}$ $\boldsymbol{r:12}$

79. $(x-2)^2+(y)^2=19$

$\boldsymbol{C:(2,0)}$ $\boldsymbol{r:\sqrt{19}}$

80. $(x-6)^2+(y-15)^2=40$

$\boldsymbol{C:(6,15)}$ $\boldsymbol{r:2\sqrt{10}}$

Write the standard form of the equation for the given information.

81. center (-2, -4) radius 9

$\boldsymbol{(x+2)^2+(y+4)^2=81}$

82. center (-3, 3) radius 11

$\boldsymbol{(x+3)^2+(y-3)^2=121}$

83. center (5, 8) radius 12

$\boldsymbol{(x-5)^2+(y-8)^2=144}$

84. center (0 , 8) diameter 16

$\boldsymbol{x^2+(y-8)^2=64}$

85. center (-4,6) and point on the circle (-2, -8)

$\boldsymbol{(x+4)^2+(y-6)^2=200}$

86. diameter with endpoints (5, 14) and (11, -8)

$\boldsymbol{(x-8)^2+(y-3)^2=130}$

87. center (4, 9) and tangent to the y-axis

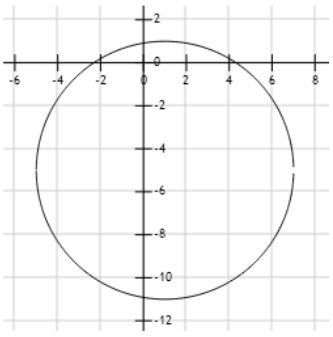
$\boldsymbol{(x-4)^2+(y-9)^2=16}$

Write the standard form of the equation, identify the Center and Radius, then graph.

88. $x^2-2x+y^2+10y=10$

$\boldsymbol{(x-1)^2+(y+5)^2=36}$

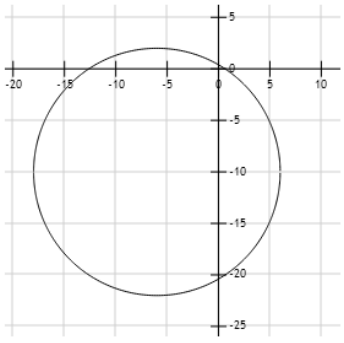
$\boldsymbol{C:(1,-5)}$ $\boldsymbol{r:6}$



89. $x^2+12x+y^2+20y=8$

$\boldsymbol{(x+6)^2+(y+10)^2=144}$

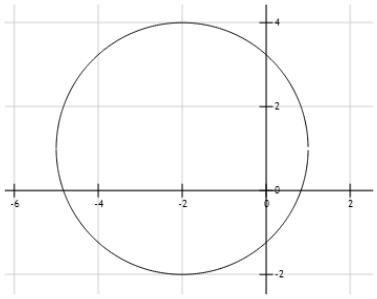
$\boldsymbol{C:(-6,-10)}$ $\boldsymbol{r:12}$



90. $4x^2+16x+4y^2-8y=16$

$\boldsymbol{(x+2)^2+(y-1)^2=9}$

$\boldsymbol{C:(-2,1)}$ $\boldsymbol{r:3}$



Ellipses – Class Work

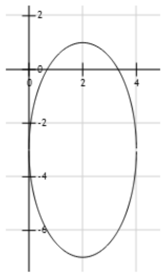
State whether the ellipse is vertical or horizontal, and the length of the major and minor axes. Identify the ellipse's center, vertices, and foci. Graph the ellipse.

91. $\frac{(x-2)^2}{4}+\frac{(y+3)^2}{16}=1$

$\boldsymbol{C: (2,-3)}$ ***Vertical*** ***Major*: 8** ***Minor*: 4**

$\boldsymbol{V: (2,1)(2,-7)(0,-3)\ (4,-3)}$

$\boldsymbol{F: (2,0.46)(2,-6.46)}$

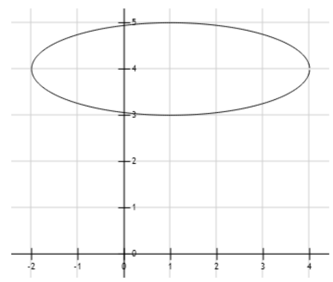


92. $\frac{(x-1)^2}{9}+\frac{(y-4)^2}{1}=1$

$\boldsymbol{C: (1,4)}$ ***Horizontal*** ***Major*: 6** ***Minor*: 2**

$\boldsymbol{V: (4,4)(-2,4)(1,5)\ (1,3)}$

$\boldsymbol{F: (3.83,4)(-1.83,4)}$

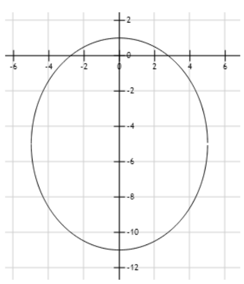


93. $\frac{(x)^2}{25}+\frac{(y+5)^2}{36}=1$

$\boldsymbol{C: (0,-5)}$ ***Vertical*** ***Major*: 12** ***Minor*: 10**

$\boldsymbol{V: (0,1)(0,-11)(5,-5)\ (-5,-5)}$

$\boldsymbol{F: (0,-1.68)(0,-8.32)}$

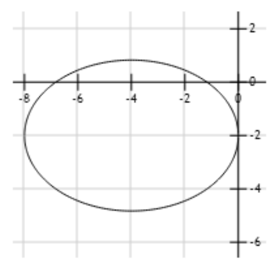


94. $\frac{(x+4)^2}{16}+\frac{(y+2)^2}{8}=1$

$\boldsymbol{C: (-4,-2)}$ ***Horizontal*** ***Major*: 8** ***Minor*: 5.66**

$\boldsymbol{V: (0,-2)(-8,-2)(-4,0.83)\ (-4,-4.83)}$

$\boldsymbol{F: (-1.17,-2)(-6.83,-2)}$

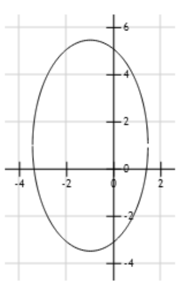


95. $\frac{(x+1)^2}{6}+\frac{(y-1)^2}{20}=1$

$\boldsymbol{C: (-1,1)}$ ***Vertical*** ***Major*: 8.94** ***Minor*: 4.9**

$\boldsymbol{V: (-1,5.47)(-1,-3.47)(1.45,1)\ (-3.45,1)}$

$\boldsymbol{F: (-1,4.74)(-1,-2.74)}$

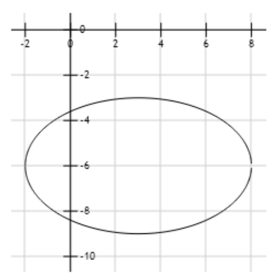


96. $\frac{(x-3)^2}{25}+\frac{(y+6)^2}{9}=1$

$\boldsymbol{C: (3,-6)}$ ***Horizontal*** ***Major*: 10** ***Minor*: 6**

$\boldsymbol{V: (8,-6)(-2,-6)(3,-3)\ (3,-9)}$

$\boldsymbol{F: (7,-6)(-1,-6)}$



Write the equation of the ellipse in standard form with the following properties.

97. $x^2 + 4x + 2y^2 - 8y = 20$

$$\frac{(x+2)^2}{32} + \frac{(y-2)^2}{16} = 1$$

98. $4x^2 - 8x + 3y^2 + 18y = 5$

$$\frac{(x-2)^2}{12} + \frac{(y+3)^2}{16} = 1$$

99. Center (1,4), a horizontal major axis of 10 and a minor axis of 6.

$$\frac{(x-1)^2}{25} + \frac{(y-4)^2}{9} = 1$$

100. Foci (2,5) and (2,11) with a minor axis of 10

$$\frac{(x-2)^2}{25} + \frac{(y-8)^2}{34} = 1$$

101. Foci (-2,4) and (-6,4) with a major axis of 18

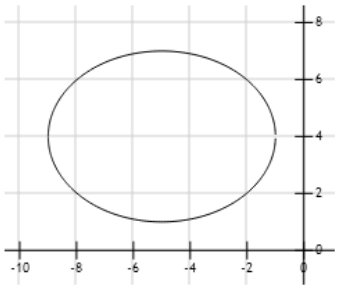
$$\frac{(x+4)^2}{81} + \frac{(y-4)^2}{77} = 1$$

Ellipses – Homework

State whether the ellipse is vertical or horizontal, and the length of the major and minor axes. Identify the ellipse's center, vertices, and foci. Graph the ellipse.

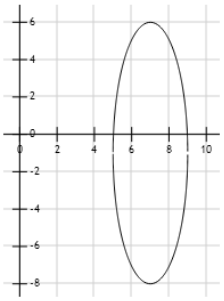
102. $\frac{(x+5)^2}{16}+\frac{(y-4)^2}{9}=1$

$\boldsymbol{C}:(-5,4)\ \boldsymbol{Horizontal\ Major}:8\ \boldsymbol{Minor}:6$
$\boldsymbol{V}:(-1,4)(-9,4)(-5,7)\ (-5,1)$
$\boldsymbol{F}:(-2.35,4)(-7.65,4)$



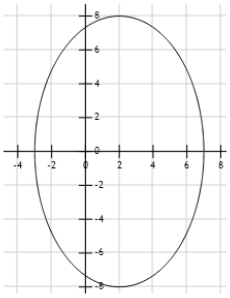
103. $\frac{(x-7)^2}{4}+\frac{(y+1)^2}{49}=1$

$\boldsymbol{C}:(7,-1)\ \ \boldsymbol{Vertical}\ \ \boldsymbol{Major}:14\ \ \boldsymbol{Minor}:4$
$\boldsymbol{V}:(7,6)(7,-8)(9,-1)\ (5,-1)$
$\boldsymbol{F}:(7,5.71)(7,-7.71)$



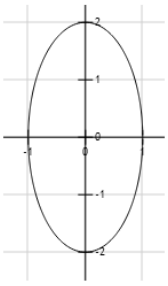
104. $\frac{(x-2)^2}{25}+\frac{(y)^2}{64}=1$

$\boldsymbol{C}:(2,0)\ \ \boldsymbol{Vertical}\ \ \boldsymbol{Major}:16\ \ \boldsymbol{Minor}:10$
$\boldsymbol{V}:(2,8)(2,-8)(7,0)\ (-3,0)$
$\boldsymbol{F}:(2,6.25)(2,-6.25)$



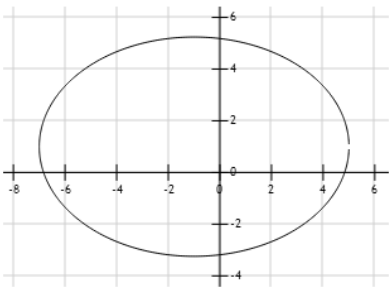
105. $\frac{(x)^2}{1}+\frac{(y)^2}{4}=1$

$\boldsymbol{C}:(0,0)\ \ \boldsymbol{Vertical}\ \ \boldsymbol{Major}:8\ \ \boldsymbol{Minor}:1$
$\boldsymbol{V}:(0,2)(0,-2)(1,0)\ (-1,0)$
$\boldsymbol{F}:(0,1.73)(0,-1.73)$



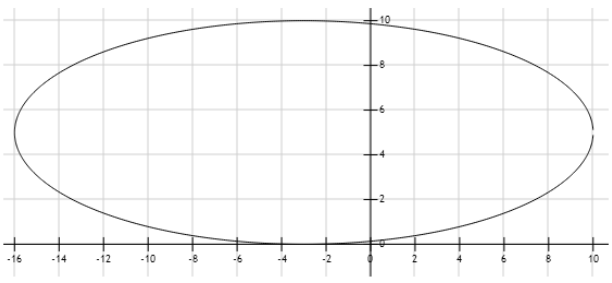
106. $\frac{(x+1)^2}{36}+\frac{(y-1)^2}{18}=1$

$\boldsymbol{C}:(-1,1)\ \boldsymbol{Horizontal\ Major}:12\ \boldsymbol{Minor}:8.49$
$\boldsymbol{V}:(5,1)(-7,1)(-1,5.24)\ (-1,-3.24)$
$\boldsymbol{F}:(-5.24,1)(3.24,1)$



107. $\frac{(x+3)^2}{169}+\frac{(y-5)^2}{25}=1$

$\boldsymbol{C}:(-3,5)\ \boldsymbol{Horizontal\ Major}:26\ \boldsymbol{Minor}:10$
$\boldsymbol{V}:(10,5)(-16,5)(-3,10)\ (-3,0)$
$\boldsymbol{F}:(9,5)(-15,5)$



Write the equation of the ellipse in standard form with the following properties.

108. $x^2 + 10x + 2y^2 - 12y = -1$

$$\frac{(x+5)^2}{42} + \frac{(y-3)^2}{21} = 1$$

109. $3x^2 - 12x + 4y^2 + 16y = 8$

$$\frac{(x-2)^2}{12} + \frac{(y+2)^2}{9} = 1$$

110. Center (-1,2), a vertical major axis of 8 and a minor axis of 4.

$$\frac{(x+1)^2}{4} + \frac{(y-2)^2}{16} = 1$$

111. Foci (3, 5) and (3,11) with a minor axis of 8

$$\frac{(x-3)^2}{16} + \frac{(y-8)^2}{25} = 1$$

112. Foci (-2, 6) and (-8, 6) with a major axis of 14

$$\frac{(x+5)^2}{49} + \frac{(y-6)^2}{40} = 1$$

Hyperbolas – Class Work

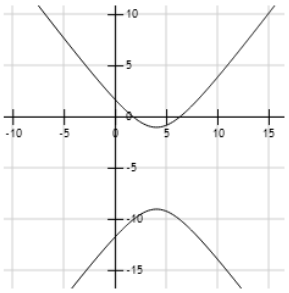
State whether the hyperbola is vertical or horizontal, identify the center, vertices, foci, and the slopes of the asymptotes. Graph the hyperbola.

113. $\frac{(y+5)^2}{16} - \frac{(x-4)^2}{9} = 1$

$\boldsymbol{Vert}\quad \boldsymbol{C}: (\mathbf{4}, -\mathbf{5})\qquad \boldsymbol{Asy}: \pm\frac{\mathbf{4}}{\mathbf{3}}$

$\boldsymbol{V}: (\mathbf{4}, -\mathbf{1})(\mathbf{4}, -\mathbf{9})$

$\boldsymbol{F}: (\mathbf{4}, \mathbf{0})(\mathbf{4}, -\mathbf{10})$

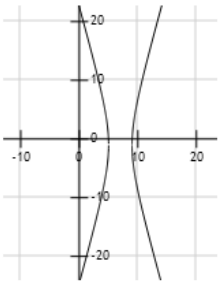


114. $\frac{(x-7)^2}{4} - \frac{(y+1)^2}{49} = 1$

$\boldsymbol{Horiz}\quad \boldsymbol{C}: (\mathbf{7}, -\mathbf{1})\qquad \boldsymbol{Asy}: \pm\frac{\mathbf{7}}{\mathbf{2}}$

$\boldsymbol{V}: (\mathbf{9}, -\mathbf{1})(\mathbf{5}, -\mathbf{1})$

$\boldsymbol{F}: (\mathbf{14.28}, -\mathbf{1})(-\mathbf{0.28}, -\mathbf{1})$

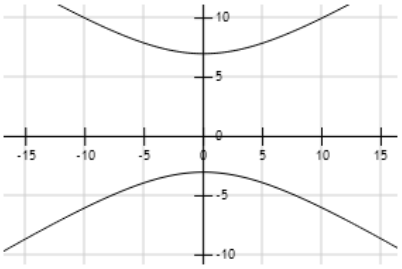


115. $\frac{(y-2)^2}{25} - \frac{(x)^2}{64} = 1$

$\boldsymbol{Vert}\quad \boldsymbol{C}: (\mathbf{0}, \mathbf{2})\qquad \boldsymbol{Asy}: \pm\frac{\mathbf{5}}{\mathbf{8}}$

$\boldsymbol{V}: (\mathbf{0}, \mathbf{7})(\mathbf{0}, -\mathbf{3})$

$\boldsymbol{F}: (\mathbf{0}, \mathbf{11.43})(\mathbf{0}, -\mathbf{7.43})$

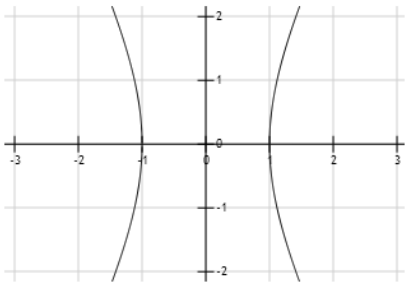


116. $\frac{(x)^2}{1} - \frac{(y)^2}{4} = 1$

$\boldsymbol{Horiz}\quad \boldsymbol{C}: (\mathbf{0}, \mathbf{0})\qquad \boldsymbol{Asy}: \pm\mathbf{2}$

$\boldsymbol{V}: (\mathbf{1}, \mathbf{0})(-\mathbf{1}, \mathbf{0})$

$\boldsymbol{F}: (\mathbf{5.24}, \mathbf{0})(-\mathbf{5.24}, \mathbf{0})$

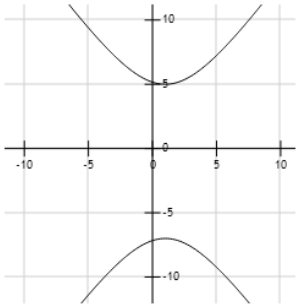


117. $\frac{(y+1)^2}{36} - \frac{(x-1)^2}{18} = 1$

$\boldsymbol{Vert}\quad \boldsymbol{C}: (\mathbf{1}, -\mathbf{1})\qquad \boldsymbol{Asy}: \pm\sqrt{\mathbf{2}}$

$\boldsymbol{V}: (\mathbf{1}, \mathbf{5})(\mathbf{1}, -\mathbf{7})$

$\boldsymbol{F}: (\mathbf{1}, \mathbf{6.35})(\mathbf{1}, -\mathbf{8.35})$



Write the equation of the hyperbola in standard form.

118. $x^2 + 4x - 2y^2 - 8y = 20$

$$\frac{(x+2)^2}{16} - \frac{(y+2)^2}{8} = 1$$

119. $3y^2 + 18y - 4x^2 - 8x = 1$

$$\frac{(y+3)^2}{8} - \frac{(x+1)^2}{6} = 1$$

120. Opens horizontally, with center (3,7) and asymptotes with slope $m = \pm\frac{2}{5}$

$$\frac{(x-3)^2}{25} - \frac{(y-7)^2}{4} = 1$$

121. Opens vertically, with asymptotes $y = \frac{3}{2}x + 8$ and $y = -\frac{3}{2}x - 4$

$$\frac{(y-2)^2}{9} - \frac{(x+4)^2}{4} = 1$$

Hyperbolas – Homework

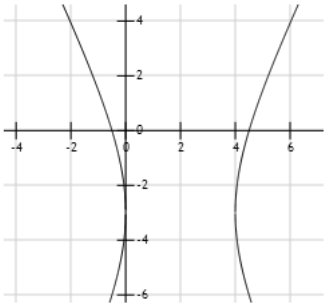
State whether the hyperbola is vertical or horizontal, identify the center, vertices, foci, and the slopes of the asymptotes. Graph the hyperbola.

122. $\frac{(x-2)^2}{4} - \frac{(y+3)^2}{16} = 1$

$\boldsymbol{Horiz}\ \ \boldsymbol{C}: (\mathbf{2}, -\mathbf{3})\quad \boldsymbol{Asy}: \pm\mathbf{2}$

$\boldsymbol{V}: (\mathbf{4}, -\mathbf{3})(\mathbf{0}, -\mathbf{3})$

$\boldsymbol{F}: (\mathbf{6.47}, -\mathbf{3})(-\mathbf{2.47}, -\mathbf{3})$

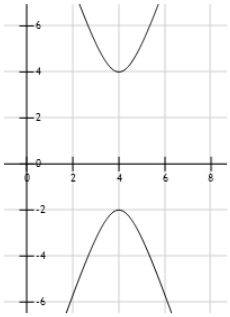


123. $\frac{(y-1)^2}{9} - \frac{(x-4)^2}{1} = 1$

$\boldsymbol{Vert}\ \ \boldsymbol{C}: (\mathbf{4}, \mathbf{1})\quad \boldsymbol{Asy}: \pm\mathbf{3}$

$\boldsymbol{V}: (\mathbf{4}, \mathbf{4})(\mathbf{4}, -\mathbf{2})$

$\boldsymbol{F}: (\mathbf{4}, \mathbf{4.16})(\mathbf{4}, -\mathbf{2.16})$

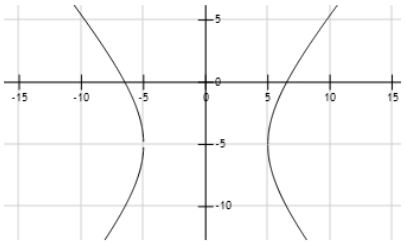


124. $\frac{(x)^2}{25} - \frac{(y+5)^2}{36} = 1$

$\boldsymbol{Horiz}\ \ \boldsymbol{C}: (\mathbf{0}, -\mathbf{5})\quad \boldsymbol{Asy}: \pm\frac{\mathbf{6}}{\mathbf{5}}$

$\boldsymbol{V}: (\mathbf{5}, -\mathbf{5})(-\mathbf{5}, -\mathbf{5})$

$\boldsymbol{F}: (\mathbf{7.81}, -\mathbf{5})(-\mathbf{7.81}, -\mathbf{5})$

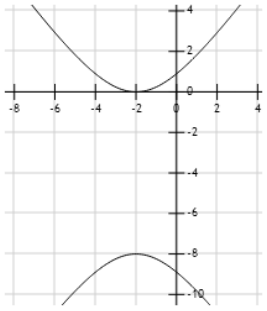


125. $\frac{(y+4)^2}{16} - \frac{(x+2)^2}{8} = 1$

$\boldsymbol{Vert}\ \ \boldsymbol{C}: (-\mathbf{2}, -\mathbf{4})\quad \boldsymbol{Asy}: \pm\sqrt{\mathbf{2}}$

$\boldsymbol{V}: (-\mathbf{2}, \mathbf{0})(-\mathbf{2}, -\mathbf{8})$

$\boldsymbol{F}: (-\mathbf{2}, \mathbf{0.9})(-\mathbf{2}, -\mathbf{8.9})$

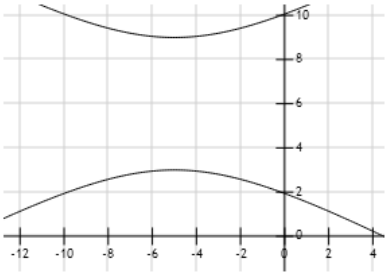


126. $\frac{(y-6)^2}{9} - \frac{(x+5)^2}{30} = 1$

$\boldsymbol{Vert}\ \ \boldsymbol{C}: (-\mathbf{5}, \mathbf{6})\quad \boldsymbol{Asy}: \pm\frac{\sqrt{\mathbf{30}}}{\mathbf{10}}$

$\boldsymbol{V}: (-\mathbf{5}, \mathbf{9})(-\mathbf{5}, \mathbf{3})$

$\boldsymbol{F}: (-\mathbf{5}, \mathbf{12.25})(-\mathbf{5}, -\mathbf{0.25})$



Write the equation of the hyperbola in standard form.

127. $4y^2 - 24y - 5x^2 + 20x = 4$

$$\frac{(y-3)^2}{5} - \frac{(x-2)^2}{4} = 1$$

128. $6y^2 + 36y - x^2 - 14x = 1$

$$\frac{(y+3)^2}{1} - \frac{(x+7)^2}{6} = 1$$

129. Opens vertically, with center (-4,1) and asymptotes with slope $m = \pm\frac{3}{7}$

$$\frac{(y-1)^2}{9} - \frac{(x+4)^2}{49} = 1$$

130. Opens horizontally, with asymptotes $y = \frac{4}{9}x + 10$ and $y = -\frac{4}{9}x - 14$

$$\frac{(x+27)^2}{16} - \frac{(y+2)^2}{81} = 1$$

Recognizing Conic Sections from the General Form – Class Work

Identify the conic section and write the equation in standard form. State all pertinent information.

131. $y^2 + 6y + x^2 + 10x = 15$

Circle

$(y+3)^2 + (x+5)^2 = 49$

$C: (-5, -3)$

$r = 7$

132. $y^2 + 8y - x^2 + 12x = 24$

Vertical Hyperbola

$\frac{(y+4)^2}{4} - \frac{(x-6)^2}{4} = 1$

$C: (6, -4)$

$V: (6, -2)(6, -6)$

$F: (6, -1.17)(6, -6.83)$

$Asy: \pm 1$

133. $4y^2 + 16y + 3x^2 - 18x = 5$

Horizontal Ellipse

$\frac{(y+2)^2}{12} + \frac{(x-3)^2}{16} = 1$

$C: (3, -2)$

$Major: 8$

$Minor: 6.92$

$V: (7, -2)(-1, -2)$

$(3, 1.46)(3, -5.46)$

$F: (5, -2)(1, -2)$

134. $y^2 + 2y - x^2 + 8x = y^2 + 12$

Vertical Parabola

$(x-2)^2 = -2(y-4)$

$\downarrow \quad V: (2, 4)$

$F: (2, 3.5)$

$D: y = 4.5$

$A: x = 2$

135. $2x^2 - 20x + 2y^2 + 16y = -10$

Circle

$(x-5)^2 + (y+4)^2 = 36$

$C: (5, -4)$

$r = 6$

136. $4x^2 - 24x - 2y^2 + 8y = -4$

Horizontal Hyperbola

$\frac{(x-3)^2}{6} - \frac{(y-2)^2}{12} = 1$

$C: (3, 2)$

$V: (5.45, 2)(0.55, 2)$

$F: (7.24, 2)(-1.24, 2)$

$Asy: \pm\sqrt{2}$

Recognizing Conic Sections from the General Form – Homework

Identify the conic section and write the equation in standard form. State all pertinent information.

137. $4y^2 + 8y + 2x^2 + 12x = 10$

Horizontal Ellipse

$\frac{(y+1)^2}{8} + \frac{(x+3)^2}{16} = 1$

$C: (-3, -1)$

$Major: 8$

$Minor: 5.66$

$V: (1, -1)(-7, -1)$

$(-3, 1.83)(-3, -3.83)$

$F: (5, -2)(1, -2)$

138. $y^2 + 2y - x^2 + 8x = 16$

Vertical Hyperbola

$\frac{(y+1)^2}{1} - \frac{(x-4)^2}{1} = 1$

$C: (4, -1)$

$V: (4, 0)(4, -2)$

$F: (4, 0.41)(4, -2.41)$

$Asy: \pm 1$

139. $4y^2 + 16y + 4x^2 - 24x = 12$

Circle

$(y + 2)^2 + (x - 3)^2 = 16$

$C: (3, -2)$

$r = 4$

140. $y^2 + 2y + x^2 + 12x = 2y^2 + 12$

Horizontal Parabola

$(y + 1)^2 = -12(x - 1)$

$\leftarrow \quad V: (1, -1)$

$F: (-2, -1)$

$D: x = 4$

$A: y = -1$

141. $x^2 - 20x - 2y^2 + 16y = -6$

Horizontal Hyperbola

$\frac{(x-10)^2}{64} - \frac{(y-4)^2}{32} = 1$

$C: (10, 4)$

$V: (18, 4)(2, 4)$

$F: (19.8, 4)(0.2, 4)$

$Asy: \pm \frac{\sqrt{2}}{2}$

142. $6x^2 - 24x + 4y^2 + 8y = -4$

Vertical Ellipse

$\frac{(x-2)^2}{4} + \frac{(y+1)^2}{6} = 1$

$C: (2, -1)$

$Major: 4.9$

$Minor: 4$

$V: (2, 1.45)(2, -3.45)$

$(4, -1)(0, -1)$

$F: (2, 0.41)(2, -2.41)$

Unit Review - Multiple Choice

1. The distance from A(2,y) to B(-1,7) is 5. Find y.
   a. 3
   b. 4
   c. 11
   d. A and C

   D

2. M is the midpoint of EF. Find F given E(3,4) and M(5, -2).
   a. (4,1)
   b. (4,3)
   c. (7,-8)
   d. (1,10)

   C

3. What is the vertex of the parabola $(y-9)^2 = -4(x-2)$
   a. (9,-2)
   b. (-2,2)
   c. (2,-2)
   d. (2,9)

   D

4. Write the following equations in standard form $2y^2 + 12y - x + 2 = 0$
   a. $(y+6)^2 = \frac{1}{2}(x-2)$
   b. $(y+3)^2 = \frac{1}{2}(x+7)$
   c. $(y+3)^2 = \frac{1}{2}(x+10)$
   d. $(y+3)^2 = \frac{1}{2}(x+16)$

   D

5. Identify the focus of $(y-3)^2 - 8(x-2)$
   a. F(0,3)
   b. F(4,3)
   c. F(2,1)
   d. F(2,5)

   A

6. Write the equations of the directrix and axis of symmetry of a parabola with vertex (4,-2) and focus (4,4).
   a. Directrix: y= -8; Axis of Symmetry: x=4
   b. Directrix: y= -10; Axis of Symmetry: x=4
   c. Directrix: x= -8; Axis of Symmetry: y=4
   d. Directrix: x= -10; Axis of Symmetry: y=4

   A

7. Write the equation of the parabola with vertex (4,-2) and focus (4,4).
   a. $(x-4)^2 = 16(y+2)$
   b. $(x-4)^2 = 8(y+2)$
   c. $(x-4)^2 = 24(y+2)$
   d. $(y+2)^2 = 12(x-4)$

   C

8. What are the center and the radius of the following circle: $(x-7)^2 + (y+6)^2 = 4$
   a. (-7,6); r=4
   b. (7,-6); r=16
   c. (-7,6); r= 8
   d. (7,-6); r= 2

   D

9. Write the equation of the circle with a diameter with endpoints (6, 12) and (16, -8).
   a. $(x-11)^2 + (y-6)^2 = 125$
   b. $(x-11)^2 + (y+6)^2 = 11.2$
   c. $(x-11)^2 + (y-2)^2 = 125$
   d. $(x-11)^2 + (y-2)^2 = 11.2$

   C

10. Identify the ellipse's center and foci: $\frac{(x+4)^2}{16}+\frac{(y-1)^2}{36}=1$
    a. C(-4,1); Foci: $(-4\pm4\sqrt{5},1)$
    b. C(4,-1); Foci: $(4\pm4\sqrt{5},-1)$
    c. C(-4,1); Foci: $(-4,1\pm4\sqrt{5})$
    d. C(4,-1); Foci: $(4,1\pm4\sqrt{5})$

C

11. State the length of the major and minor axes of $\frac{(x+4)^2}{16}+\frac{(y-1)^2}{36}=1$
    a. Major: 4; Minor: 6
    b. Major: 6; Minor: 4
    c. Major: 36; Minor: 16
    d. Major: 12; Minor: 8

D

12. Write the equation in standard form $4y^2-24y-2x^2+20x=22$
    a. $\frac{(y-3)^2}{2}-\frac{(x-5)^2}{4}=1$
    b. $\frac{(y-3)^2}{2}-\frac{(x+5)^2}{4}=1$
    c. $\frac{(y-3)^2}{27}-\frac{(x-5)^2}{54}=1$
    d. $\frac{(y-3)^2}{27}-\frac{(x+5)^2}{54}=1$

A

13. What is the slope of the asymptotes for the hyperbola $\frac{(y+4)^2}{16}-\frac{(x+2)^2}{8}=1$
    a. $y=\pm2$
    b. $y=\pm\frac{1}{2}$
    c. $y=\pm\frac{\sqrt{2}}{2}$
    d. $y=\pm\sqrt{2}$

D

14. Write the equation in standard form $x^2+12x+3y^2-12y=-3$
    a. $(x+6)^2+3(y-2)^2=45$
    b. $\frac{(x+6)^2}{45}+\frac{(y-2)^2}{15}=1$
    c. $(x+6)^2+3(y-2)^2=23$
    d. $\frac{(x+6)^2}{23}+\frac{3(y-2)^2}{23}=1$

B

15. Identify the type of conic section: $y^2-4y-x^2+6x=12$
    a. Circle
    b. Ellipse
    c. Hyperbola
    d. Parabola

D

16. Identify the type of conic section. $4y^2+16y+4x^2-24x=12$
    a. Hyperbola
    b. Circle
    c. Parabola
    d. Ellipse

B

Short Answer

Identify the conic section, graph, and write in standard form. State all pertinent information: (Parabolas – direction, vertex, focus, directrix, axis of symmetry; Circles – center, radius; Ellipse – direction, center, vertices, foci, major axis, minor axis; Hyperbola – direction, center, vertices, foci, slope of asymptotes)

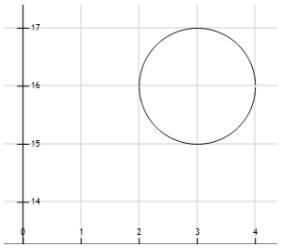
1. $x^2 - 6x + y^2 - 32y = -264$

Circle

$(x-3)^2 + (y-16)^2 = 1$

$C: (3, 16)$

$r = 1$



2. $y^2 - 6y - 4x + 5 = 0$

Parabola - Right

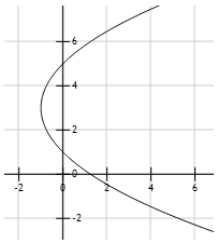
$(y-3)^2 = 4(x+1)$

$V: (-1, 3)$

$F: \left(-\frac{3}{4}, 3\right)$

$D: x = -1\frac{1}{4}$

$A: y = 3$



3. $16y^2 - 32y - 9x^2 - 18x = 137$

Hyperbola - Vertical

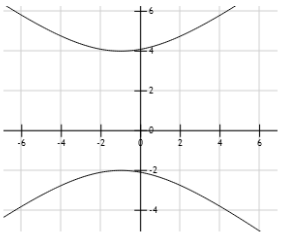
$\frac{(y-1)^2}{9} - \frac{(x+1)^2}{16} = 1$

$C: (1, -1)$

$V: (-1, 4)(-1, -2)$

$F: (-1, 6)(-1, -4)$

$A: \pm\frac{3}{4}$



4. $4x^2 - 24x + 49y^2 - 882y = -3809$

Ellipse - Horizontal

$\frac{(x-3)^2}{49} + \frac{(y-9)^2}{4} = 1$

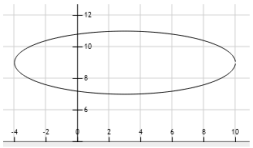
$C: (3, 9)$

$V: (10, 9)(-4, 9)(3, 7)(3, 11)$

$F: (10.28, 9)(-4.28, 9)$

$Major: 14$

$Minor: 4$



5. $x^2 - 6x + 8y - 31 = 0$

Parabola - Down

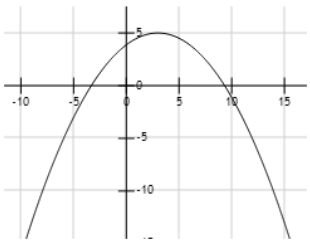
$(x-3)^2 = -8(y-5)$

$V: (3, 5)$

$F: (3, 3)$

$D: y = 7$

$A: x = 3$



6. $81x^2 - 1296x + 16y^2 + 64y = -3952$

Ellipse - Vertical

$\frac{(x-4)^2}{16} + \frac{(y+2)^2}{81} = 1$

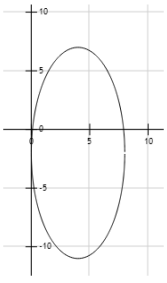
$C: (4, -2)$

$V: (4, 7)(4, -11)(8, -2)(0, -2)$

$F: (4, 7.85)(4, -11.85)$

$Major: 18$

$Minor: 8$



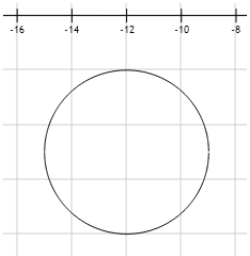
7. $x^2 + 24x + y^2 + 10y = -160$

Circle

$(x+12)^2 + (y+5)^2 = 9$

$C: (-12, -5)$

$r = 3$



8. $4x^2 - 25y^2 + 100y = 200$

Hyperbola - Horizc

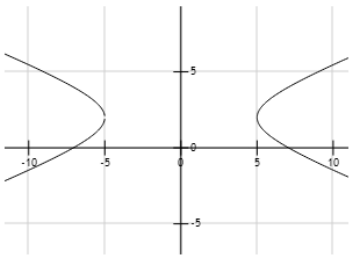
$\frac{x^2}{25} - \frac{(y-2)^2}{4} = 1$

$C: (0, 2)$

$V: (5, 2)(-5, 2)$

$F: (5.4, 2)(-5.4, 2)$

$A: \pm\frac{2}{5}$



Extended Response

1. A parabola has vertex (3, 4) and focus (4, 4)
   a. What direction does the parabola open?

   Right

   b. What are the equations of the axis of symmetry and the directrix?

   $$\boldsymbol{D: x = 2}$$
   $$\boldsymbol{A: y = 4}$$

   c. Write the equation of the parabola.

   $$\boldsymbol{(y-4)^2 = 4(x-3)}$$

2. Consider a circle and a parabola.
   a. At how many points can they intersect?

   $$\boldsymbol{0, 1, 2, 3, or\ 4}$$

   b. If the circle has equation $x^2 + y^2 = 4$ and the parabola has equation $y = x^2$, what are the point(s) of intersection?

   $$\boldsymbol{(1.25, 1.56)\ and\ (-1.25, 1.56)}$$

   c. If the parabola were reflected over the x-axis, what would be the point(s) of intersection?

   $$\boldsymbol{(1.25, -1.56)\ and\ (-1.25, -1.56)}$$