Progressive Mathematics Initiative®

This material is made freely available at www.njctl.org and is intended for the non-commercial use of students and teachers. It may not be used for any commercial purpose without the written permission of NJCTL.

We, at the New Jersey Education Association, are proud founders and supporters of NJCTL, an independent non-profit organization with the mission of empowering teachers to lead school improvement for the benefit of all students.
Table of Contents

Polygons
Properties of Parallelograms
Proving Quadrilaterals are Parallelograms
Rhombi, Rectangles and Squares
Trapezoids
Kites
Constructing Quadrilaterals
Families of Quadrilaterals
Proofs
Coordinate Proofs
PARCC Released Questions & Standards
Polygons
Polygons

A **polygon** is a closed figure made of line segments connected end to end at vertices.

Since it is made of line segments, none its sides are curved.

The sides of simple polygons intersect each other at the common endpoints and do not extend beyond the endpoints.

The sides of complex polygons intersect each other and extend to intersect with other sides.

However, we will only be considering simple polygons in this course and will refer to them as "polygons."

Would a circle be considered a polygon? Why or why not?
This course will only consider simple polygons, which we will refer to as "polygons."
**Types of Polygons**

Polygons are named by their number of sides, angles or vertices; which are all equal in number.

<table>
<thead>
<tr>
<th>Number of Sides, Angles or Vertices</th>
<th>Type of Polygon</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>triangle</td>
</tr>
<tr>
<td>4</td>
<td>quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td>pentagon</td>
</tr>
<tr>
<td>6</td>
<td>hexagon</td>
</tr>
<tr>
<td>7</td>
<td>heptagon</td>
</tr>
<tr>
<td>8</td>
<td>octagon</td>
</tr>
<tr>
<td>9</td>
<td>nonagon</td>
</tr>
<tr>
<td>10</td>
<td>decagon</td>
</tr>
<tr>
<td>11</td>
<td>11-gon</td>
</tr>
<tr>
<td>12</td>
<td>dodecagon</td>
</tr>
<tr>
<td>n</td>
<td>n-gon</td>
</tr>
</tbody>
</table>
Convex or Concave Polygons

Convex Polygons

All interior angles are less than 180°.

No side, if extended, will cross into the interior.
Convex or Concave Polygons

Concave Polygons

At least one interior angle is at least 180°.

At least two sides, if extended, will cross into the interior.

A way to remember the difference is that concave shapes "cave in".
1. The figure below is a concave polygon.

   True

   False
2 Identify the polygon.

A Pentagon
B Octagon
C Quadrilateral
D Hexagon
E Decagon
F Triangle
3. Is the polygon convex or concave?

A. Convex
B. Concave
4 Is the polygon convex or concave?

A Convex
B Concave
Equilateral Polygons

A polygon is equilateral if all its sides are congruent.
A polygon is equiangular if all its angles are congruent.
Regular Polygons

A polygon is regular if it is equilateral and equiangular.
5. Describe the polygon. (Choose all that apply)

A. Pentagon
B. Octagon
C. Quadrilateral
D. Hexagon
E. Triangle
F. Convex
G. Concave
H. Equilateral
I. Equiangular
J. Regular

Answer: E, F, H, I, J
6. Describe the polygon. (Choose all that apply)

A. Pentagon
B. Octagon
C. Quadrilateral
D. Hexagon
E. Triangle
F. Convex
G. Concave
H. Equilateral
I. Equiangular
J. Regular
7 Describe the polygon. (Choose all that apply)

A  Pentagon
B  Octagon
C  Quadrilateral
D  Hexagon
E  Triangle
F  Convex
G  Concave
H  Equilateral
I  Equiangular
J  Regular

Answer: C, F, I
Angle Measures of Polygons

In earlier units, we spent a lot of time studying one specific polygon: the triangle.

We won't have to spend that same amount of time on each new polygon we study.

Instead, we will be able to use what we learned about triangles to save a lot of time and effort.
Angle Measures of Polygons

Every polygon is comprised of a number of triangles. The sum of the interior angles of each triangle is 180°. A triangle is (obviously) comprised of one triangle, and therefore the sum of its interior angles is 180°.

What is the sum of the interior angles of this polygon?
Angle Measures of Polygons

Every polygon can be thought of as consisting of some number of triangles.

A quadrilateral is comprised of two triangles.

What is the sum of the interior angles of a quadrilateral?
Angle Measures of Polygons

Every polygon can be thought of as consisting of some number of triangles.

A pentagon is comprised of three triangles.

What is the sum of the interior angles of a pentagon?
Angle Measures of Polygons

Every polygon can be thought of as consisting of some number of triangles.

An octagon is comprised of six triangles.

What is the sum of the interior angles of an octagon?
Angle Measures of Polygons

Fill in the final cell of this table with an expression for the number of triangular regions of a polygon based on the number of its sides.

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Number of triangular regions</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>n</td>
<td></td>
</tr>
</tbody>
</table>
Angle Measures of Polygons

The sum of the interior angles of a triangle is 180 degrees. This will also be true of each triangular region of a polygon. Using that information, complete this table.

<table>
<thead>
<tr>
<th>Number of sides</th>
<th>Number of triangular regions</th>
<th>Sum of Interior Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>180°</td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td>360°</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>540°</td>
</tr>
<tr>
<td>6</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8 What is the sum of the interior angles of this polygon?
9 How many triangular regions can be drawn in this polygon?
10 What is the sum of the interior angles of this polygon?
11 How many triangular regions can be drawn in this polygon?
12 What is the sum of the interior angles of this polygon?
13 Which of the following road signs has interior angles that add up to 360°?

A  Yield Sign
B  No Passing Sign
C  National Forest Sign
D  Stop Sign
E  No Crossing Sign
The sum of the measures of the interior angles of a **convex** polygon with \( n \) sides is 180(n-2).

Complete the table.

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Number of Sides</th>
<th>Sum of the measures of the interior angles.</th>
</tr>
</thead>
<tbody>
<tr>
<td>hexagon</td>
<td>6</td>
<td>180(6-2) = 720°</td>
</tr>
<tr>
<td>heptagon</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>octagon</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>nonagon</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>decagon</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>11-gon</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>dodecagon</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>
14 What is the measurement of each angle in the diagram below?

\[
\begin{align*}
\text{L} & \quad (3x+4)^\circ \\
\text{M} & \quad 146^\circ \\
\text{P} & \quad (2x+3)^\circ \\
\text{N} & \quad x^\circ \\
\text{P} & \quad (3x)^\circ
\end{align*}
\]

Answer

\[
9x + 153 = 540
\]

\[
x = 43
\]
Interior Angles of Equiangular Polygons

Regular polygons are equiangular: their angles are equal.

So, it must be true that each angle must be equal to the sum of the interior angles divided by the number of angles.

\[
m\angle = \frac{\text{Sum of the Angles}}{\text{Number of Angles}}
\]

\[
m\angle = \frac{(n - 2)180^\circ}{n}
\]
The measure of *each interior angle* of an regular polygon is given by:

$$ m \angle = \frac{(n-2)(180^\circ)}{n} $$

<table>
<thead>
<tr>
<th>regular polygon</th>
<th>number of sides</th>
<th>sum of interior angles</th>
<th>measure of each angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
<td>3</td>
<td>180°</td>
<td></td>
</tr>
<tr>
<td>quadrilateral</td>
<td>4</td>
<td>360°</td>
<td></td>
</tr>
<tr>
<td>pentagon</td>
<td>5</td>
<td>540°</td>
<td></td>
</tr>
<tr>
<td>hexagon</td>
<td>6</td>
<td>720°</td>
<td></td>
</tr>
<tr>
<td>octagon</td>
<td>8</td>
<td>1080°</td>
<td></td>
</tr>
<tr>
<td>decagon</td>
<td>10</td>
<td>1440°</td>
<td></td>
</tr>
</tbody>
</table>

**Interior Angles of Regular Polygons**
15 If the stop sign is a regular octagon, what is the measure of each interior angle?
16. What is the sum of the measures of the interior angles of a convex 20-gon?

A. 2880°
B. 3060°
C. 3240°
D. 3420°
17 What is the measure of each interior angle of a regular 20-gon?

A 162°
B 3240°
C 180°
D 60°
18. What is the measure of each interior angle of a regular 16-gon?

A. 2520°
B. 2880°
C. 3240°
D. 157.5°
19 What is the value of x?
Polygon Exterior Angle Theorem

The sum of the measures of the exterior angles of a convex polygon, *one at each vertex*, is 360°.

In this case, \( \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5 = 360° \)
Proof of Exterior Angles Theorem

**Given:** Any polygon has \( n \) exterior angles, where \( n \) is the number of its sides, interior angles, or vertices

**Prove:** The sum of those exterior angles is 360°

We will prove this for any polygon, not for a specific polygon.

To do that, it will help if we introduce one piece of notation: the Greek letter Sigma, \( \Sigma \).

\( \Sigma \) represents the phrase "the sum of." So, the phrase,

"the sum of the interior angles" becomes "\( \Sigma \) interior \( \angle \)s."
Proof of Exterior Angles Theorem

**Given:** Any polygon has $n$ interior angles, where $n$ is the number of its sides, interior angles, or vertices

**Prove:** $\sum$ external $\angle s = 360^\circ$

This proof is based on the fact that each exterior angle is supplementary to an interior angle.

And, that the sum of the measures of the interior angle of a polygon with $n$ sides is $(n-2) \times 180^\circ$,

Or, with our Sigma notation: $\Sigma$ internal $\angle s = (n-2) \times 180^\circ$
External angles are formed by extending one side of a polygon.

Take a look at $\angle 1$ in the diagram.

The external and internal angles at a vertex form a linear pair.

What is the sum of their measures?
Polygon Exterior Angle Theorem

Any polygon with $n$ sides has $n$ vertices and $n$ internal $\angle$s.

That also means if we include one external $\angle$ at each vertex, that there are $n$ linear pairs.

Since 1 linear pair has a measure of 180°, then $n$ of them must have a measure of $n \times 180°$.

The sum of the measures of a polygon's linear pairs of internal and external $\angle$s must be $n \times 180°$. 
Each linear pair includes an external $\angle$ and an internal $\angle$. Adding up all of those means that:

The sum of all the external and internal $\angle$s of a polygon yields a measure of $n \times 180^\circ$.

Symbolically:

$\Sigma$ external $\angle$s  + $\Sigma$ internal $\angle$s  = $n \times 180^\circ$
Polygon Exterior Angle Theorem

\[ \Sigma \text{external} \angle s + \Sigma \text{internal} \angle s = n \times 180^\circ \]

But recall that the internal angle theorem tell us that:

\[ \Sigma \text{internal} \angle s = (n-2) \times 180^\circ \]

Substituting that in yields

\[ \Sigma \text{external} \angle s + [(n-2) \times 180^\circ] = n \times 180^\circ \]

Multiplying out the second term yields:
Polygon Exterior Angle Theorem

\[ \Sigma \text{ external } \angle s \ + [n \times 180^\circ - 2 \times 180^\circ] = n \times 180^\circ \]

Subtracting \( n \times 180^\circ - 2 \times 180^\circ \) from both sides yields

\[ \Sigma \text{ external } \angle s = 2 \times 180^\circ \]

\[ \Sigma \text{ external } \angle s = 360^\circ \]

Notice that \( n \) is not in this formula, so this result holds for all polygons with any number of sides.

This is an important, and surprising, result.
**Proof of Exterior Angles Theorem**

**Given:** Any polygon has $n$ interior $\angle$s, one at each vertex, where $n$ is the number of sides, vertices or angles

**Prove:** The sum of those $n$ exterior $\angle$s = $360^\circ$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Any polygon has $n$ interior $\angle$s</td>
<td>Given</td>
</tr>
<tr>
<td>2. Each interior $\angle$ and exterior $\angle$ are a linear pair</td>
<td>Given in diagram</td>
</tr>
<tr>
<td>3. Each interior $\angle$ is supplementary to an exterior $\angle$</td>
<td>Linear pair angles are supplementary</td>
</tr>
<tr>
<td>4. The sum of the measures of each linear pair is $180^\circ$</td>
<td>Definition of supplementary</td>
</tr>
<tr>
<td>5. There are $n$ linear pairs</td>
<td>Substitution</td>
</tr>
<tr>
<td>6. The sum of the linear pairs = $n \times 180^\circ$</td>
<td>Multiplication Property</td>
</tr>
<tr>
<td></td>
<td>sum of exterior $\angle$s + interior $\angle$s</td>
</tr>
</tbody>
</table>
### Proof of Exterior Angles Theorem

**Given:** Any polygon has $n$ interior $\angle$s, one at each vertex, where $n$ is the number of sides, vertices or angles

**Prove:** The sum of those $n$ exterior $\angle$s $= 360^\circ$

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 The sum of interior $\angle$s is $(n - 2) \times 180^\circ$</td>
<td>Interior Angle Theorem</td>
</tr>
<tr>
<td>8 $n \times 180^\circ = \text{sum of exterior } \angle s + (n - 2) \times 180^\circ$</td>
<td>Substitution Property</td>
</tr>
<tr>
<td>9 The sum of the exterior $\angle$s $= n \times 180^\circ - [(n - 2) \times 180^\circ]$</td>
<td>Subtraction Property</td>
</tr>
<tr>
<td>10 The sum of the exterior $\angle$s $= 180^\circ[n - (n - 2)]$</td>
<td>Subtraction Property</td>
</tr>
<tr>
<td>11 The sum of the exterior $\angle$s $= 180^\circ(n - n + 2)$</td>
<td>Distributive Property</td>
</tr>
<tr>
<td>12 The sum of the exterior $\angle$s $= 180^\circ(2) = 360^\circ$</td>
<td>Simplifying</td>
</tr>
</tbody>
</table>
The measure of each exterior angle of a regular polygon with \( n \) sides is \( \frac{360^\circ}{n} \).

The sum of the external angles of any polygon is \( 360^\circ \).

The number of external angles of a polygon equals \( n \).

All angles are equal in a regular polygon, so their external angles must be as well.

So, each external angle of a regular polygon equals \( \frac{360^\circ}{n} \).
Polygon Exterior Angle Theorem Corollary

The measure of each exterior angle of a regular polygon with \( n \) sides is \( \frac{360^\circ}{n} \)

This is a regular hexagon.

\[ n = 6 \]
\[ a = \frac{360}{6} \]
\[ a = 60^\circ \]
20. What is the sum of the measures of the exterior angles of a heptagon?

A. 180°  
B. 360°  
C. 540°  
D. 720°
21 If a heptagon is regular, what is the measure of each exterior angle?

A  72°
B  60°
C  51.43°
D  45°
22. What is the sum of the measures of the exterior angles of a pentagon?
23 What is the measure of each exterior angle of a regular pentagon?
24 The measure of each interior angle of a regular convex polygon is 172°. How many sides does the polygon have?
The measure of each interior angle of a regular convex polygon is $174^\circ$. Find the number of sides of the polygon.

A  64
B  62
C  58
D  60
26 The measure of each interior angle of a regular convex polygon is 162°. Find the number of sides of the polygon.
Properties of Parallelograms

Return to the Table of Contents
Click on the link below and complete the two labs before the Parallelogram lesson.

Lab - Investigating Parallelograms

Lab - Quadrilaterals Party
Parallelograms

By definition, the opposite sides of a parallelogram are parallel.

In parallelogram DEFG,

\[ \overline{DG} \parallel ? \]

\[ \overline{DE} \parallel ? \]
Theorem

The opposite sides of a parallelogram are congruent.

If ABCD is a parallelogram,

then \( \overline{AB} \cong \overline{DC} \) and \( \overline{DA} \cong \overline{CB} \)
Proof of Theorem

The opposite sides a parallelogram are congruent.

Since we are proving that this theorem is true, we are not using it directly in the proof. Instead, we need to use other properties/theorems that we know.

Construct diagonal $\overline{AC}$ as shown and consider it a transversal to sides $\overline{AD}$ and $\overline{BC}$, which we know are $\parallel$.

What does that tell about the pairs of angles:
$\angle ACD \& \angle CAB$ and $\angle ACB \& \angle CAD$

Why?
Given that $\angle ACD \cong \angle CAB$ and $\angle ACB \cong \angle CAD$

Name a pair of congruent $\triangle$s
For what reason are they congruent?
Given that $\triangle ACD \cong \triangle CAB$
Which pairs of sides are must be congruent?
Why?
27 The opposite sides of a parallelogram are

A Congruent

B Parallel

C Both a and b
Theorem

The opposite angles of a parallelogram are congruent.

If ABCD is a parallelogram, then
\[ m\angle A = m\angle C \]
and
\[ m\angle B = m\angle D \]
Proof of Theorem

The opposite angles of a parallelogram are congruent.

As we did earlier, construct diagonal $\overline{AC}$.

We earlier proved $\triangle ACD \cong \triangle CAB$

What does this tell you about $\angle B$ and $\angle D$?

Why?
28 $\Delta DAB \cong \Delta BCD$. Prove your answer.
29 Based on the previous question, \( m\angle A = \)?

Prove your answer.

A \( m\angle B \)

B \( m\angle C \)

C \( m\angle D \)

D None of the above
Theorem

The consecutive angles of a parallelogram are supplementary.

If ABCD is a parallelogram, then $x^\circ + y^\circ = 180^\circ$
Proof of Theorem

The consecutive angles of a parallelogram are supplementary.

Consider $\overline{AD}$ and $\overline{BC}$ to be transversals of $\overline{AB}$ and $\overline{DC}$, which are $\parallel$.

What are these pairs of angles called: $\angle A$ & $\angle D$ and $\angle B$ & $\angle C$?

What is the relationship of each pair of angles like that?
Proof of Theorem

The consecutive angles of a parallelogram are supplementary.

Repeat the same process, but this time consider \( \overline{AB} \) and \( \overline{DC} \) to be transversals of \( \overline{AD} \) and \( \overline{BC} \), which are \( \parallel \).

What are these pairs of angles called: \( \angle A \) & \( \angle B \) and \( \angle D \) & \( \angle C \)?

What is the relationship of each pair of angles like that?
30. ABCD is a parallelogram. Find the value of x.
31 ABCD is a parallelogram. Find the value of y.
32 ABCD is a parallelogram. Find the value of z.
33 ABCD is a parallelogram. Find the value of $w$. 

\[
\begin{array}{c}
A & 2y \\
B & w \\
C & (5z) \\
D & 12 \\
\end{array}
\]
34 DEFG is a parallelogram. Find the value of w.
35 DEFG is a parallelogram. Find the value of x.
36 DEFG is a parallelogram. Find the value of $y$. 

Answer: 30
37 DEFG is a parallelogram. Find the value of z.
Theorem

The diagonals of a parallelogram bisect each other.

If ABCD is a parallelogram,

then $\overline{AE} \cong \overline{EC}$ and $\overline{BE} \cong \overline{ED}$
Proof of Theorem

The diagonals of a parallelogram bisect each other.

Since we are proving that this theorem is true, we are not using it directly in the proof. Instead, we need to use other properties/theorems that we know.

Steps to follow:

1) Identify pairs of triangles in the above diagram that you need to prove congruent to prove this theorem.

2) Then, using what you know about the relationships between pairs of angles and pairs of sides in a parallelogram and the properties of the angles formed by parallel lines & transversals, prove that the triangles are congruent.

3) Then prove the theorem by stating that corresponding parts of congruent triangles are congruent (CPCTC).
38 LMNP is a parallelogram. The diagonals bisect each other. Find QN
39 LMNP is a parallelogram. Find MP
40 BEAR is a parallelogram. Find the value of x.
41 BEAR is a parallelogram. Find the value of $y$.

Answer

$4y = RS = 8$

$y = 2$
42 BEAR is a parallelogram. Find ER.
43 In a parallelogram, the opposite sides are ________ parallel.

A sometimes
B always
C never
44 The opposite angles of a parallelogram are __________.

A  bisect
B  congruent
C  parallel
D  supplementary
45. The consecutive angles of a parallelogram are _________.

A. bisect
B. congruent
C. parallel
D. supplementary
46. The diagonals of a parallelogram ______ each other.

A. bisect
B. congruent
C. parallel
D. supplementary

Answer
47 The opposite sides of a parallelogram are __________.

A  bisect
B  congruent
C  parallel
D  supplementary
48 MATH is a parallelogram. AH = 12 and MR = 7. Find RT.

A 6
B 7
C 8
D 9
49 MATH is a parallelogram. Find AR.

A 6
B 7
C 8
D 9
50 MATH is a parallelogram. Find $m \angle H$. 

\[ (3y + 8) \]
51 MATH is a parallelogram. Find the value of $x$.

Answer

\[ x^2 - 24 = 2x \]
\[ x^2 - 2x - 24 = 0 \]
\[ (x - 6)(x + 4) = 0 \]
\[ x = 6 \text{ or } x = -4 \]

Since $HT = 2x$, and $2(-4) = -8$, which is not a positive length, the solution $x = -4$ can be ruled out (crossed off above).
52 MATH is a parallelogram. Find the value of $y$. 

\[ (3y + 8)° \]
Proving Quadrilaterals are Parallelograms

Return to the Table of Contents
Theorem

If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

In quadrilateral ABCD,

\[ \overline{AB} \cong \overline{DC} \text{ and } \overline{AD} \cong \overline{BC}, \]

so ABCD is a parallelogram.
Theorem

If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

In quadrilateral ABCD,

\[ \angle A \cong \angle D \text{ and } \angle B \cong \angle C, \]

so ABCD is a parallelogram.
53 PQRS is a Parallelogram. Explain your reasoning.

True

False

Answer

True, PQRS is a parallelogram. The opposite sides are congruent.
54 Is PQRS a parallelogram? Explain your reasoning.

Yes

No

Because PQRS is a quadrilateral, \( m\angle Q + m\angle R = 180° \). But, we can't assume that \( \angle Q \) and \( \angle R \) are right angles. We can't prove PQRS is a parallelogram.
55 Is this quadrilateral definitely a parallelogram?

Yes

No

![Diagram of quadrilateral with angles 78° and 136°]
56. Is this quadrilateral definitely a parallelogram?

Yes

No
57 Is this quadrilateral definitely a parallelogram?

Yes

No
Theorem

If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram.

In quadrilateral ABCD,
\[ m \angle A + m \angle B = 180^\circ \]
and \[ m \angle B + m \angle C = 180^\circ, \]
so ABCD is a parallelogram.
Theorem

If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

In quadrilateral ABCD, \( \overline{AE} \cong \overline{CE} \) and \( \overline{DE} \cong \overline{BE} \), so ABCD is a quadrilateral.
Theorem

If one pair of sides of a quadrilateral is parallel and congruent, then the quadrilateral is a parallelogram.

In quadrilateral ABCD,
\[ \overline{AD} \cong \overline{CB} \] and \[ \overline{AD} \parallel \overline{BC}, \]
so ABCD is a parallelogram.
58 Is this quadrilateral definitely a parallelogram?

Yes
No

Answer: No
59. Is this quadrilateral definitely a parallelogram?

Yes

No
60 Is this quadrilateral definitely a parallelogram?

Yes

No
61 Is this quadrilateral definitely a parallelogram?

Yes
No
62 Three interior angles of a quadrilateral measure $67^\circ$, $67^\circ$ and $113^\circ$. Is this enough information to tell whether the quadrilateral is a parallelogram? Explain.

Yes

No

Answer

NO, the question did not state the position of the measurements in the quadrilateral. We cannot assume their position.
In a parallelogram...

the opposite sides are _________________ and ______________, the opposite angles are ________________, the consecutive angles are _______________, and the diagonals ______________ each other.
Fill in the blank

parallel  perpendicular  supplementary

bisect  congruent

To prove a quadrilateral is a parallelogram...

both pairs of opposite sides of a quadrilateral must be ______________,
both pairs of opposite angles of a quadrilateral must be ______________,
an angle of the quadrilateral must be ______________ to its consecutive angles, the diagonals of the quadrilateral ____________ each other, or one pair of opposite sides of a quadrilateral are ______________ and ____________.

Answer
63. Which theorem proves the quadrilateral is a parallelogram?

A. The opposite angles are congruent.
B. The opposite sides are congruent.
C. An angle in the quadrilateral is supplementary to its consecutive angles.
D. The diagonals bisect each other.
E. One pair of opposite sides are congruent and parallel.
F. Not enough information.
64 Which theorem proves this quadrilateral is a parallelogram?

A The opposite angles are congruent.
B The opposite sides are congruent.
C An angle in the quadrilateral is supplementary to its consecutive angles.
D The diagonals bisect each other.
E One pair of opposite sides are congruent and parallel.
F Not enough information.

Answer
65 Which theorem proves the quadrilateral is a parallelogram?

A  The opposite angles are congruent.
B  The opposite sides are congruent.
C  An angle in the quadrilateral is supplementary to its consecutive angles.
D  The diagonals bisect each other.
E  One pair of opposite sides are congruent and parallel.
F  Not enough information.

Answer: B
Rhombi, Rectangles and Squares
Three Special Parallelograms

All the same properties of a parallelogram apply to the rhombus, rectangle, and square.
A rhombus is a quadrilateral with four congruent sides.

If $AB \cong BC \cong CD \cong DA$, then $ABCD$ is a rhombus.
66 What is the value of y that will make the quadrilateral a rhombus?

- A 7.25
- B 12
- C 20
- D 25

Answer: C
What is the value of $y$ that will make this parallelogram a rhombus?

A. 7.25
B. 12
C. 20
D. 25
Theorem
The diagonals of a rhombus are perpendicular.

If ABCD is a rhombus, then \( \overline{AC} \perp \overline{BD} \).
Theorem

The diagonals of a rhombus are perpendicular.

Since $ABCD$ is a parallelogram, its diagonals bisect each other.

$\overline{AO} \cong \overline{CO}$ and $\overline{BO} \cong \overline{DO}$. 
The diagonals of a rhombus are perpendicular.

By definition, the sides of a rhombus are of equal measure, which means that $AB = BC = CD = DA$.

Then by SSS,
$\triangle AOB \cong \triangle BOC \cong \triangle COD \cong \triangle DOA$

And due to corresponding parts of $\cong$ $\triangle$s being $\cong$ (or CPCTC)

$\angle AOB \cong \angle COB \cong \angle COD \cong \angle AOD$

If the adjacent angles formed by intersecting lines are congruent, then the lines are perpendicular.
Theorem

If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

If ABCD is a rhombus, then

\[ \angle DAC \cong \angle BAC \cong \angle BCA \cong \angle DCA \]

and

\[ \angle ADB \cong \angle CDB \cong \angle ABD \cong \angle CBD \]
Theorem

If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles.

We proved earlier by SSS, that
\[ \triangle AOB \cong \triangle BOC \cong \triangle COD \cong \triangle AOD \]

Corresponding parts of \( \cong \) \( \Delta \)s are \( \cong \); so
\[ \angle ABO \cong \angle CBO \cong \angle ADO \cong \angle CDO \]
AND
\[ \angle DAO \cong \angle BAO \cong \angle BCO \cong \angle DCO \]
So each diagonal bisects a pair of opposite angles
68 EFGH is a rhombus. Find the value of x.
69 EFGH is a rhombus. Find the value of $y$. 

[Diagram of a rhombus EFGH with sides labeled $2x - 6$, $72^\circ$, $5y$, and $10$, and angles $z^\circ$.]
70 EFGH is a rhombus. Find the value of z.
71 This quadrilateral is a rhombus. Find the value of x.
72 This quadrilateral is a rhombus. Find the value of \( y \).
73 This quadrilateral is a rhombus. Find the value of $z$. 

Answer
74 This is a rhombus. Find the value of $x$. 

\[ x^\circ \]
75 This is a rhombus. Find the value of $x$. 

\[
\frac{1}{3}x - 3
\]
76 This is a rhombus. Find the value of \( x \).
77 HJKL is a rhombus. Find the length of $HJ$. 

![Diagram of rhombus HJKL with diagonals shown]

Answer
Rectangles

A rectangle is a quadrilateral with only right angles.

If ABCD is a rectangle, then $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are right angles.

If $\angle A$, $\angle B$, $\angle C$ and $\angle D$ are right angles, then ABCD is a rectangle.
What value of $y$ will make the quadrilateral a rectangle?

\[
\text{(6y)}°
\]

12
The diagonals of a rectangle are congruent.

If ABCD is a rectangle, then $\overline{AC} \cong \overline{BD}$.
Also, since a rectangle is a parallelogram,
$\overline{AM} \cong \overline{BM} \cong \overline{CM} \cong \overline{DM}$.
The diagonals of a rectangle are congruent.

Since ABCD is a rectangle (and therefore a parallelogram) \( \overline{AB} \cong \overline{DC} \)

Because of the reflexive property, \( \overline{BC} \cong \overline{BC} \).

Since ABCD is a rectangle \( m\angle B = m\angle C = 90^\circ \)

Then \( \triangle ABC \cong \triangle DCB \) by SAS

And, \( \overline{AC} \cong \overline{DB} \) since they are corresponding parts of \( \cong \) \( \triangle s \).
79 RECT is a rectangle. Find the value of $x$. 

\[ 2x - 5 = 13 \]
\[ 2x = 18 \]
\[ x = 9 \]
RECT is a rectangle. Find the value of y.
81 RSTU is a rectangle. Find the value of $z$. 

\[
\text{(8z)°}
\]
82 RSTU is a rectangle. Find the value of z.
A square is a quadrilateral with equal sides and equal angles. It is, therefore both a rhombus and a rectangle.

A square has all the properties of a rectangle and rhombus.
This quadrilateral is a square. Find the value of $x$. 

Answer
84 This quadrilateral is a square. Find the value of y.
85  This quadrilateral is a square. Find the value of $z$. 

Answer
86 This quadrilateral is a square. Find the value of x.
87 This quadrilateral is a square. Find the value of y.
88 This quadrilateral is a square. Find the value of $z$. 

\[ \sqrt{y^2} \quad 10 - 3y \quad (12z)° \quad (x^2 + 9)° \]
89 This quadrilateral is a square. Find the value of y.

$(18y)°$
90. This quadrilateral is a rhombus. Find the value of $x$. 

Answer
91 The quadrilateral is parallelogram. Find the value of $x$. 

\[112^\circ\] 

\[(4x)^\circ\]
92  The quadrilateral is a rectangle. Find the value of x.
Fill in the Table with the Correct Definition

<table>
<thead>
<tr>
<th></th>
<th>rhombus</th>
<th>rectangle</th>
<th>square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Opposite sides are</td>
<td></td>
<td></td>
<td>Diagonals bisect opposite (\angle)'s</td>
</tr>
<tr>
<td>Diagonals are (\perp)</td>
<td>Has 4 right (\angle)'s</td>
<td>Diagonals are (\cong)</td>
<td></td>
</tr>
</tbody>
</table>
Lab - Quadrilaterals in the Coordinate Plane
Trapezoids
A trapezoid is a quadrilateral with one pair of parallel sides.

The parallel sides are called bases.

The nonparallel sides are called legs.

A trapezoid also has two pairs of base angles.
Isosceles Trapezoid

An isosceles trapezoid is a trapezoid with congruent legs.
Theorem

If a trapezoid is isosceles, then each pair of base angles are congruent.

If ABCD is an isosceles trapezoid, then \( \angle A \cong \angle B \) and \( \angle C \cong \angle D \).
Theorem

If a trapezoid is isosceles, then each pair of base angles are congruent.

Given: ABCD is an isosceles trapezoid
Prove: \( \angle A \cong \angle B \) and \( \angle C \cong \angle D \)
Theorem

If a trapezoid is isosceles, then each pair of base angles are congruent.

**Given:** ABCD is an isosceles trapezoid

**Prove:** \( \angle A \cong \angle B \) and \( \angle C \cong \angle D \).

continued...
### Theorem

**Given:** ABCD is an isosceles trapezoid  
**Prove:** \( \angle A \cong \angle B \) and \( \angle C \cong \angle D \).

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ABCD is an isosceles trapezoid</td>
<td></td>
</tr>
<tr>
<td>2 ( \overline{AB} \parallel \overline{DC} ), ( \overline{AD} \cong \overline{BC} )</td>
<td></td>
</tr>
<tr>
<td>3 Draw segments from A and B perpendicular to ( \overline{AB} ) and ( \overline{DC} ). These segments are ( \overline{AE} ) and ( \overline{BF} ).</td>
<td>Geometric Construction</td>
</tr>
<tr>
<td>4 ( \angle EAB, \angle AEF, \angle BFE, \angle FBA, \angle BFC ) and ( \angle AED ) are right angles</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>All right angles are ( \cong )</td>
</tr>
<tr>
<td>6</td>
<td>Definition of congruent and right angles</td>
</tr>
</tbody>
</table>

---

continued...
### Theorem

<table>
<thead>
<tr>
<th></th>
<th>Statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>ABFE is a rectangle</td>
</tr>
<tr>
<td>8</td>
<td>$AE \cong BF$</td>
</tr>
<tr>
<td>9</td>
<td>$\triangle AED \cong \triangle BFC$</td>
</tr>
</tbody>
</table>
| 10 | $\angle C \cong \angle D$
    | $\angle DAE \cong \angle CBF$ |
| 11 | $m\angle DAE = m\angle CBF$ |
| 12 | $m\angle DAE + 90 = m\angle CBF + 90$ |
| 13 | $m\angle DAE + m\angle EAB =$
    | $m\angle CBF + m\angle FBA$ |
| 14 | $m\angle DAE + m\angle EAB = m\angle DAB$
    | $m\angle CBF + m\angle FBA = m\angle CBA$ |
| 15 | $m\angle DAB = m\angle CBA$ |
| 16 | $\angle DAB \cong \angle CBA$, or $\angle A \cong \angle B$ |
Theorem

If a trapezoid has at least one pair of congruent base angles, then the trapezoid is isosceles.

If in trapezoid ABCD, $\angle A \cong \angle B$. Then, ABCD is an isosceles trapezoid.
Theorem
If a trapezoid has at least one pair of congruent base angles, then the trapezoid is isosceles.

Case 1 - Longer Base ∠s are ≅

**Given:** In trapezoid ABCD, ∠C ≅ ∠D
**Prove:** ABCD is an isosceles trapezoid

continued...
<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 In trapezoid ABCD, $\angle C \cong \angle D$</td>
<td></td>
</tr>
<tr>
<td>2 $AB \parallel DC$</td>
<td></td>
</tr>
<tr>
<td>3 Draw segments from A and B perpendicular to AB and DC. These segments are AE and BF.</td>
<td>Geometric Construction</td>
</tr>
<tr>
<td>4 $\angle EAB$, $\angle AEF$, $\angle BFE$, $\angle FBA$, $\angle BFC$ and $\angle AED$ are right angles</td>
<td></td>
</tr>
<tr>
<td>5 All right angles are $\cong$</td>
<td></td>
</tr>
<tr>
<td>6 ABFE is a rectangle</td>
<td></td>
</tr>
<tr>
<td>7 $AE \cong BF$</td>
<td></td>
</tr>
<tr>
<td>8 $\triangle AED \cong \triangle BFC$</td>
<td></td>
</tr>
<tr>
<td>9 $AD \cong BC$</td>
<td></td>
</tr>
<tr>
<td>10 ABCD is an isosceles trapezoid</td>
<td></td>
</tr>
</tbody>
</table>
Theorem

If a trapezoid has at least one pair of congruent base angles, then the trapezoid is isosceles.

Case 2 - \( \angle s \) of shorter base are \( \cong \)

In this case, extend the shorter base and draw a perpendicular line from each vertex of the longer base to the extended base.
Theorem

If a trapezoid has at least one pair of congruent base angles, then the trapezoid is isosceles.

Case 2 - $\angle$s of shorter base are $\cong$

In this case, extend the shorter base and draw a perpendicular line from each vertex of the longer base to the extended base, as shown.

Can you now see how to extend the approach we used for the first case to this second case?
Case 2 - \( \angle \)s of shorter base are \( \cong \)

**Given:** In trapezoid ABCD, \( \angle A \cong \angle B \\

**Prove:** ABCD is an isosceles trapezoid

Steps to follow:
1) Include the constructions of our new perpendicular segments \( CE \) & \( DF \) in your proof.
2) Identify pairs of triangles in the above diagram that you need to prove congruent to prove this theorem.
3) Then, using what you know about the relationships between pairs of angles and sides in isosceles trapezoids and rectangles, prove that the triangles are congruent.
4) Then prove the theorem by stating that corresponding parts of congruent triangles are congruent (CPCTC).
93 The quadrilateral is an isosceles trapezoid. Find the value of $x$.

A 3
B 5
C 7
D 9
The quadrilateral is an isosceles trapezoid. Find the value of $x$.

A 3
B 5
C 7
D 9

$64^\circ$ $(9x + 1)^\circ$
Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

In trapezoid ABCD,

If \( AC \cong BD \).
Then, ABCD is isosceles.

If ABCD is isosceles
Then, \( AC \cong BD \).
# Theorem

A trapezoid is isosceles if and only if its diagonals are congruent.

**Given:** Trapezoid ABCD is isosceles  
**Prove:** \( AC \cong BD \)

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Trapezoid ABCD is isosceles</td>
<td></td>
</tr>
<tr>
<td>2 ( AB \parallel DC, AD \cong BC, \angle A \cong \angle B, \angle C \cong \angle D )</td>
<td></td>
</tr>
<tr>
<td>4 ( CD \cong CD )</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>SAS</td>
</tr>
<tr>
<td>6 ( AC \cong BD )</td>
<td></td>
</tr>
</tbody>
</table>
95 PQRS is a trapezoid. Find the $m \angle S$.

The sum of the interior angles of a quadrilateral is $360^\circ$.
The parallel lines in a trapezoid create pairs of consecutive interior angles.

$m \angle P + m \angle S = 180^\circ$ and $m \angle Q + m \angle R = 180^\circ$

$(6w + 2) + 112 = 180$
$6w + 114 = 180$
$w = 11$
96 PQRS is a trapezoid. Find the \( m \angle R \).
97  PQRS is an isosceles trapezoid. Find the $m\angle Q$. 

![Diagram of PQRS is an isosceles trapezoid with angles labeled.]

\[
\begin{align*}
\angle P & = 123^\circ \\
\angle Q & = (9w - 3)^\circ \\
\angle S & = (4w + 1)^\circ \\
\end{align*}
\]
98. PQRS is an isosceles trapezoid. Find the m\(\angle R\).
99 PQRS is an isosceles trapezoid. Find the m∠S.
100 The trapezoid is isosceles. Find the value of $x$. 
101 The trapezoid is isosceles. Find the value of $x$. 

Answer: $x = 143$
102 In trapezoid HIJK, can $HI$ and $KJ$ have slopes that are opposite reciprocals?

Yes
No
Midsegment of a Trapezoid

The midsegment of a trapezoid is a segment that joins the midpoints of the legs.

Click on the link below and complete the lab.

Lab - Midsegments of a Trapezoid
Theorem

The midsegment of a trapezoid is parallel to both the bases.

\[ \overline{AB} \parallel \overline{EF} \parallel \overline{DC} \]
Proof of Theorem

The midsegment of a trapezoid is parallel to both the bases.

It's given that E is the midpoint of $\overline{AD}$; F is the midpoint of $\overline{BC}$; and $\overline{AB} \parallel \overline{DC}$.

Since $\overline{AB} \parallel \overline{DC}$, they have the same slope and therefore the distance between them, along any perpendicular line, will be the same.

Any line that connects the midpoints of the legs will also be the same distance from each base if measured along a perpendicular line.

Which means that the lines that connect the midpoints will have the same slope as the bases.

If the slopes are the same, the lines are parallel.

That means that $\overline{AB} \parallel \overline{EF} \parallel \overline{DC}$, where $\overline{EF}$ is the midsegment of the trapezoid.
Theorem

The length of the midsegment is half the sum of the bases.

$$EF = 0.5(AB + DC)$$
Proof of Theorem
The length of the midsegment is half the sum of the bases.

Given: E is the midpoint of \( \overline{AD} \)
F is the midpoint of \( \overline{BC} \)
\( \overline{AB} \parallel \overline{DC} \parallel \overline{EF} \)

Prove: \( EF = 0.5(AB + DC) \)

continued...
## Proof of Theorem

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
</table>
| 1 E is the midpoint of $\overline{AD}$  
F is the midpoint of $\overline{BC}$  
$\overline{AB} \parallel \overline{DC} \parallel \overline{EF}$ |  |
| 2 $\angle AEG \equiv \angle ADH$, $\angle BFI \equiv \angle BCJ$ |  |
| 3 $\angle EAG \equiv \angle DAH$, $\angle FBI \equiv \angle CBJ$ |  |
| 4 $\triangle AEG \sim \triangle ADH$, $\triangle BFI \sim \triangle BCJ$ |  |
| 5 $\frac{AD}{AE} = \frac{AH}{AG} = \frac{DH}{EG}$, $\frac{BC}{BF} = \frac{BJ}{BI} = \frac{CJ}{IF}$ |  |
| 6 $AD = 2AE$, $BC = 2BF$ | The midpoints E and F divide $\overline{AD}$ and $\overline{BC}$ into 2 congruent segments, so $\overline{AD}$ and $\overline{BC}$ are twice the size of $\overline{AE}$ and $\overline{BF}$ respectfully. |
| 7 $\frac{AD}{AE} = 2$, $\frac{BC}{BF} = 2$ |  |
| 8 The scale factor in both of our similar triangles is 2. | continued... |
### Proof of Theorem

<table>
<thead>
<tr>
<th>Statement</th>
<th>Reason</th>
</tr>
</thead>
<tbody>
<tr>
<td>9 If $EG = x$, then $DH = 2x$. If $IF = y$, then $CH = 2y$.</td>
<td></td>
</tr>
<tr>
<td>10 $EF = AB + x + y$</td>
<td></td>
</tr>
<tr>
<td>11 $2EF = 2AB + 2x + 2y$</td>
<td></td>
</tr>
<tr>
<td>12 $DC = AB + 2x + 2y$</td>
<td></td>
</tr>
<tr>
<td>13 $2x + 2y = DC - AB$</td>
<td></td>
</tr>
<tr>
<td>14 $2EF = 2AB + DC - AB$</td>
<td></td>
</tr>
<tr>
<td>15 $2EF = AB + DC$</td>
<td></td>
</tr>
<tr>
<td>16 $EF = 0.5(AB + DC)$</td>
<td></td>
</tr>
</tbody>
</table>
103  PQRS is a trapezoid. Find LM.
104 PQRS is a trapezoid. Find PS.
105 PQRS is an trapezoid. ML is the midsegment. Find the value of x.
106 PQRS is an trapezoid. \( ML \) is the midsegment. Find the value of \( y \).
107 PQRS is an trapezoid. $\overline{ML}$ is the midsegment. Find the value of $z$. 
108 \ EF \text{ is the midsegment of trapezoid HIJK. Find the value of } x.
109 $\overline{EF}$ is the midsegment of trapezoid HIJK. Find the value of $x$. 

Answer 1
110 Which of the following is true of every trapezoid? Choose all that apply.

A  Exactly 2 sides are congruent.
B  Exactly one pair of sides are parallel.
C  The diagonals are perpendicular.
D  There are 2 pairs of base angles.

Answer: B and D
Kites

Return to the Table of Contents
Kites

A kite is a quadrilateral with two pairs of adjacent congruent sides whose opposite sides are not congruent.

Lab - Properties of Kites
Theorem

If a quadrilateral is a kite, then it has one pair of congruent opposite angles.

In kite ABCD, \( \angle B \cong \angle D \)

and \( \angle A \not\cong \angle C \)
Proof of Theorem

If a quadrilateral is a kite, then it has one pair of congruent opposite angles.

Draw line AC and use the Reflexive Property to say that $\overline{AC} \cong \overline{AC}$.

$\triangle ABC \cong \triangle ADC$ by SSS.

$\angle B \cong \angle D$ since corresponding parts of $\cong \triangle s$ are $\cong$.

$m\angle A \neq m\angle C$ since $AD \neq CD$.

In kite $ABCD$, $m\angle B = m\angle D$ and $m\angle A \neq m\angle C$.
111  LMNP is a kite. Find the value of x.

\[ m\angle L + m\angle M + m\angle N + m\angle P = 360° \]

\[ 72 + (x^2 - 1) + (x^2 - 1) + 48 = 360 \]

\[ 2x^2 + 118 = 360 \]

\[ 2x^2 = 242 \]

\[ x^2 = 121 \]

\[ x = \pm 11 \]
112 READ is a kite. $\overline{RE} \cong ____.$

A  $\overline{EA}$
B  $\overline{AD}$
C  $\overline{DR}$
113 READ is a kite. $\angle A \cong ____$. 

A $\angle E$
B $\angle D$
C $\angle R$
114 Find the value of $z$ in the kite.
115 Find the value of x in the kite.
116 Find the value of x.
Theorem

The diagonals of a kite are perpendicular.

In kite ABCD
\[ AC \perp BD \]
Proof of Theorem

The diagonals of a kite are perpendicular.

Given that $\overline{AD} \cong \overline{AB}$ and $\overline{DC} \cong \overline{BC}$

$\overline{AC} \cong \overline{AC}$ and $\overline{AE} \cong \overline{AE}$ by reflexive property

$\triangle ADC \cong \triangle ABC$ by SSS

$\angle DAE \cong \angle BAE$ by CPCTC

$\triangle ADE \cong \triangle ABE$ by SAS

$\angle AED \cong \angle AEB$ by CPCTC

$\overline{AE}$ is perpendicular to $\overline{BD}$ since intersecting lines that create $\cong$ adjacent angles are perpendicular.

In kite $ABCD$

$\overline{AC} \perp \overline{BD}$
117 Find the value of $x$ in the kite.
118 Find the value of y in the kite.

(12y)°
Constructing Quadrilaterals

Return to the Table of Contents
Construction of a Parallelogram

1. Draw two intersecting lines.
Construction of a Parallelogram

2. Place a compass at the point of intersection and draw an arc where it intersects one line in two places.
Construction of a Parallelogram

3. Keeping the compass at the point of intersection, either enlarge, or shorten the radius of the compass and draw an arc where it intersects the other line in 2 places.
Construction of a Parallelogram

4. Connect the four points where the arcs intersect the lines.
Which property of parallelograms does this construction technique depend upon?
Construction of a Parallelogram

Try this!
Construct a parallelogram using the lines below.

1)
Construction of a Parallelogram

Try this!
Construct a parallelogram using the lines below.

2)
Videos Demonstrating the Constructions of a Parallelogram using Dynamic Geometric Software

Click here to see parallelogram video: compass & straightedge

Click here to see parallelogram video: Menu options
Construction of a Rectangle

1. Draw two intersecting lines.
Construction of a Rectangle

2. Place a compass at the point of intersection and draw an arc where it intersects each line, keeping the radius of the compass the same all the way around.
Construction of a Rectangle

3. Connect the four points where the arcs intersect the lines.
Construction of a Rectangle

Which properties of rectangles does this construction technique depend upon?
Construction of a Rectangle

Try this!
Construct a rectangle using the lines below.

3)
Construction of a Rectangle

Try this!
Construct a rectangle using the lines below.

4)
Videos Demonstrating the Constructions of a Rectangle using Dynamic Geometric Software

Click here to see rectangle video: compass & straightedge

Click here to see rectangle video: Menu options
Construction of a Rhombus

1. Construct a segment of any length.
Construction of a Rhombus

2. Find the perpendicular bisector and midpoint of a line segment by constructing the locus between two points.

a. Place the point of the compass on A and open it so the pencil is more than halfway, it does not matter how far, to point B.
2. Find the perpendicular bisector and midpoint of a line segment by constructing the locus between two points.

b) Keeping the compass locked, place the point of the compass on B and draw an arc so that it intersects the one you just drew.
Construction of a Rhombus

2. Find the perpendicular bisector and midpoint of a line segment by constructing the locus between two points.

c. Draw a line through the arc intersections. The intersection point created with the two perpendicular lines is the center of your rhombus.
Construction of a Rhombus

3. Place the tip of your compass on your center point (midpoint) C. Extend your compass to any length. Draw 2 arcs where the compass intersects the perpendicular bisector.

Note: If you wanted to, you could use the intersection points of the 2 arcs from step #2 as your extra points.
Construction of a Rhombus

4. Connect the intersection points of the 2 arcs and the perpendicular bisector from step #3 with the endpoints of your original segment.
Construction of a Rhombus
Which properties of rhombii (pl. for rhombus) does this construction technique depend upon?
Videos Demonstrating the Constructions of a Rhombus using Dynamic Geometric Software

Click here to see rhombus video: compass & straightedge

Click here to see rhombus video: Menu options
Construction of a Rhombus

Try this!
Construct a rhombus using the segment below.

5)
Construction of a Rhombus

Try this!
Construct a rhombus using the segment below.

6)
Construction of a Square

1. Construct a segment of any length.
Construction of a Square

2. Find the perpendicular bisector and midpoint of a line segment by constructing the locus between two points.

a. Place the point of the compass on A and open it so the pencil is more than halfway, it does not matter how far, to point B. Draw an arc.
2. Find the perpendicular bisector and midpoint of a line segment by constructing the locus between two points.

b) Keeping the compass locked, place the point of the compass on B and draw an arc so that it intersects the one you just drew
Construction of a Square

2. Find the perpendicular bisector and midpoint of a line segment by constructing the locus between two points.

c. Draw a line through the arc intersections. The intersection point created with the two perpendicular lines is the center of your circle (and future square).
Construction of a Square

3. Place your compass tip on point C and the pencil point on either A or B and construct your circle.
Construction of a Square

4. Connect the intersection points of your perpendicular bisector and the circle with the initial vertices of the line segment to create your square.
Construction of a Square

Which properties of squares does this construction technique depend upon?
Construction of a Square

Try this!
Construct a square using the segment below.

7)
Construction of a Square

Try this!
Construct a square using the segment below.

8)
Videos Demonstrating the Constructions of a Square using Dynamic Geometric Software

- Click here to see square video: compass & straightedge
- Click here to see square video: Menu options
Construction of an Isosceles Trapezoid

1. Draw two intersecting lines.
2. Place a compass at the point of intersection and draw an arc where it intersects two of the lines one time each.
3. Keeping the compass at the point of intersection, either enlarge, or shorten the radius of the compass and draw an arc where it intersects the two lines on the other side (e.g. Since I drew my 1st 2 arcs in the upper half, these 2 arcs will be in the lower half).
Construction of an Isosceles Trapezoid

4. Connect the four points where the arcs intersect the lines.
Construction of an Isosceles Trapezoid

Which properties of isosceles trapezoids does this construction technique depend upon?
Construction of an Isosceles Trapezoid

Try this!
Construct an isosceles trapezoid using the lines below.

9)
Try this!
Construct an isosceles trapezoid using the lines below.

10)
Videos Demonstrating the Constructions of an Isosceles Trapezoid using Dynamic Geometric Software

Click here to see isosceles trapezoid video: compass & straightedge

Click here to see isosceles trapezoid video: Menu options
Construction of a Kite

1. Construct a segment of any length.

![Diagram of a segment AB](image-url)
2. Place your compass tip at any point and draw 2 arcs that intersect your segment.
Construction of a Kite

3. Find the perpendicular bisector between the 2 red arcs.

a) Place your compass tip at one of the intersection points. Extend your pencil out more than half way between the 2 arcs. Draw an arc.
Construction of a Kite

3. Find the perpendicular bisector between the 2 red arcs.

b) Keeping the radius of your compass the same, place your compass tip at the other intersection point and draw an arc.
Construction of a Kite

3. Find the perpendicular bisector between the 2 red arcs.

c) Connect the intersections of the 2 arcs to construct your perpendicular line.
Construction of a Kite

4. Place the tip of your compass at the intersection point of our perpendicular lines. Extend your compass to any length. Draw 2 arcs where the compass intersects the perpendicular line.

Note: If you wanted to, you could use the intersection points of the 2 arcs from step #3 as your extra points.
Construction of a Kite

5. Connect the intersection points of your perpendicular line and the arcs from step #4 with the initial vertices of the line segment to create your kite.
Construction of a Kite

Which properties of a kite does this construction technique depend upon?
Construction of a Kite

Try this!
Construct a kite using the segment below.

11)

A

C
Construction of a Kite

Try this!
Construct a kite using the segment below.

12)
Videos Demonstrating the Constructions of a Kite using Dynamic Geometric Software

Click here to see kite video: compass & straightedge

Click here to see kite video: Menu options
Families of Quadrilaterals

Return to the Table of Contents
Complete the chart
(There can be more than one answer).

<table>
<thead>
<tr>
<th>Description</th>
<th>Special Quadrilateral(s)</th>
<th>Answer(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>An equilateral quadrilateral</td>
<td></td>
<td>click to reveal</td>
</tr>
<tr>
<td>An equiangular quadrilateral</td>
<td></td>
<td>click to reveal</td>
</tr>
<tr>
<td>The diagonals are perpendicular</td>
<td></td>
<td>click to reveal</td>
</tr>
<tr>
<td>The diagonals are congruent</td>
<td></td>
<td>click to reveal</td>
</tr>
<tr>
<td>Has at least 1 pair of parallel sides</td>
<td></td>
<td>click to reveal</td>
</tr>
</tbody>
</table>
119 A rhombus is a square.

A always
B sometimes
C never
120 A square is a rhombus.

   A  always
   B  sometimes
   C  never
121 A rectangle is a rhombus.

A always
B sometimes
C never
122 A trapezoid is isosceles.

A always
B sometimes
C never
123 A kite is a quadrilateral.

A always

B sometimes

C never
124 A parallelogram is a kite.

A always
B sometimes
C never
Families of Quadrilaterals

Sometimes, you will be given a type of quadrilateral and using its qualities find the value of a variable. After substituting the variable back into the algebraic expression(s), the quadrilateral might become a more specific type.

Using what we know about the different types of quadrilaterals, let's try these next few problems.
Quadrilateral ABCD is a parallelogram.

Determine the value of $x$.

Since we know that ABCD is a parallelogram, we know that the __________ property holds.

Using the __________ property, we can write an equation & solve it.
Quadrilateral ABCD is a parallelogram.

\[ x^2 - 75 = 2x + 5 \]

\[ x^2 - 2x - 80 = 0 \]

\[ (x - 10)(x + 8) = 0 \]

\[ x = 10 \text{ and } x = -8 \]

Do any of these answers not make sense?

Yes, since the value of \( x \) cannot be negative, \( x = -8 \) can be ruled out. \( BC = 2(-8) + 5 = -11 \) which is not possible.

Since ABCD is a parallelogram, \( AB = 25 \) because opposite sides are congruent. When substituting \( x = 10 \) back into the expressions, then \( AD = CD = BC = AB = 25 \). Therefore, ABCD is a rhombus.

Is parallelogram ABCD a more specific type of quadrilateral? Explain.

;,
125 Quadrilateral EFGH is a parallelogram. Find the value of $x$. 

Answer
Based on your answer to the previous slide, what type of quadrilateral is EFGH? Explain.

When you are finished, choose the classification of quadrilateral EFGH from the choices below.

A Rhombus
B Rectangle
C Square
Quadrilateral KLMN is a trapezoid. Find the value of x.

\[
\begin{align*}
\text{K} & \quad \text{L} \\
(7x + 30)° & \quad (10x)° \\
\text{N} & \quad \text{M} \\
(x^2 - 20)° & 
\end{align*}
\]

Answer

\[
10x + x^2 - 20 = 180 \\
x^2 + 10x - 200 = 0 \\
(x + 20)(x - 10) = 0 \\
x = 10
\]

Note: 

\[x = -20\] does not work
128 Based on your answer to the previous slide, what type of quadrilateral is KLMN? Explain.

When you are finished, determine whether trapezoid KLMN is isosceles.

A  Isosceles Trapezoid

B  Not an Isosceles Trapezoid
Extension: Inscribed Quadrilaterals

A quadrilateral is inscribed if all its vertices lie on a circle.
Inscribed Quadrilaterals

If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.

\[ m\angle D = \frac{1}{2}(mEFG) \quad \text{[green arc]} \]
\[ m\angle F = \frac{1}{2}(mEDG) \quad \text{[red arc]} \]

Since \( mEFG + mEDG = 360^\circ \),
\[ m\angle D + m\angle F = \frac{1}{2}(mEFG) + \frac{1}{2}(mEDG) \]
\[ m\angle D + m\angle F = \frac{1}{2}(mEFG + mEDG) \]
\[ m\angle D + m\angle F = \frac{1}{2}(360^\circ) \]
\[ m\angle D + m\angle F = 180^\circ \]

\[ m\angle D + m\angle F = 180^\circ \]
\[ m\angle E + m\angle G = 180^\circ \]
Example: Find the $m \angle G$. Then, find the value of $a$. 

Inscribed Quadrilaterals
129 What is $\angle L$?

A. 25°
B. 65°
C. 115°
D. 155°
What is $m\angle M$?

A  23°
B  63°
C  117°
D  153°
131 What is the value of x?

A  8
B  10
C  17
D  19
132 What is the value of y?

A 13
B 14.69
C 7
D 6.08
Can special quadrilaterals be inscribed into a circle?

Throughout this unit, we have been talking about the different quadrilaterals, their properties, and how to construct them.

- Parallelogram
- Rectangle
- Rhombus
- Square
- Trapezoid
- Kite

Can any of these quadrilaterals be inscribed into a circle?

Find out by completing the lab below.

Lab: Inscribed Quadrilaterals
133 Complete the sentence below.
A trapezoid can __________ be inscribed into a circle.

A Always
B Sometimes
C Never
134 Complete the sentence below.
A rectangle can ____________ be inscribed into a circle.

A Always
B Sometimes
C Never
135 Complete the sentence below.
    A rhombus can __________ be inscribed into a circle.

A Always
B Sometimes
C Never
136 Complete the sentence below.
A kite can __________ be inscribed into a circle.

A  Always
B  Sometimes
C  Never
Proofs

Return to the Table of Contents
Given: $\overline{TE} \cong \overline{MA}, \angle 1 \cong \angle 2$
Prove: TEAM is a parallelogram.
## Option A

<table>
<thead>
<tr>
<th>statements</th>
<th>reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) $\overline{TE} \cong \overline{MA}$, $\angle 1 \cong \angle 2$</td>
<td></td>
</tr>
<tr>
<td>2) $\overline{EM} \cong \overline{EM}$</td>
<td></td>
</tr>
<tr>
<td>3) $\triangle MTE \cong \triangle EAM$</td>
<td></td>
</tr>
<tr>
<td>4) $\overline{TM} \cong \overline{AE}$</td>
<td></td>
</tr>
<tr>
<td>5) TEAM is a parallelogram</td>
<td></td>
</tr>
</tbody>
</table>
Option B

We are given that \( \overline{TE} \cong \overline{MA} \) and \( \angle 2 \cong \angle 3 \).

\( \overline{TE} \parallel \overline{AM} \), by the alternate interior angles converse.

So, TEAM is a parallelogram because...
Given: FGHJ is a parallelogram, ∠F is a right angle
Prove: FGHJ is a rectangle
## Proofs

**Given:** FGHJ is a parallelogram, $\angle F$ is a right angle  
**Prove:** FGHJ is a rectangle

<table>
<thead>
<tr>
<th>statements</th>
<th>reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) FGHJ is a parallelogram and $\angle F$ is a right angle</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) $\angle J$ and $\angle G$ are right angles</td>
<td></td>
</tr>
<tr>
<td>3) $\angle H$ is a right angle</td>
<td></td>
</tr>
<tr>
<td>4) TEAM is a rectangle</td>
<td></td>
</tr>
</tbody>
</table>
Problems

**Given:** COLD is a quadrilateral, \( m\angle O = 140^\circ, m\angle D = 40^\circ, m\angle L = 60^\circ \)

**Prove:** COLD is a trapezoid
**Given:** COLD is a quadrilateral, \(m \angle O = 140^\circ, m \angle D = 40^\circ, m \angle L = 60^\circ\)

**Prove:** COLD is a trapezoid

<table>
<thead>
<tr>
<th>statements</th>
<th>reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) COLD is a quadrilateral, (m \angle O = 140^\circ, m \angle L = 40^\circ, m \angle D = 60^\circ)</td>
<td>1) Given</td>
</tr>
<tr>
<td>2) (m \angle O + m \angle L = 180^\circ) (m \angle L + m \angle D = 100^\circ)</td>
<td>3) Definition of Supplementary Angles</td>
</tr>
<tr>
<td>4) (\angle L) and (\angle D) are not supplementary</td>
<td></td>
</tr>
<tr>
<td>5) (\overline{CL} \parallel \overline{OD})</td>
<td>5) Consecutive Interior Angles Converse</td>
</tr>
<tr>
<td>6) COLD is a trapezoid</td>
<td></td>
</tr>
</tbody>
</table>
Try this ...

**Given:** \( \Delta FCD \cong \Delta FED \)

**Prove:** \( FD \perp CE \)
Coordinate Proofs
Quadrilateral Coordinate Proofs

Recall that Coordinate Proofs place figures on the Cartesian Plane to make use of the coordinates of key features of the figure to help prove something. Usually the distance formula, midpoint formula and slope formula are used in a coordinate proof.

We'll provide a few examples and then have you do some proofs.
Coordinate Proofs

**Given:** PQRS is a quadrilateral

**Prove:** PQRS is a kite
A kite has one unique property. The adjacent sides are congruent.

\[
\begin{align*}
PS &= (6 - 3)^2 + (-1 - (-4))^2 \\
&= 3^2 + 3^2 \\
&= 9 + 9 \\
&= 18 \\
&= 4.24 \\
\end{align*}
\]
Because $PS = PQ$ and $RS = RQ$, $PQRS$ is a kite.
Coordinate Proofs

Given: JKLM is a parallelogram
Prove: JKLM is a square
Since JKLM is a parallelogram, we know the opposite sides are parallel and congruent. We also know that a square is a rectangle and a rhombus. We need to prove the adjacent sides are congruent and perpendicular.

Let's start by proving the adjacent sides are congruent.

\[
\begin{align*}
JM &= (3 - 0)^2 + (1 - (-3))^2 \\
&= 3^2 + 4^2 \\
&= 9 + 16 \\
&= 25 \\
&= 5 \\
JK &= (-1 - 3)^2 + (4 - 1)^2 \\
&= (-4)^2 + 3^2 \\
&= 9 + 16 \\
&= 25 \\
&= 5
\end{align*}
\]
Now, let's prove that the adjacent sides are perpendicular.

\[ m_{JM} = \frac{3 - (-3)}{1 - (-3)} = \frac{6}{4} = \frac{3}{2} \]
\[ m_{JK} = \frac{-1 - (-3)}{4 - 1} = \frac{2}{3} \]

\[ m_{JM} \perp m_{JK} \]

Therefore, JKLM is a square.
Coordinate Proofs

Try this ...

**Given:** PQRS is a trapezoid

**Prove:** \( \overline{LM} \) is the midsegment

---

**Hint**

Remember, the midsegment is the segment that joins the midpoints of the legs and is parallel to the bases.

You need to show that:

1. \( SL = LP \)
2. \( QM = MR \)
3. slope of \( LM \) = slope of \( SR \)
Quadrilateral Coordinate Proofs

Sometimes, shapes might be placed in a coordinate plane, but without numbered values. Instead, algebraic expressions will be used for the coordinates. You can still use the distance formula, midpoint formula and slope formula in this type of coordinate proof.

We'll provide a few examples and then have you do some proofs.
Quadrilateral Coordinate Proofs

**Given:** EFGH is a parallelogram

**Prove:** EFGH is a rectangle
Since EFGH is a parallelogram, we know the opposite sides are parallel and congruent. Since we need to prove that EFGH is a rectangle, we can either prove that adjacent sides are perpendicular or diagonals are congruent.

Let's choose the 1st option: proving the adjacent sides are perpendicular.
Quadrilateral Coordinate Proofs

$m_{EF} = \frac{p - q}{r - r} = 0$

$m_{FG} = \frac{p - p}{r - u} = \text{undef.}$

EF $\perp$ FG and both pairs of opposite sides are $\parallel$.

EF $\cong$ GH and FG $\cong$ EH (Opposite sides are congruent)

EF $\perp$ EH and FG $\perp$ GH (Perpendicular Transversal Theorem)

Therefore, EFGH is a rectangle
Quadrilateral Coordinate Proofs

Since EFGH is a parallelogram, we know the opposite sides are parallel and congruent. Since we need to prove that EFGH is a rectangle, we can either prove that adjacent sides are perpendicular or diagonals are congruent.

This problem can also be done using the 2nd option: proving that the diagonals are congruent.
We proved that $\triangle EG \cong \triangle HF$ (Diagonals are congruent).

We also know that $\triangle EF \cong \triangle GH$ and $\triangle FG \cong \triangle EH$ (Opp. sides are congruent).

Therefore, $EFGH$ is a rectangle.

$EG = \sqrt{(p - q)^2 + (u - r)^2} = \sqrt{(p - q)^2 + (-(u - r))^2} = \sqrt{(p - q)^2 + (u - r)^2}$

$HF = \sqrt{(p - q)^2 + (r - u)^2}$
Given: JKLM is a parallelogram
Prove: JL and KM bisect each other

Quadrilateral Coordinate Proofs
Since JKLM is a parallelogram, we know the opposite sides are parallel and congruent and the diagonals bisect each other. If we can prove the last part, that the diagonals bisect each other using one of our formulas, we will achieve our goal.

Which formula would prove that the diagonals bisect each other?
Midpoint of KM: U(0, b)

Midpoint of JL: U(0, b)

Because the midpoint U is the same for both of the diagonals, it proves that KM & JL bisect each other.
If Quadrilateral STUV is a rectangle, what would be the coordinates for point U?

A  U (e, d)
B  U (d + e, e)
C  U (d, e)
D  U (d, e - d)
138 If Quadrilateral OPQR is a parallelogram, what would be the coordinates for point Q?

A  Q (b, a)

B  Q (b + c, a)

C  Q (b + c, a + c)

D  Q (b, a + c)

Answer: D
If Quadrilateral ABCD is an isosceles trapezoid, what would be the coordinates for point B?

A  B (-2f, 2h)
B  B (-2g, 2h)
C  B (-2g, -2h)
D  B (2f, 2h)

Answer: B
Since ABCD is an isosceles trapezoid, $\overline{BD}$ and $\overline{AC}$ are congruent. Use the coordinates to verify that $\overline{BD}$ and $\overline{AC}$ are congruent.

When you are finished, type in the number "1".
PARCC Released Questions

The remaining slides in this presentation contain questions from the PARCC Sample Test. After finishing unit 10, you should be able to answer these questions.

Good Luck!

Return to the Table of Contents
Let $BE = x^2 - 48$ and let $DE = 2x$. What are the lengths of $BE$ and $DE$? Justify your answer.

**Part A**

**141** Let $BE = x^2 - 48$ and let $DE = 2x$. What are the lengths of $BE$ and $DE$? Justify your answer.

**PARCC Released Question - PBA - Calculator Section - #7**
The figure shows parallelogram ABCD with $AE = 16$.

![Parallelogram ABCD with AE = 16](image)

**Part B**

142 What conclusion can be made regarding the specific classification of parallelogram $ABCD$? Justify your answer.

Once you are finished with the question, type in the word to describe the classification of parallelogram $ABCD$.

**PARCC Released Question - PBA - Calculator Section - #7**
The figure shows parallelogram PQRS on a coordinate plane. Diagonals SQ and PR intersect at point T.

Part A

143 Find the coordinates of point Q in terms of a, b, and c. Enter only your answer.

PARCC Released Question - PBA - Calculator Section - #8
Part B

144 Since PQRS is a parallelogram, \( SQ \) and \( PR \) bisect each other. Use the coordinates to verify that \( SQ \) and \( PR \) bisect each other.

Which formula(s) could be used to answer this question?

A  Distance Formula
B  Slope
C  Midpoint Formula
D  A and C only
E  A, B and C

PARCC Released Question - PBA - Calculator Section - #8
145 Since PQRS is a parallelogram, \( SQ \) and \( PR \) bisect each other. Use the coordinates to verify that \( SQ \) and \( PR \) bisect each other. When you are finished, type in the word "Done".
One method that can be used to prove that the diagonals of a parallelogram bisect each other is shown in the given partial proof.

**Given:** Quadrilateral PQRS is a parallelogram.
**Prove:** PT = RT, ST = QT
Part A

146 Which reason justifies the statement for step 3 in the proof?

A When two parallel lines are intersected by a transversal, same side interior angles are congruent.
B When two parallel lines are intersected by a transversal, alternate interior angles are congruent.
C When two parallel lines are intersected by a transversal, same side interior angles are supplementary.
D When two parallel lines are intersected by a transversal, alternate interior angles are supplementary.

PARCC Released Question - EOY - Calculator Section - #19
Part B

147 Which statement is justified by the reason for step 4 in the proof?

A  \( \overline{PQ} \cong \overline{RS} \)
B  \( \overline{PQ} \cong \overline{SP} \)
C  \( \overline{PT} \cong \overline{TR} \)
D  \( \overline{SQ} \cong \overline{PR} \)

PARCC Released Question - EOY - Calculator Section - #19
Part C
148 Which reason justifies the statement for step 5 in the proof?
   A side-side-side triangle congruence
   B side-angle-side triangle congruence
   C angle-side-angle triangle congruence
   D angle-angle-side triangle congruence
Part D

Another method of proving diagonals of a parallelogram bisect each other uses a coordinate grid.
149 What could be shown about the diagonals of parallelogram PQRS to complete the proof?

A  \( PR \) and \( SQ \) have the same length.
B  \( PR \) is a perpendicular bisector of \( SQ \).
C  \( PR \) and \( SQ \) have the same midpoint.
D  Angles formed by the intersection of \( PR \) and \( SQ \) each measure 90°.

PARCC Released Question - EOY - Calculator Section - #19
Throughout this unit, the Standards for Mathematical Practice are used.

MP1: Making sense of problems & persevere in solving them.
MP2: Reason abstractly & quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for & make use of structure.
MP8: Look for & express regularity in repeated reasoning.

Additional questions are included on the slides using the "Math Practice" Pull-tabs (e.g. a blank one is shown to the right on this slide) with a reference to the standards used.

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.