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Geometry

Analytic Geometry

2015-10-26

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click on the topic to go to that section

Throughout this unit, the Standards for Mathematical Practice are used.

MP1: Making sense of problems & persevere in solving them.
MP2: Reason abstractly & quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for & make use of structure.
MP8: Look for & express regularity in repeated reasoning.

Additional questions are included on the slides using the "Math Practice" Pull-tabs (e.g. a blank one is shown to the right on this slide) with a reference to the standards used.

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.

Math Practice

Origin of Analytic Geometry

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The Origin of Analytic Geometry

Analytic Geometry is a powerful combination of geometry and algebra.

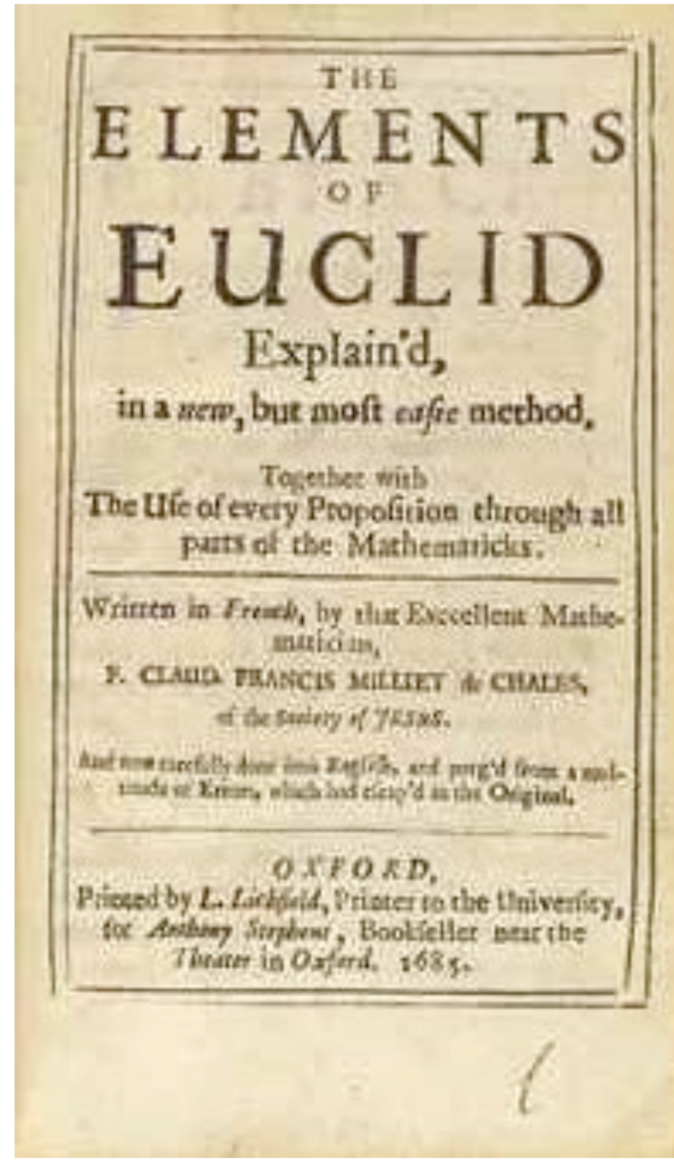
Many jobs that are looking for employees now, and will be in the future, rely on the process or results of analytic geometry.

This includes jobs in medicine, veterinary science, biology, chemistry, physics, mathematics, engineering, financial analysis, economics, technology, biotechnology, etc.

The Origin of Analytic Geometry

Euclidean Geometry

- Was developed in Greece about 2500 years ago.
- Was lost to Europe for more than a thousand years.
- Was maintained and refined during that time in the Islamic world.
- Its rediscovery was a critical part of the European Renaissance.



THE
ELEMENTS
OF
EUCLID
Explain'd,
in a new, but most easie method,
Together with
The Use of every Proposition through all parts of the Mathematicks.

Written in French, by that Excellent Mathematician,
F. CLAUD FRANCIS MILLIET de CHALES,
of the Society of JESUS.

And now carefully done into English, and purg'd from a multitude of Errors, which had creep'd in the Original.

OXFORD,
Printed by L. Lichfield, Printer to the University,
for Anthony Stephens, Bookseller near the
Theater in Oxford. 1685.

http://www.christies.com/lotfinder/books-manuscripts/euclid-milliet-dechales-claude-francois-milliet-5541389-details.aspx

The Origin of Analytic Geometry

Algebra

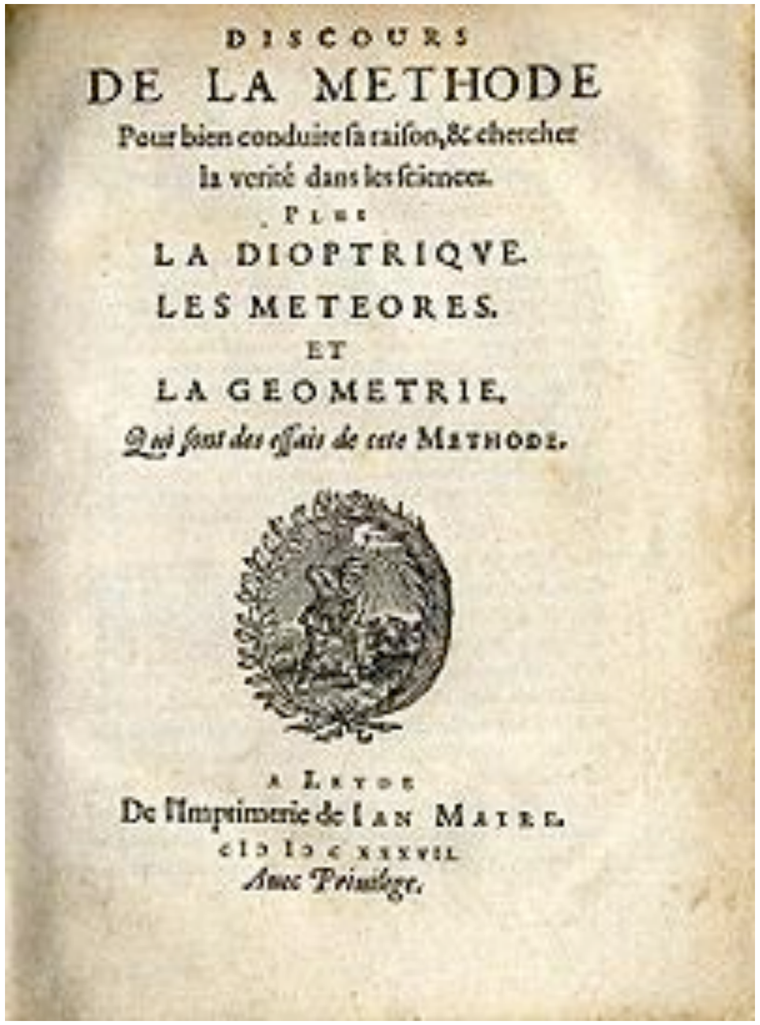
- Started by Diophantus in Alexandria about 1700 years ago.
- Ongoing contributions from Babylon, Syria, Greece and Indians.
- Named from the Arabic word al-jabr which was used by al-Khwarizmi in the title of his 7th century book.

http://en.wikipedia.org/wiki/Mathematics_in_medieval_Islam

The Origin of Analytic Geometry

Analytic Geometry

- A powerful combination of algebra and geometry.
- Independently developed, and published in 1637, by Rene Descartes and Pierre de Fermat in France.
- The Cartesian Plane is named for Descartes.



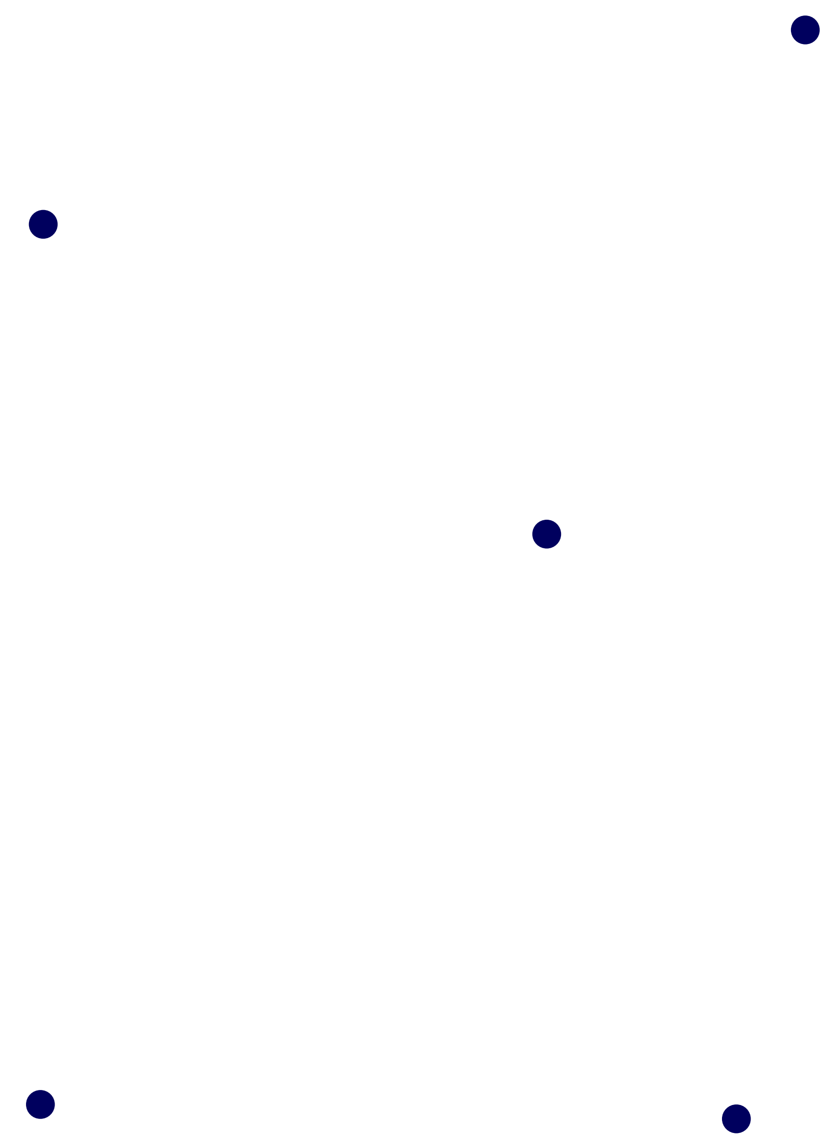
DISCOURS
DE LA METHODE
Pour bien conduire ſa raiſon, & chercher
la verité dans les ſciences.
PLUS
LA DIOPTRIQVE.
LES METEORES.
ET
LA GEOMETRIE.
Qui ſont des eſſais de cete METHODE.

A LEYDE
De l'Imprimerie de IAN MAIRE.
CIↃ IↃ C XXXVII.
Avec Privilege.

The Origin of Analytic Geometry

How would you describe to someone the location of these five points so they could draw them on another piece of paper without seeing your drawing?

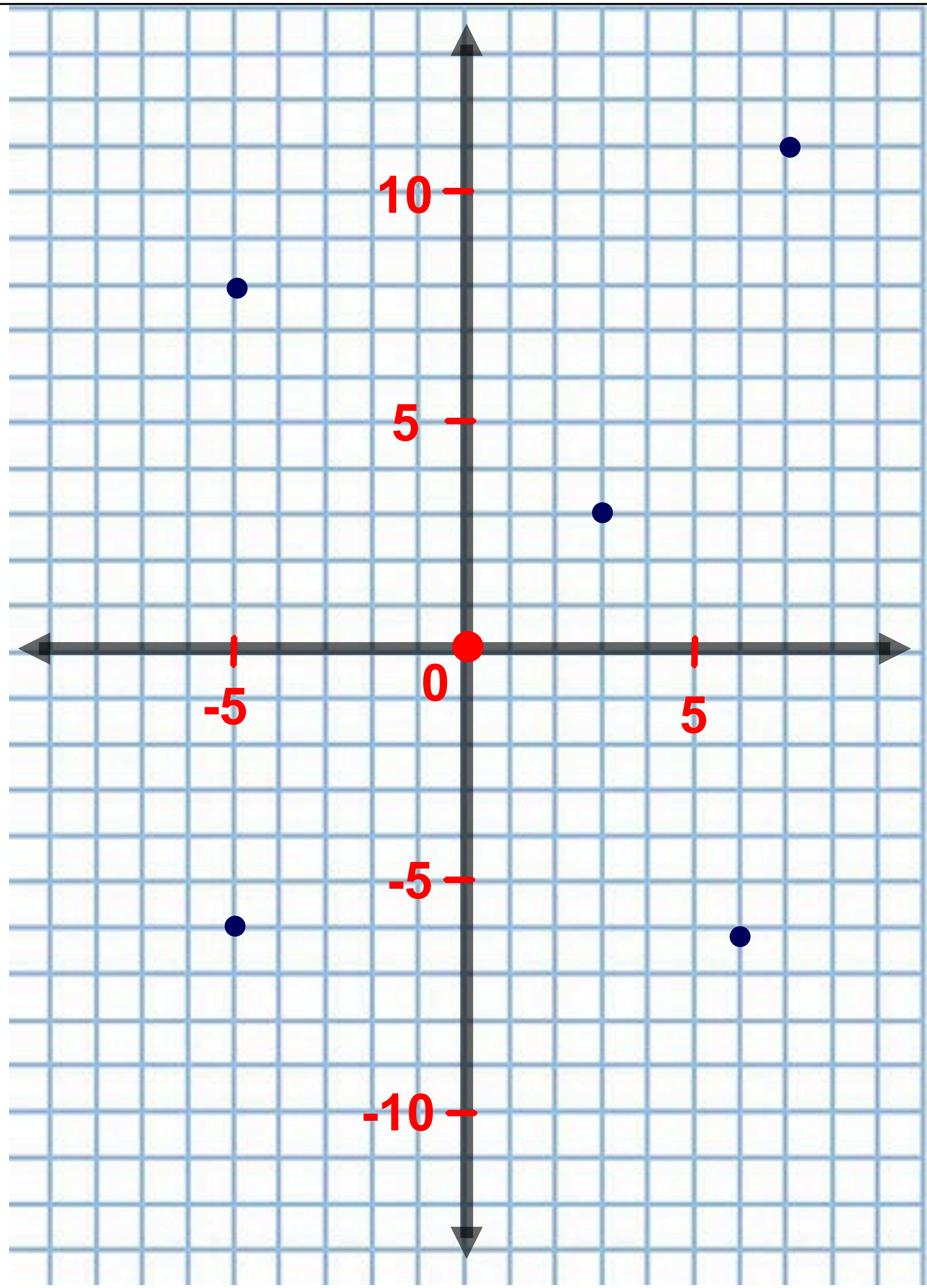
Discuss.



Math Practice

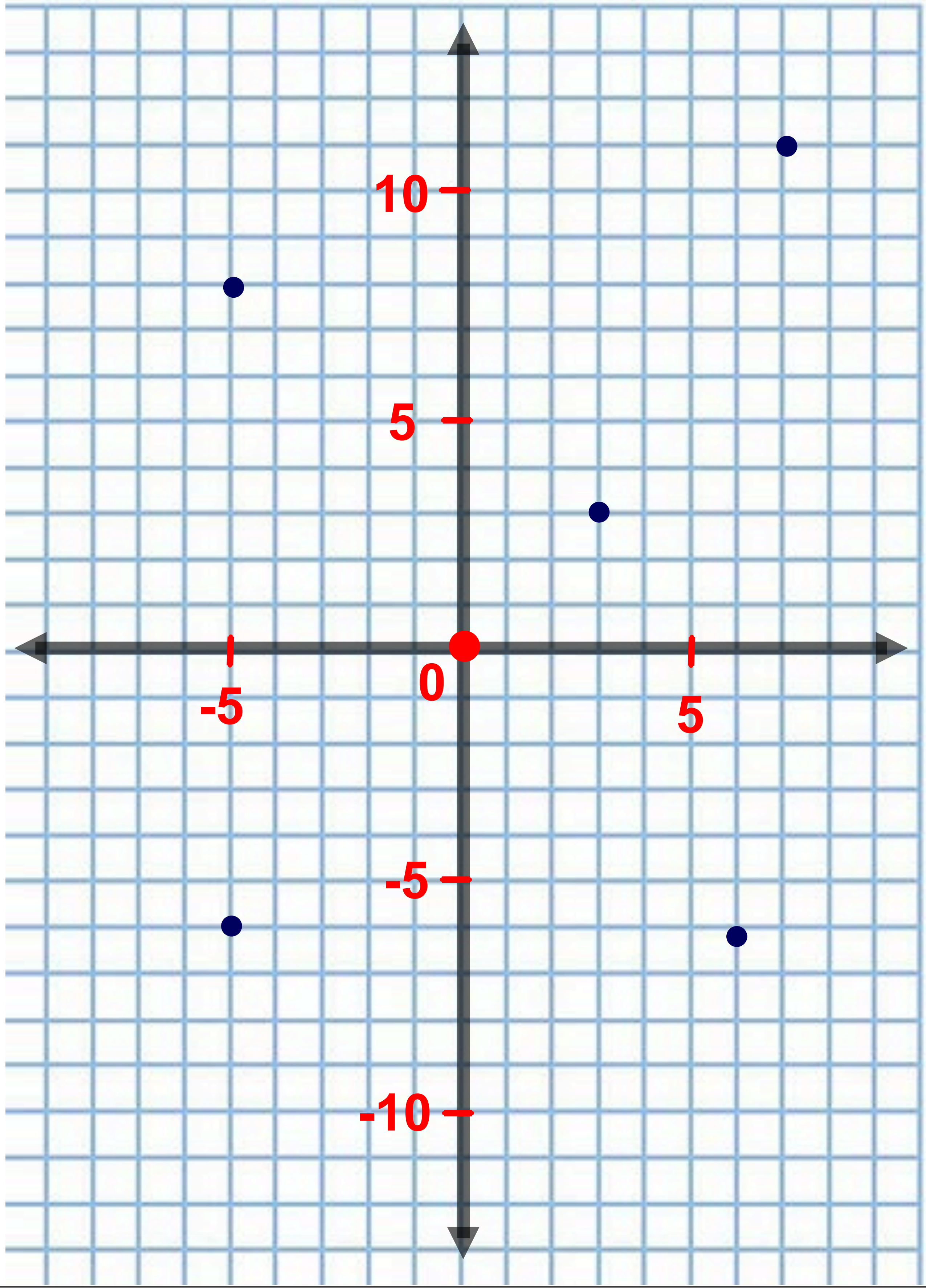
The Origin of Analytic Geometry

Adding this Cartesian coordinate plane makes that task simple since the location of each point can be given by just two numbers: an x- and y-coordinate, written as the ordered pair (x,y).



The Origin of Analytic Geometry

With the Cartesian Plane providing a numerical description of locations on the plane, geometric figures can be analyzed using algebra.



The Distance Formula

Lab: Derivation of the Distance Formula

Teacher Note

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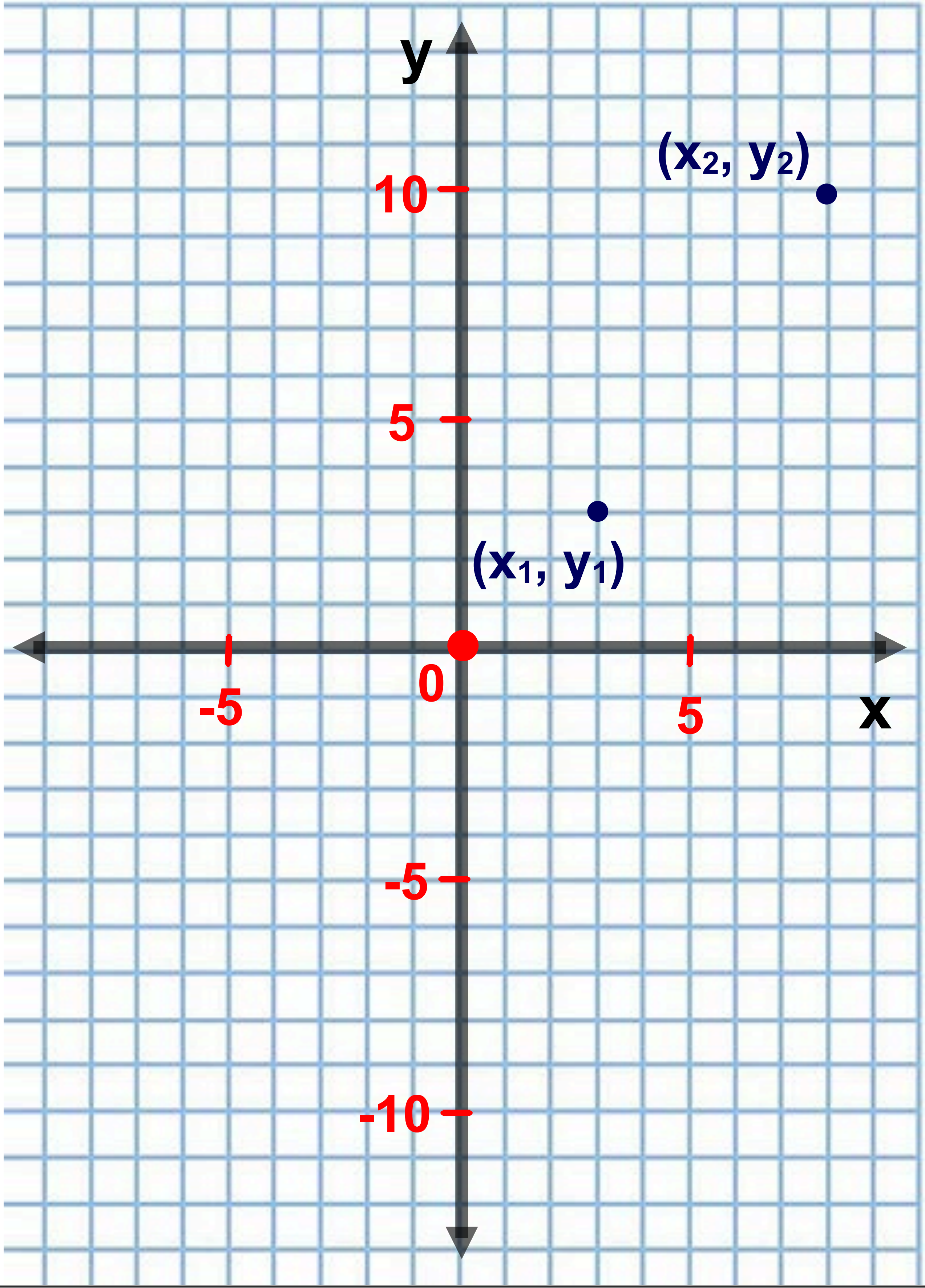
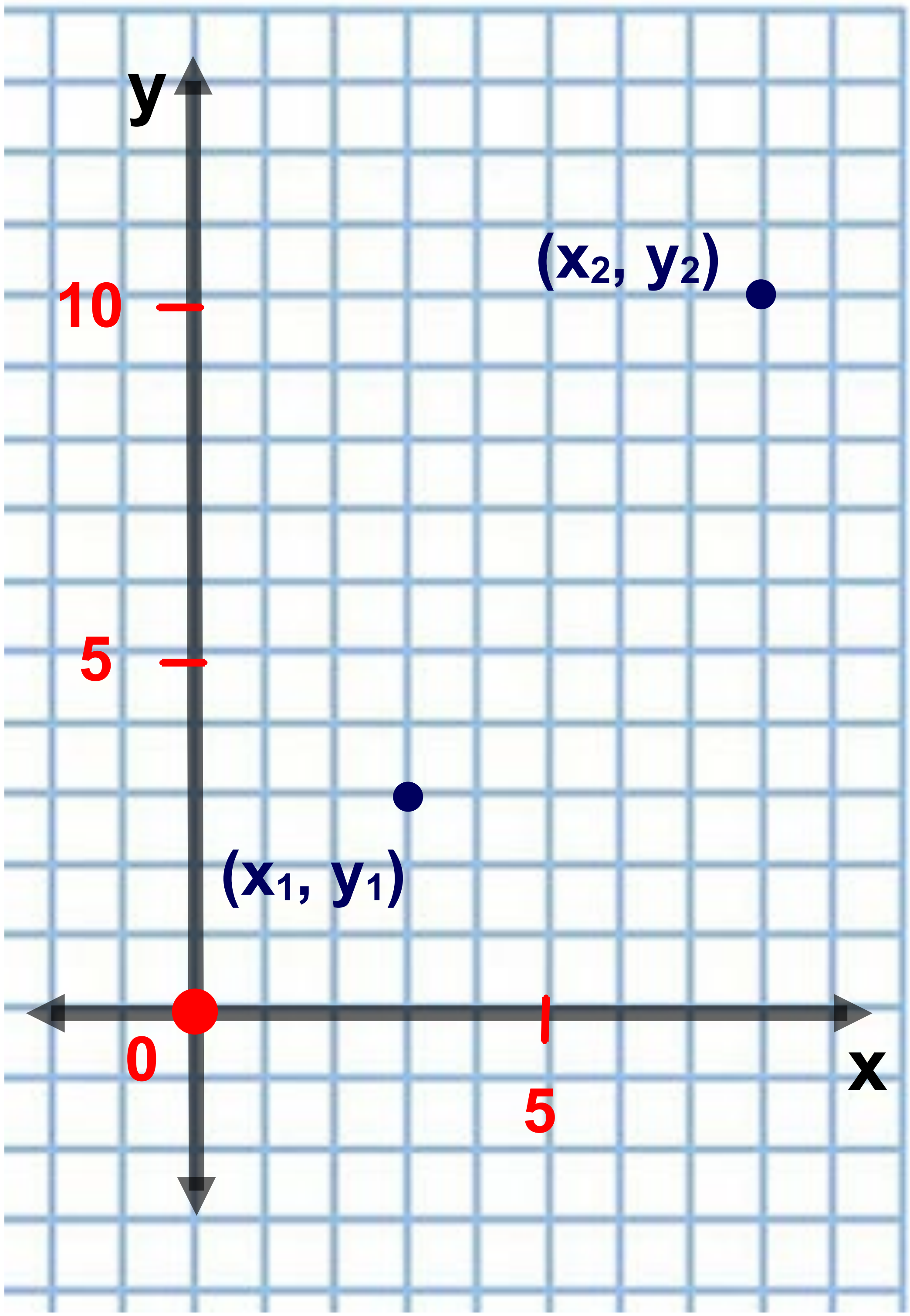


The Distance Formula

Let's derive the formula to find the distance between any two points: call the points (x_1, y_1) and (x_2, y_2).

First, let's zoom in so we have more room to work.

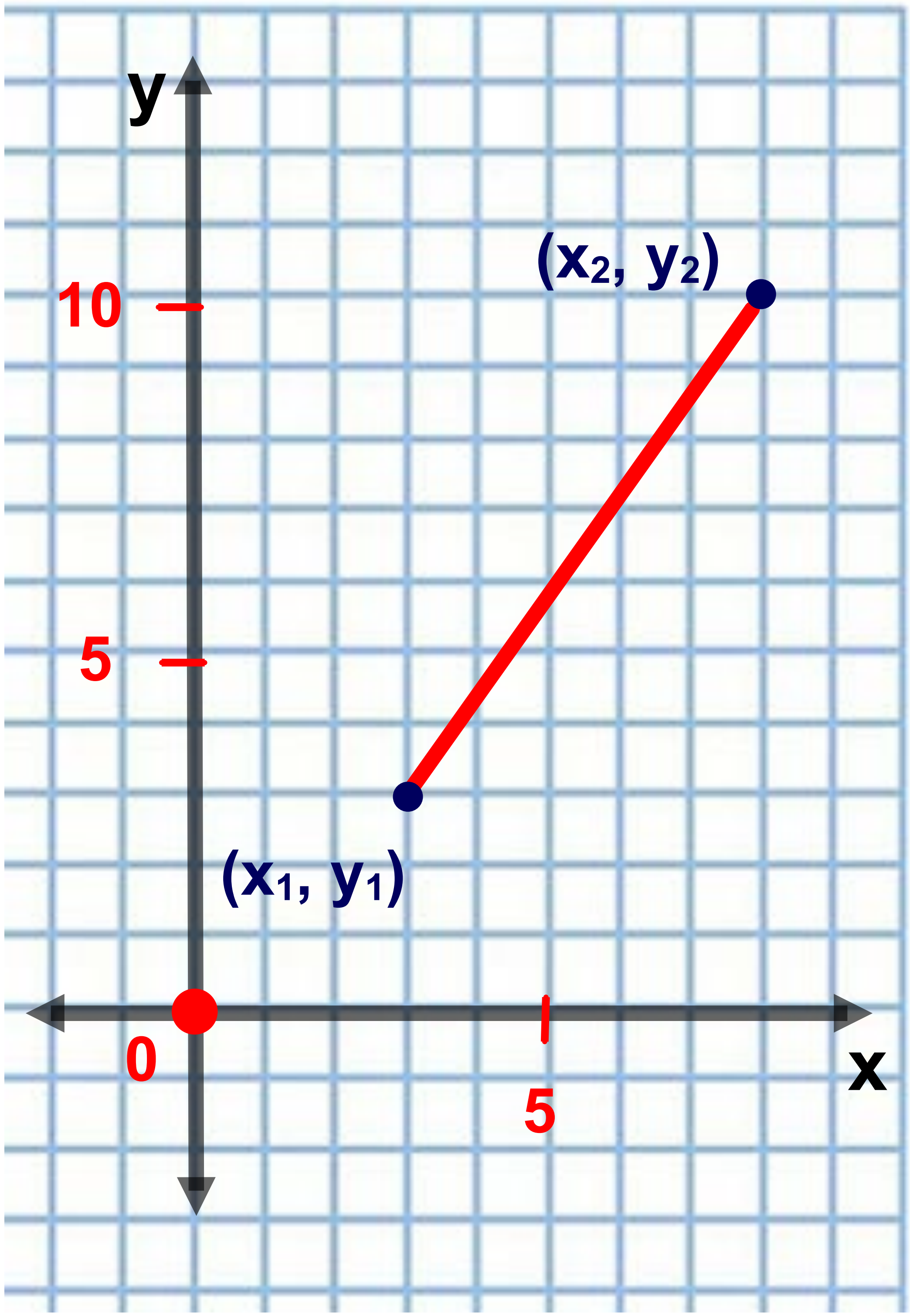
The Distance Formula



Now, we'll construct two paths between the points.

The first path will be directly between them.

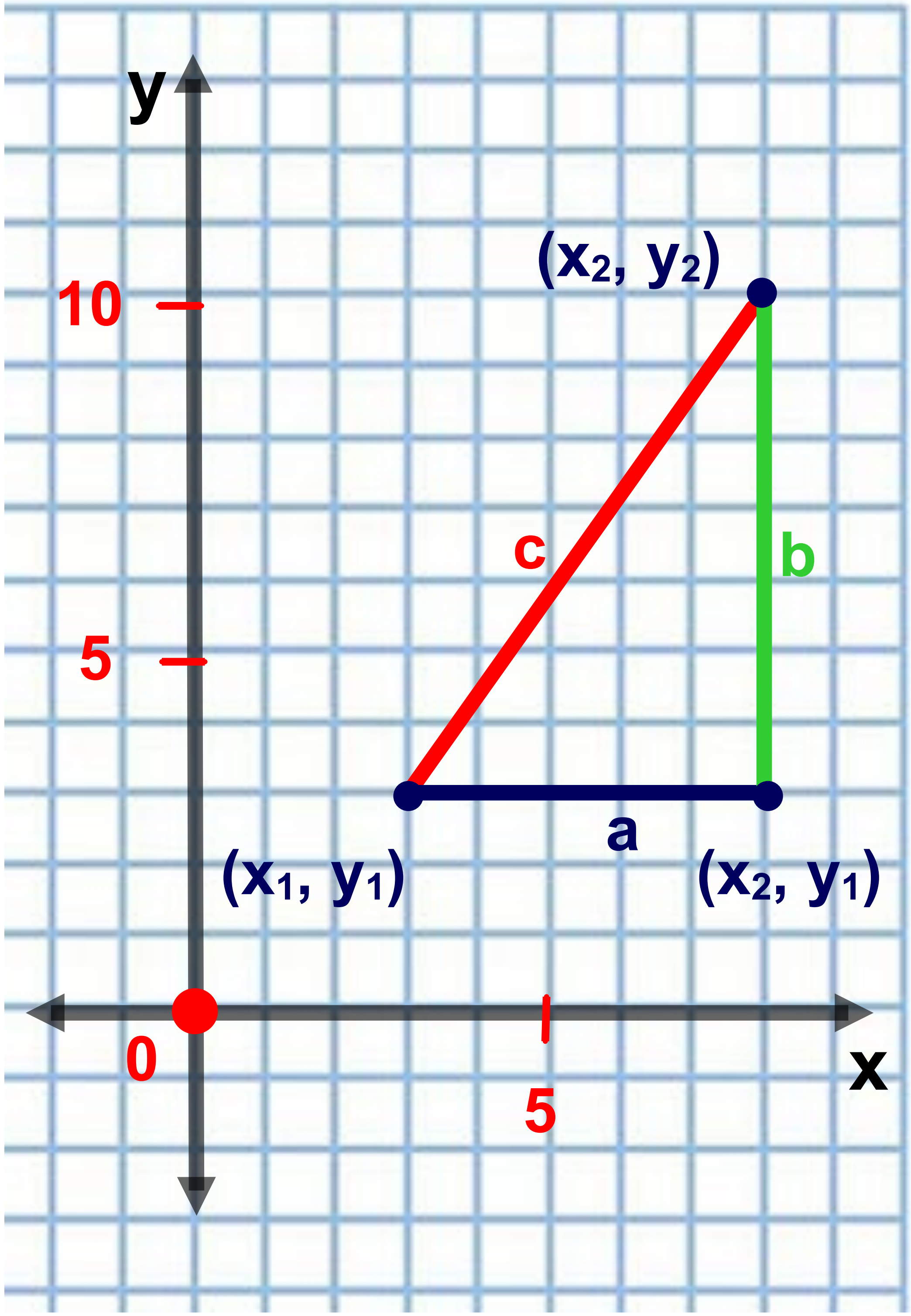
The Distance Formula



The second path will have a segment parallel the x-axis and a segment parallel to the y-axis.

That will be a big help since it's easy to read the length of each of those from the axis.

The Distance Formula



We now have a right triangle.

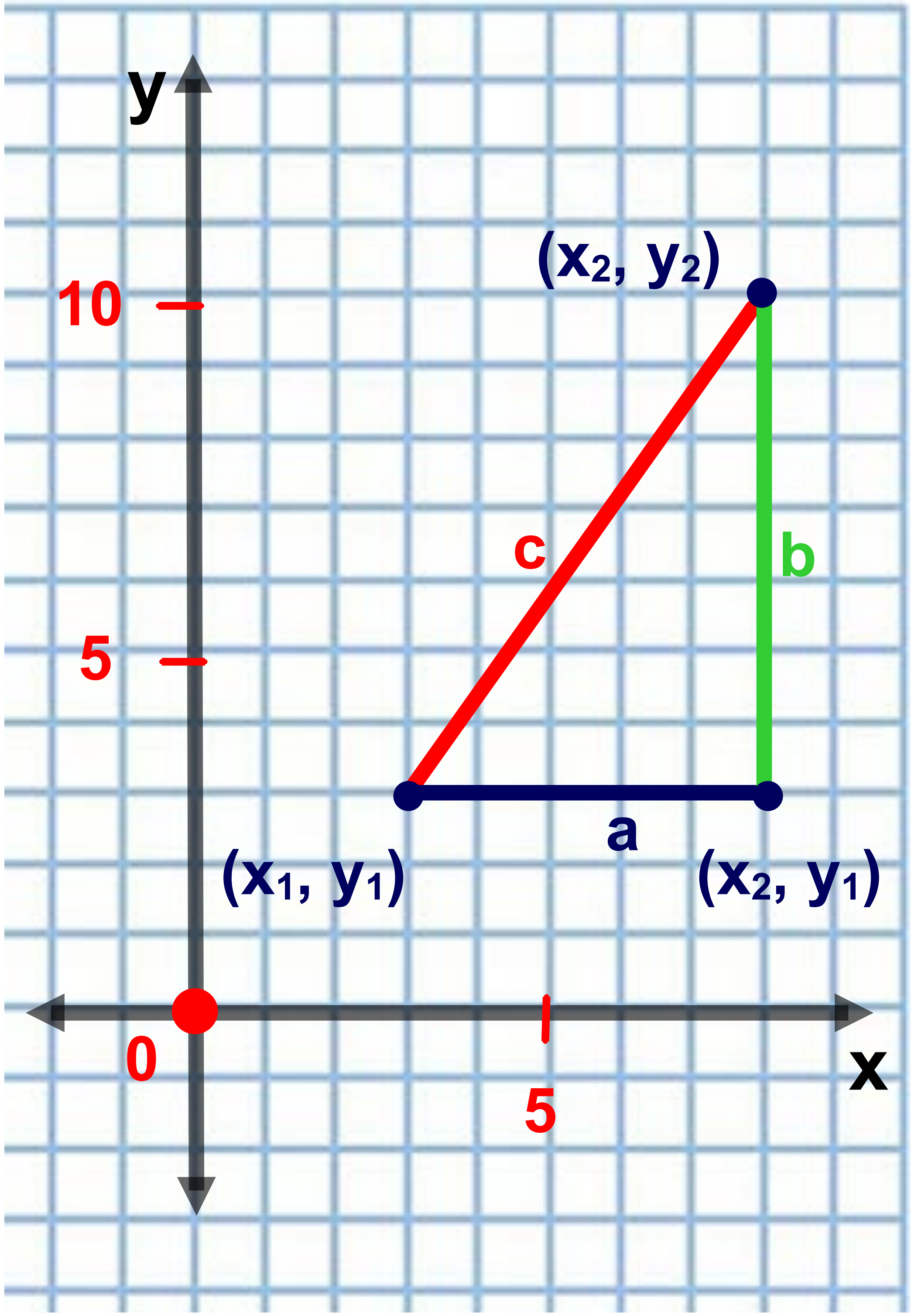
For convenience, let's label the two legs "a" and "b" and the hypotenuse "c."

Let's also label the point where the two legs meet by its coordinates: (x_2, y_1).

Take a moment to see that those are the coordinates of that vertex of the triangle.

Which formula relates the lengths of the sides of a right triangle?

The Distance Formula



Did you get this?

$c^2 = a^2 + b^2$

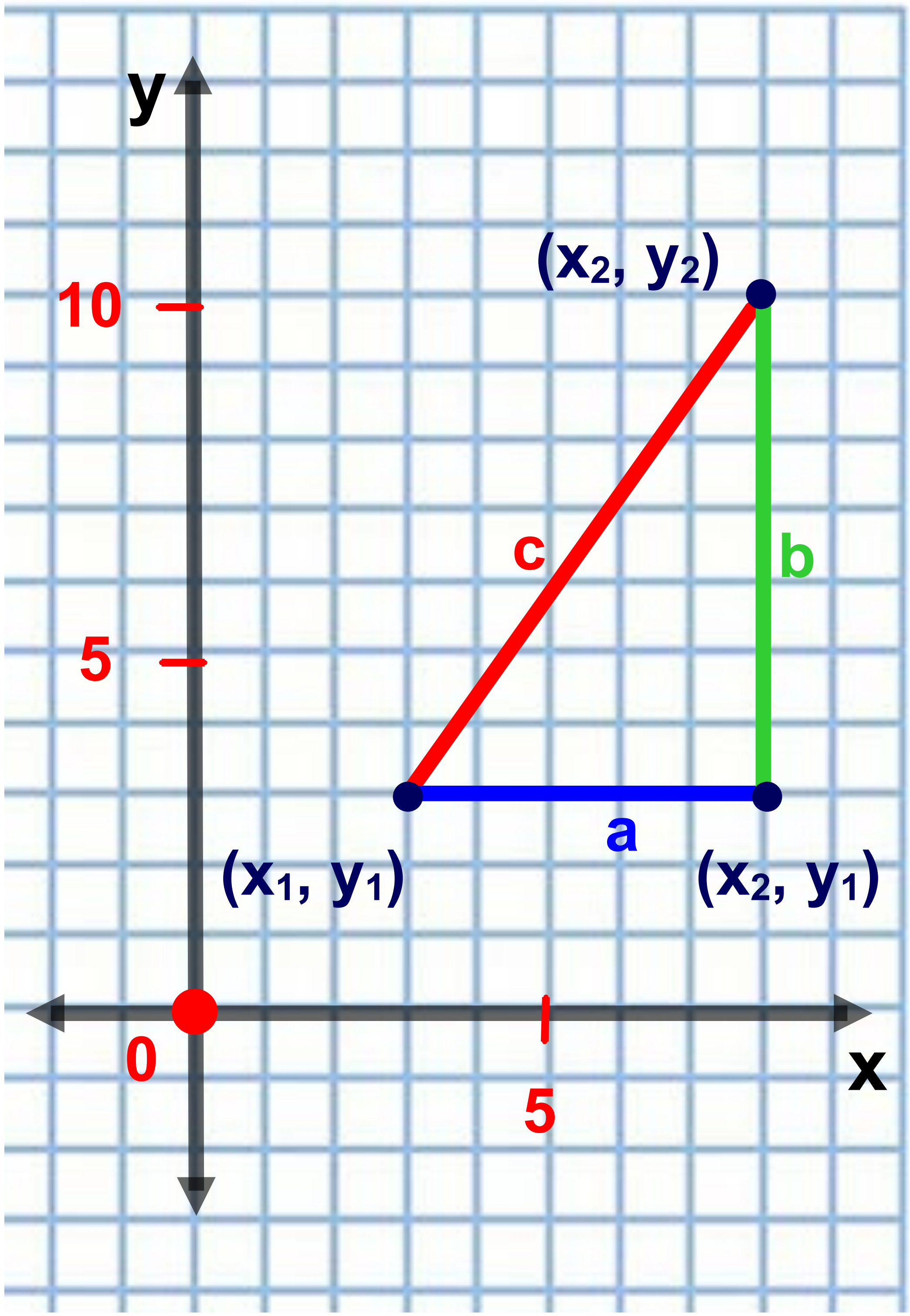
The next step is to write expressions for the lengths of sides a and b based on the x and y subscript coordinates.

Use the distance along each axis to find those.

The coordinates that we just added for the third vertex should help.

Math Practice

The Distance Formula



Did you get these?

$a = |x_2 - x_1|$ AND $b = |y_2 - y_1|$

We could equally well write

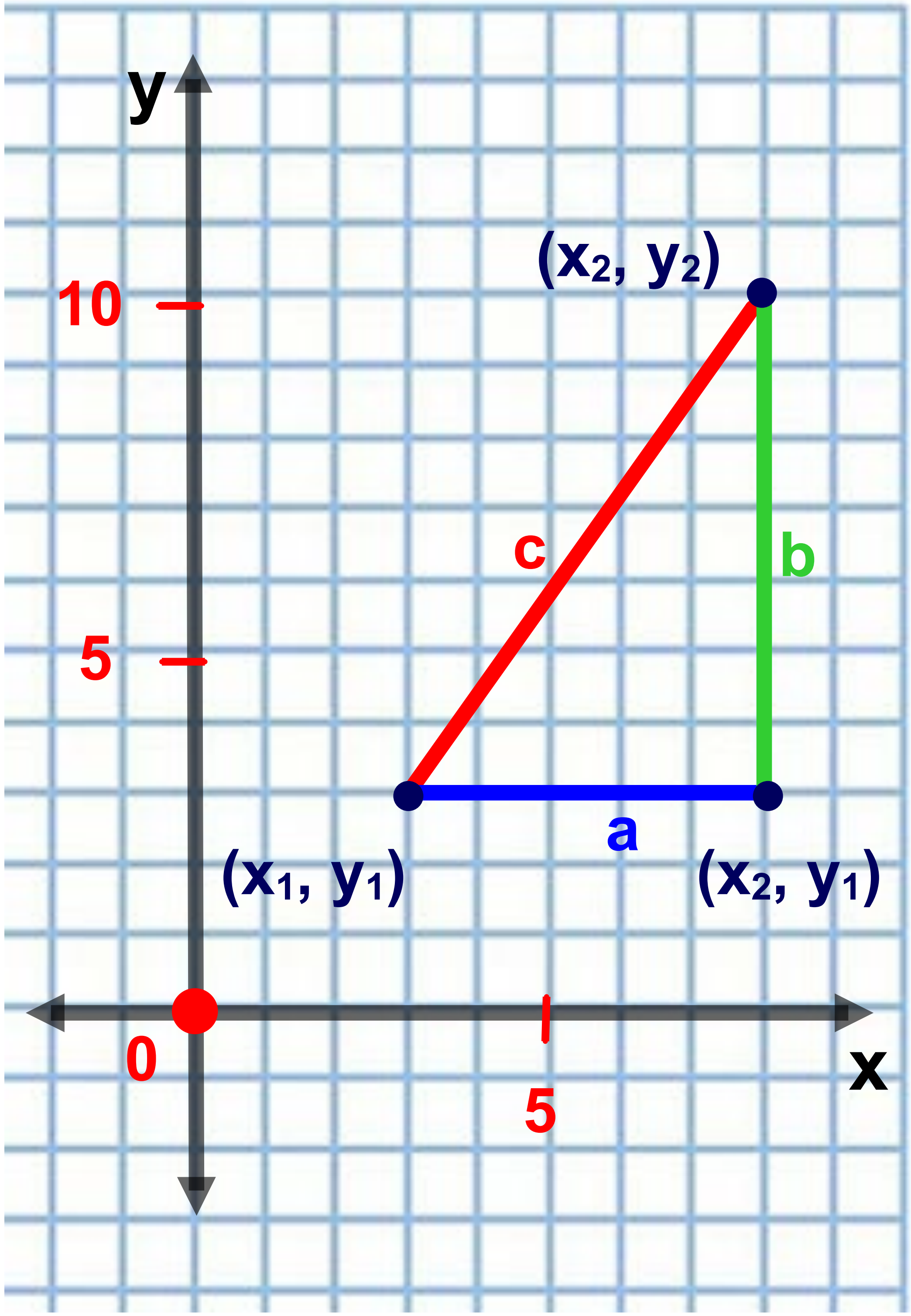
$a = |x_1 - x_2|$ AND $b = |y_1 - y_2|$

We use absolute values since we are just concerned with lengths, which are always positive. That's why the order doesn't matter and all of the above are OK.

Substitute one pair of these into

$\mathbf{c^2 = a^2 + b^2}$

The Distance Formula



We'll use the first pair:

$c^2 = (|x_2 - x_1|)^2 + (|y_2 - y_1|)^2$

Since the quantities are squared, we don't need to indicate absolute value, the result will always be positive anyway.

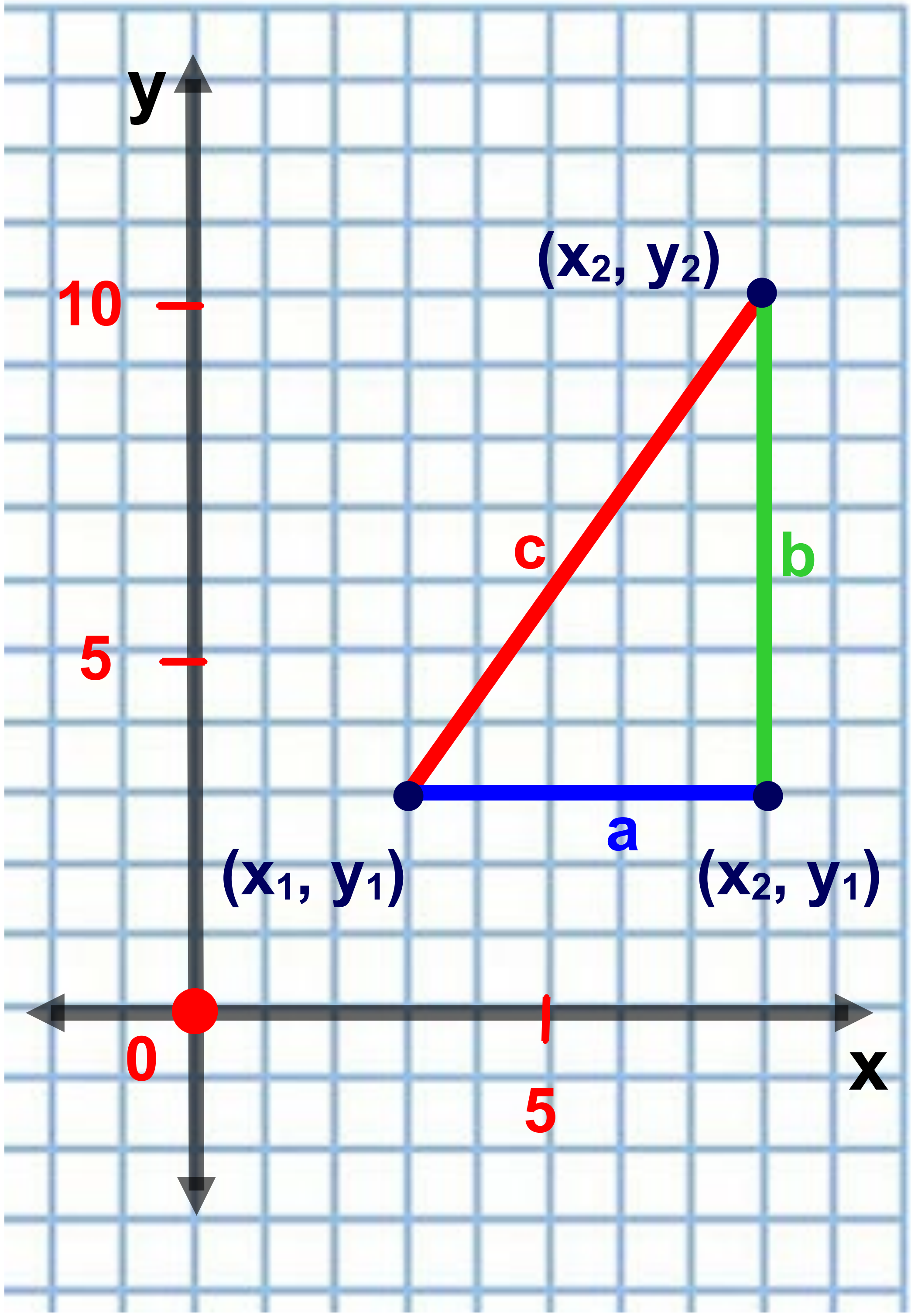
$c^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Since "c" is the distance between the points, we can use "d" for distance and say that

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

If we solve this equation for d, what will be the final formula?

The Distance Formula



$$d = ((x_2 - x_1)^2 + (y_2 - y_1)^2)^{1/2}$$

OR

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

1 What is the distance between the points: (4, 8) and (7, 3)? Round your answer to the nearest hundredth.

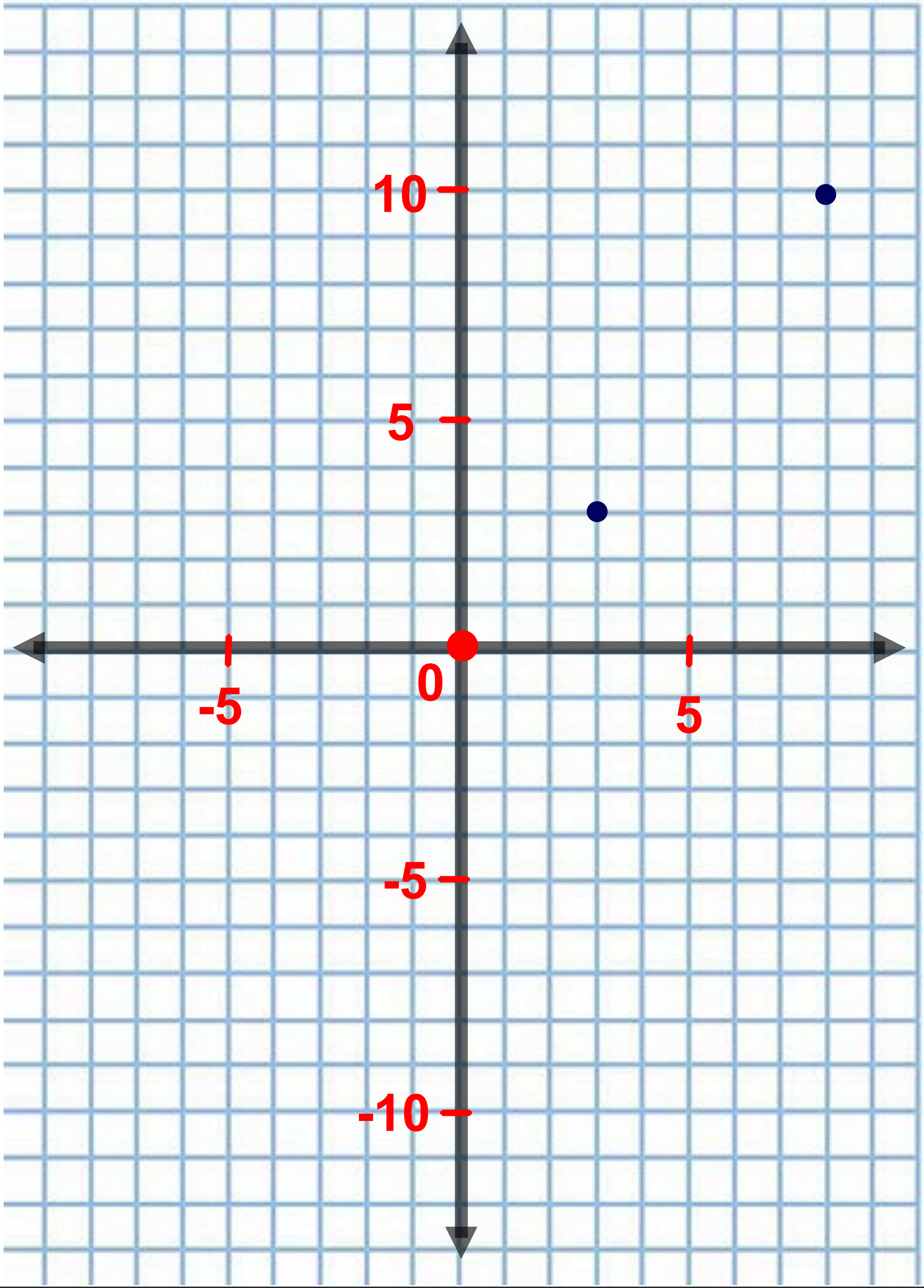
Answer

2 What is the distance between the points: (-4, 8) and (7, -3)? Round your answer to the nearest hundredth.

Answer

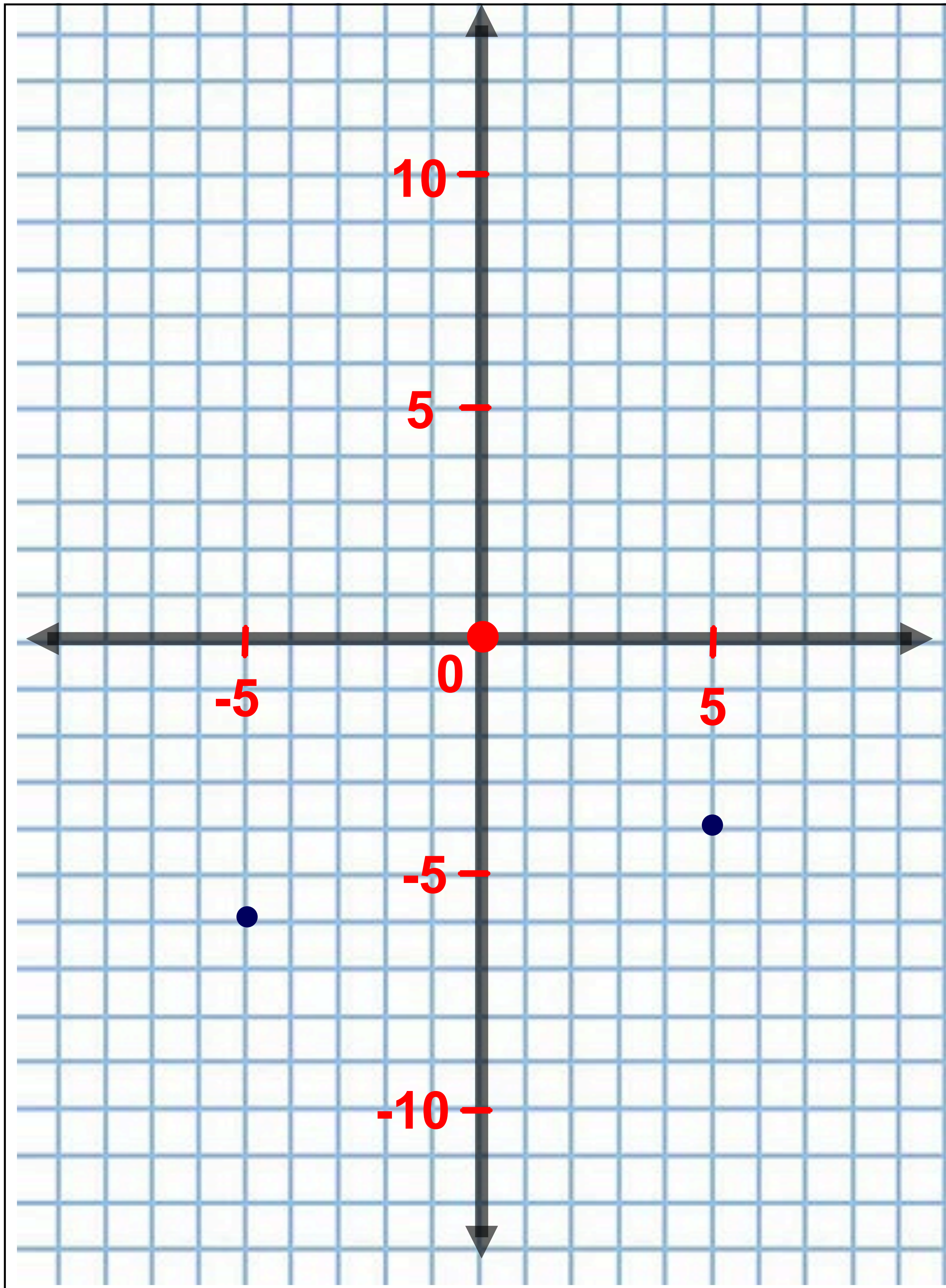
3 What is the distance between the points: (-2, -5) and (-7, 3)? Round your answer to the nearest hundredth.

Answer



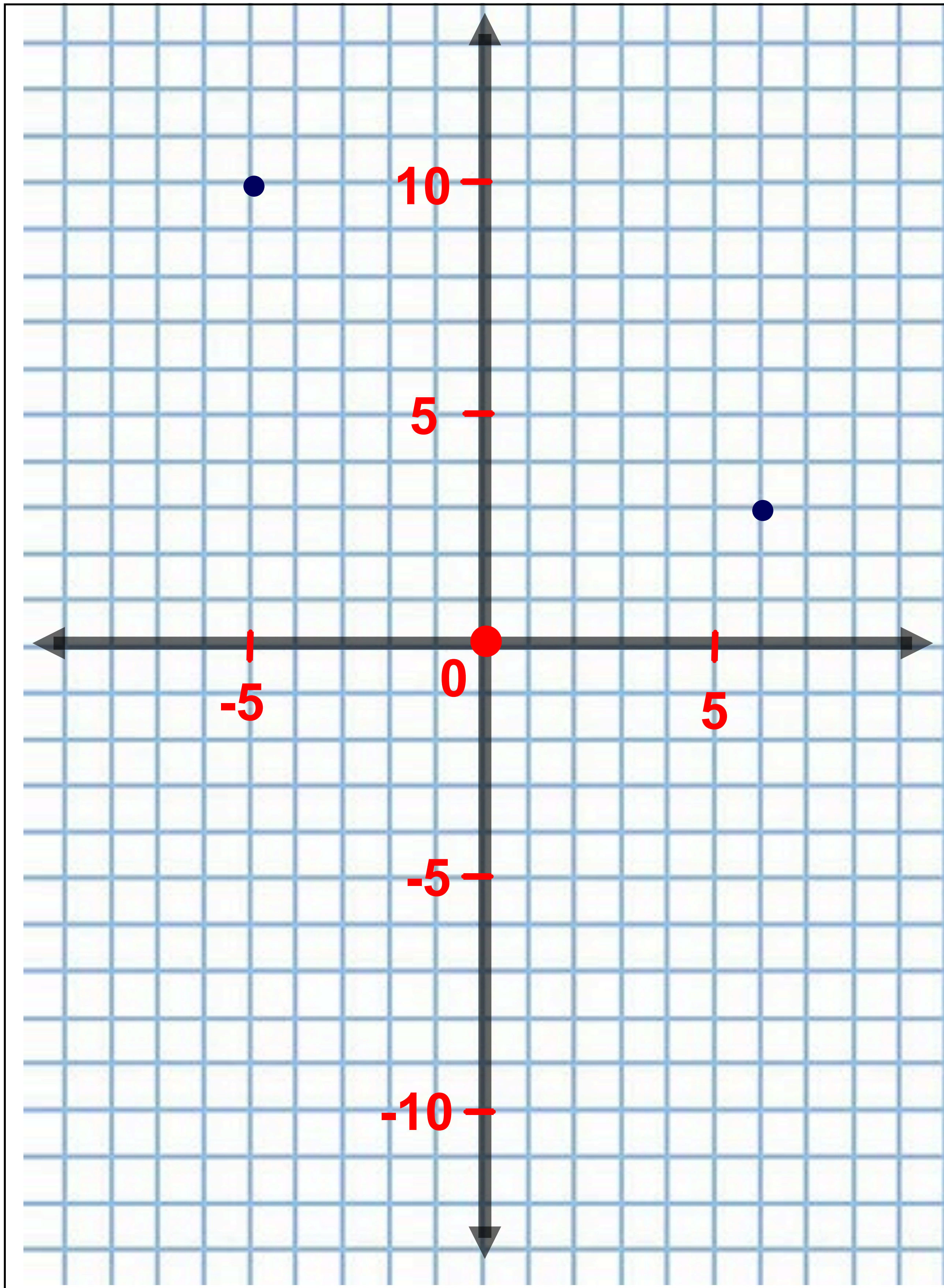
4 What is the distance between the indicated points? Round your answer to the nearest hundredth.

Answer



5 What is the distance between the indicated points? Round your answer to the nearest hundredth.

Answer



6 What is the distance between the indicated points? Round your answer to the nearest hundredth.

Answer

The Midpoint Formula

Lab - Midpoint Formula

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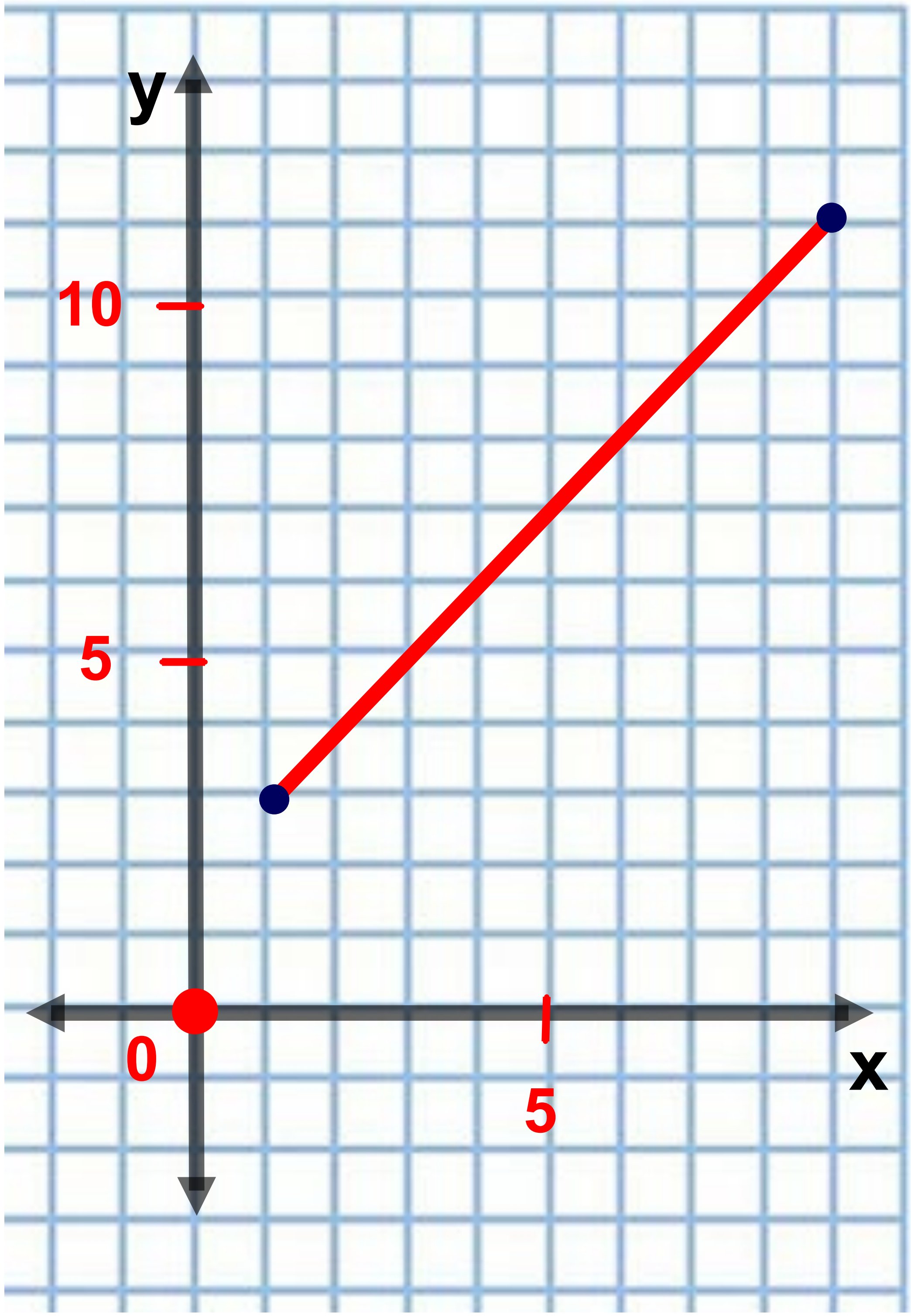
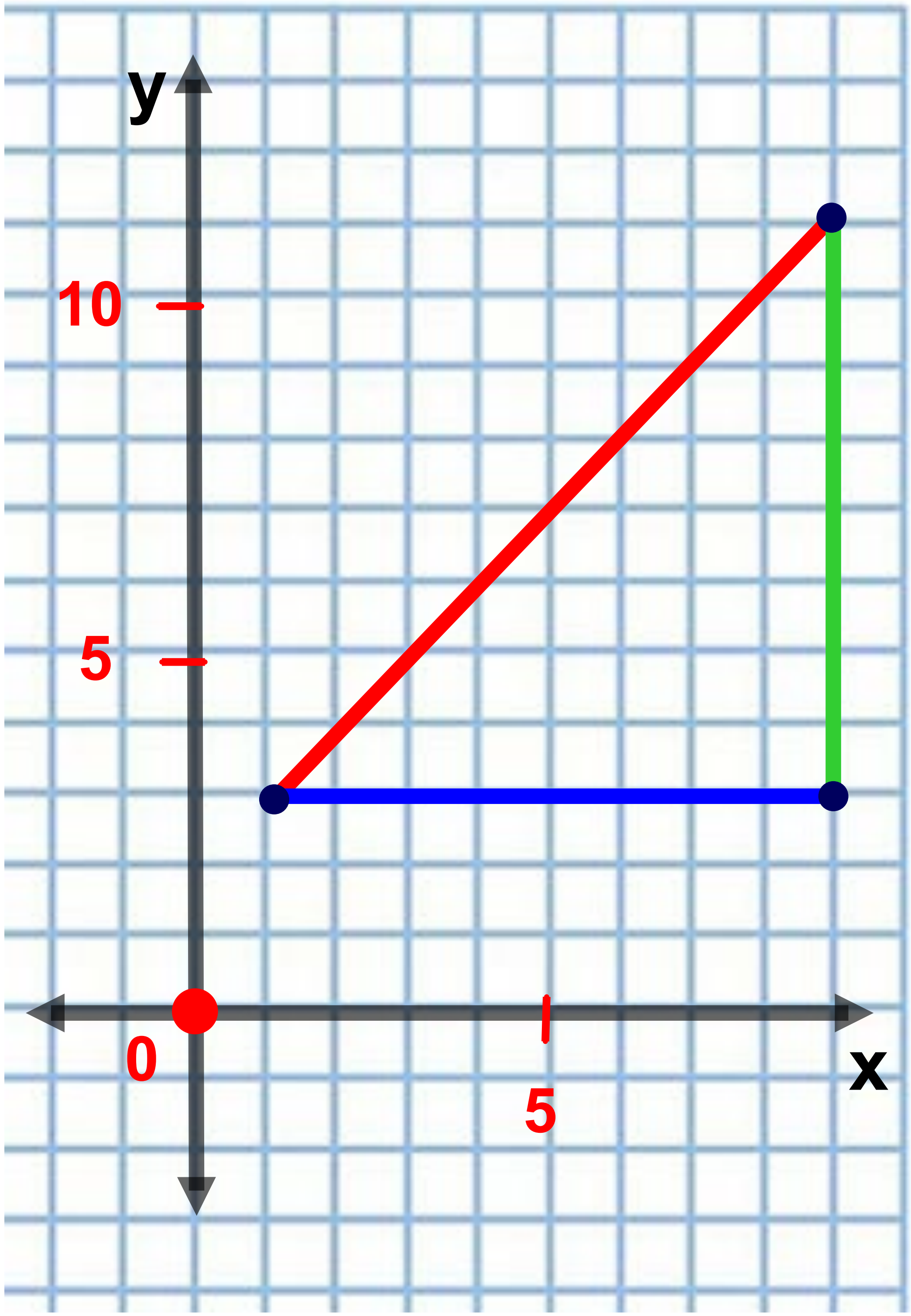
The Midpoint Formula



Another question which is easily solved using analytic geometry is to find the midpoint of a line.

Once again, we make use of the fact that it's easy to determine distance parallel to an axis, so let's add those lines.

The Midpoint Formula

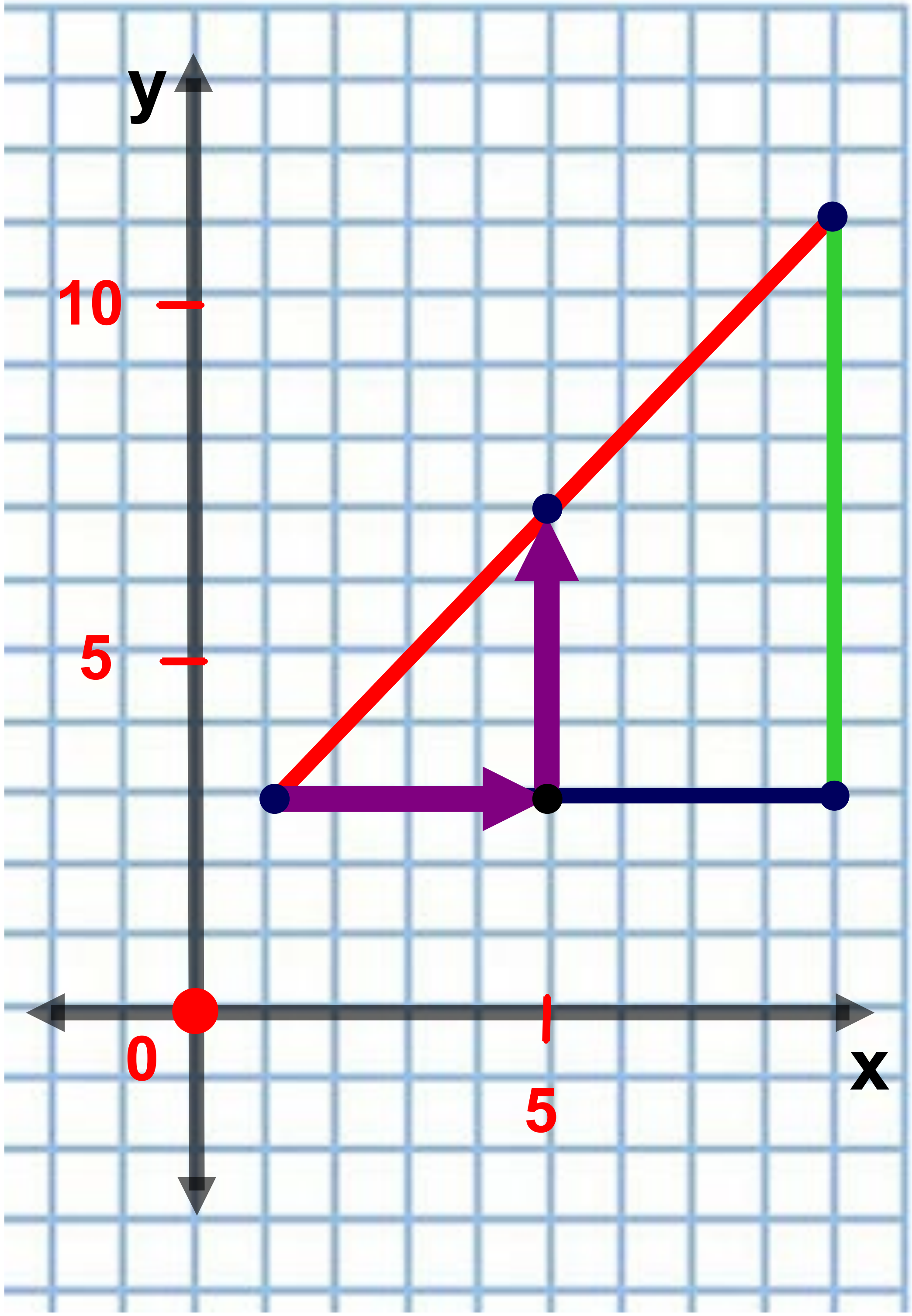


The **x-coordinate** for the midpoint between (x_1, y_1) and (x_2, y_2) is **halfway between x_1 and x_2.**

Similarly, the **y-coordinate** of that midpoint will be **halfway between y_1 and y_2.**

If you're provided a graph of a line, and asked to mark the midpoint, you can often do that without much calculating.

The Midpoint Formula



In this graph, $x_1 = 1$ and $x_2 = 9$.

They are 8 units apart, so just go 4 units along the x-axis and go up until you intersect the line.

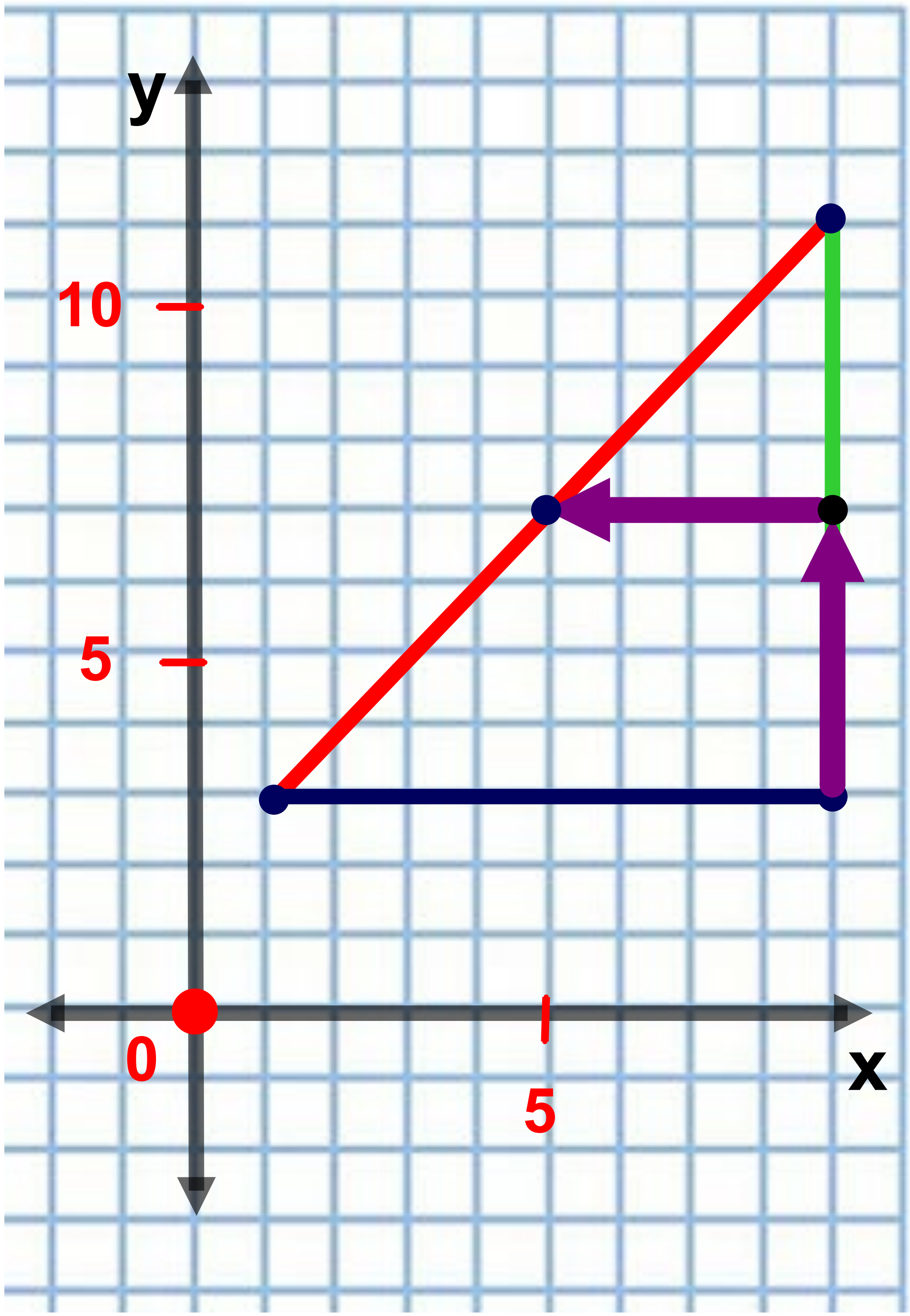
That will give you the midpoint.

In this case, that is at (5, 7), which can be read from the graph.

We would get the same answer if we had done this along the y-axis, as we do on the next slide.

Math Practice

The Midpoint Formula



The y-coordinates of the two points are 11 and 3, so they are also 8 units apart.

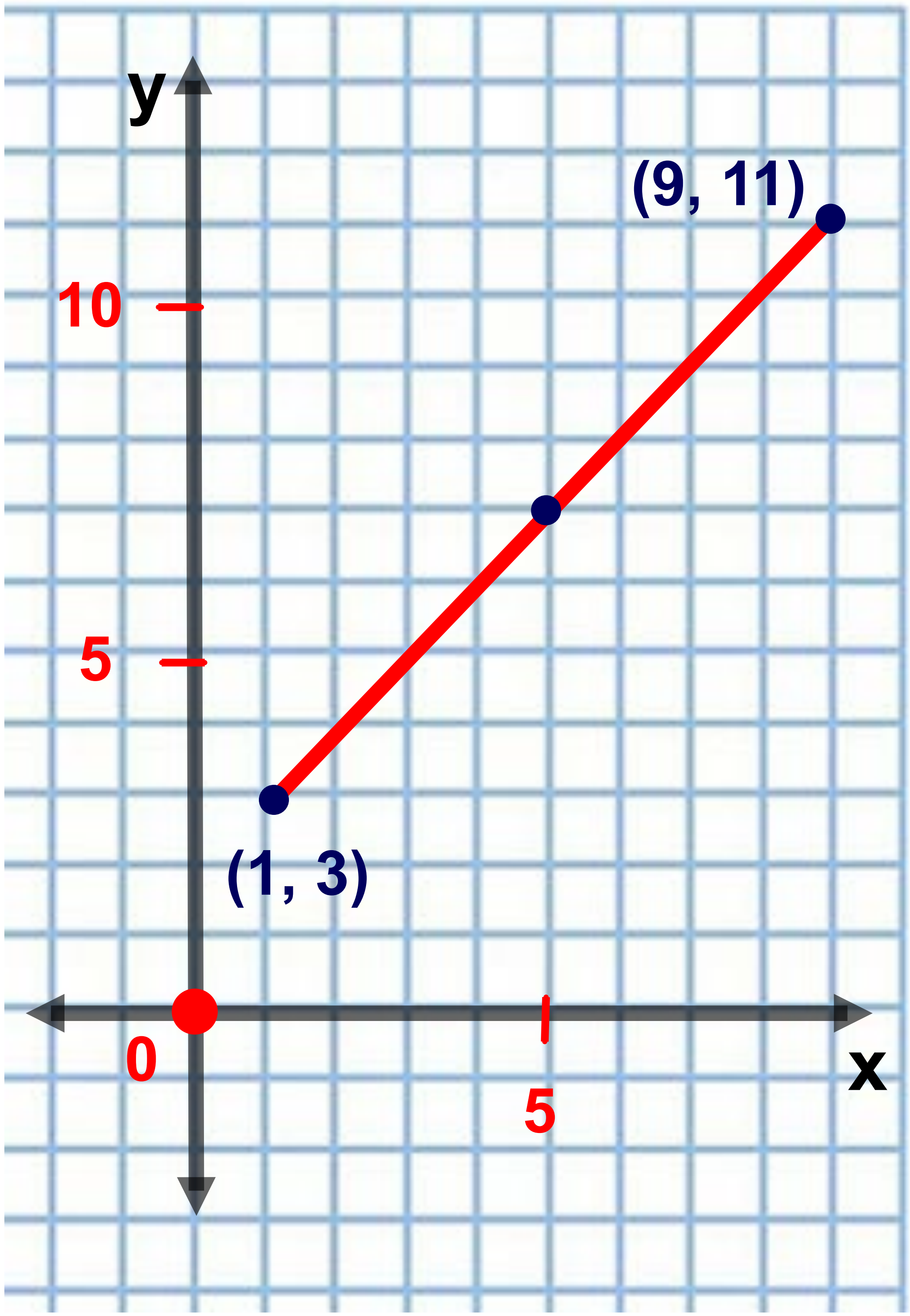
So, just go up 4 from the lower y-coordinate and then across to the line to also get (5, 7) to be the midpoint.

So, the order doesn't matter and if you have the graph and the numbers are easy to read, you may as well use it.

If you are not given a graph or the lines don't fall so that the values are easy to read, you can do a quick calculation to get the same result.

Math Practice

The Midpoint Formula



We can calculate the x-coordinate midway between that of the two given points by finding their average.

The same for the y-coordinate.

Just add the two values and divide by two.

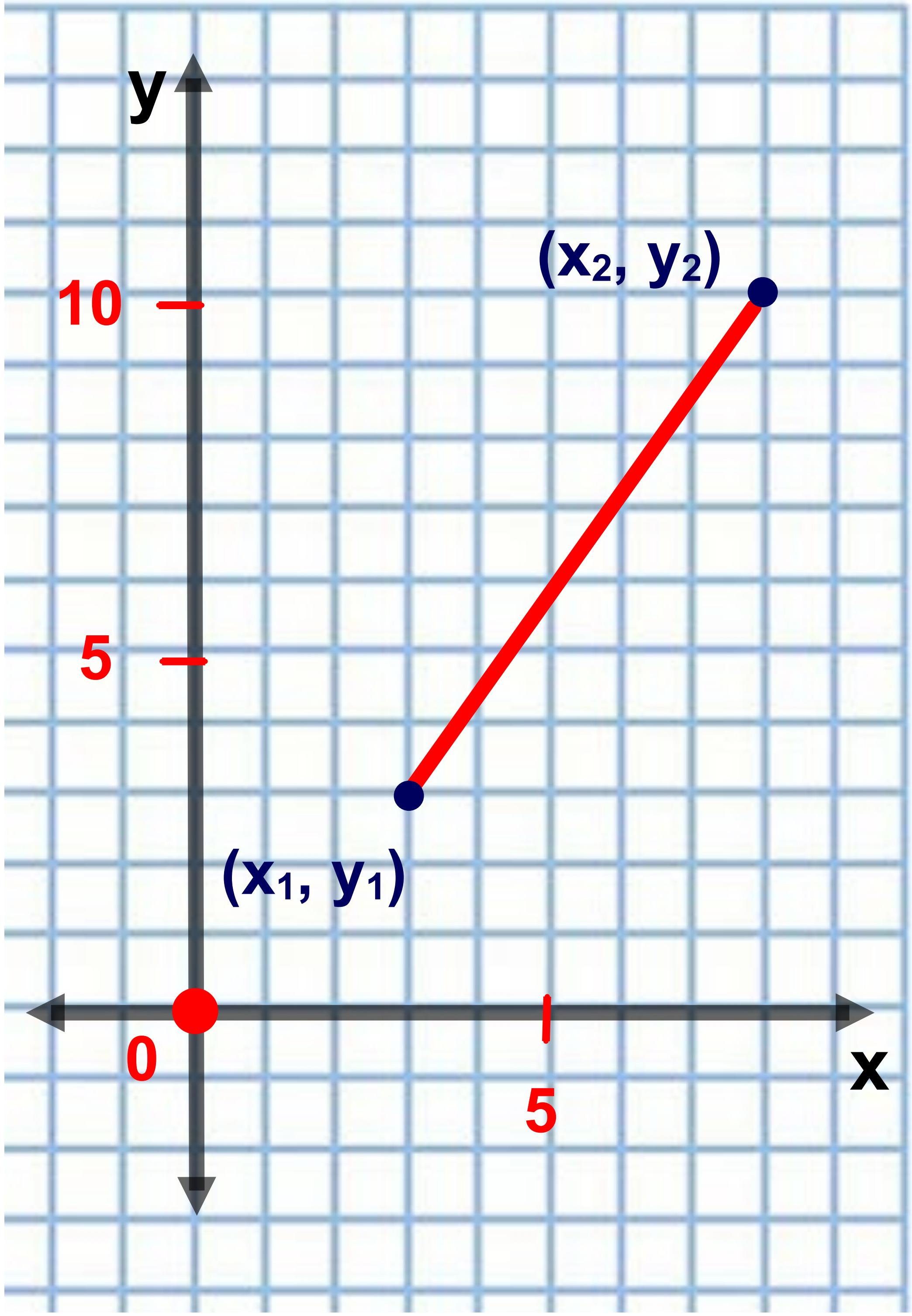
The x-coordinate of the midpoint is (1 + 9)/2 = 5

The y-coordinate of the midpoint is (3 + 11)/2 = 7

That's the same answer: (5, 7)

Math Practice

The Midpoint Formula



The more general solution is given below for any points: (x_1, y_1) and (x_2, y_2).

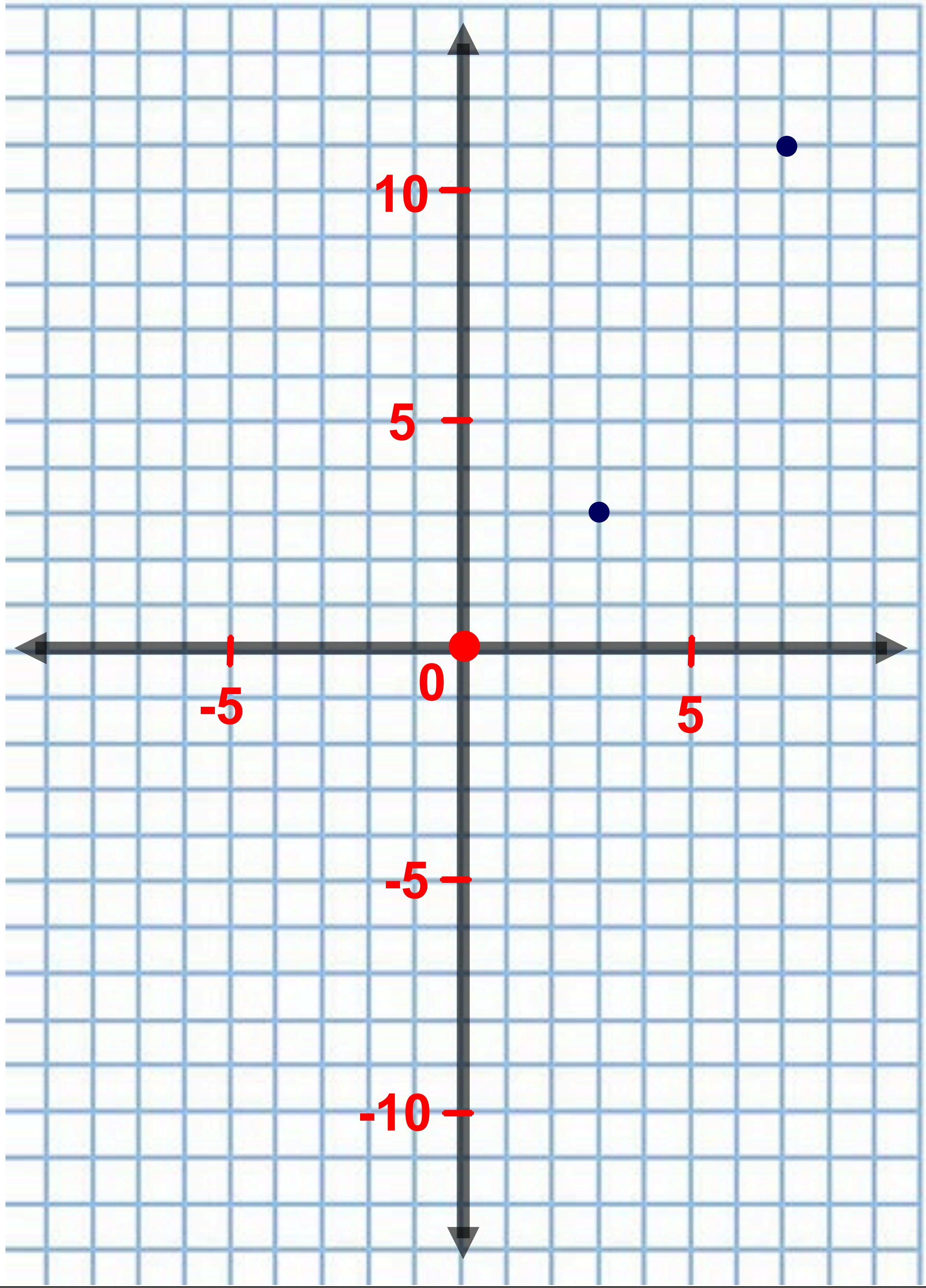
The x-coordinate of the midpoint is given by
$x_M = (x_1 + x_2) / 2$

The y-coordinate of the midpoint is given by
$y_M = (y_1 + y_2) / 2$

So, the coordinates of the midpoint are:

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$$

7 What is the midpoint between the indicated points?



A (4, 9)

B (-5, -4)

C (5, 6)

D (5, 7)

Answer

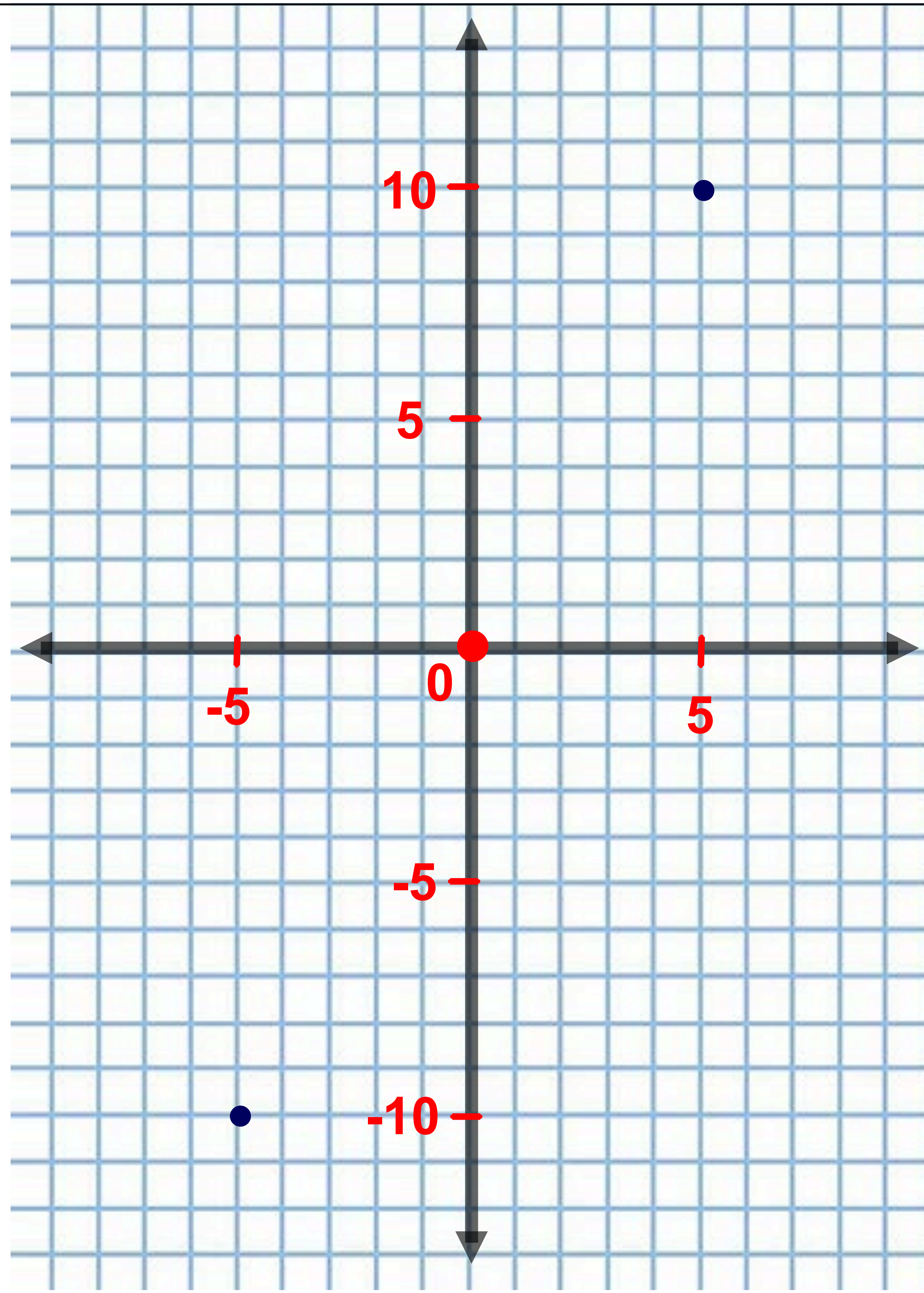
8 What is the midpoint between the indicated points?

A (0, 0)

B (5, 10)

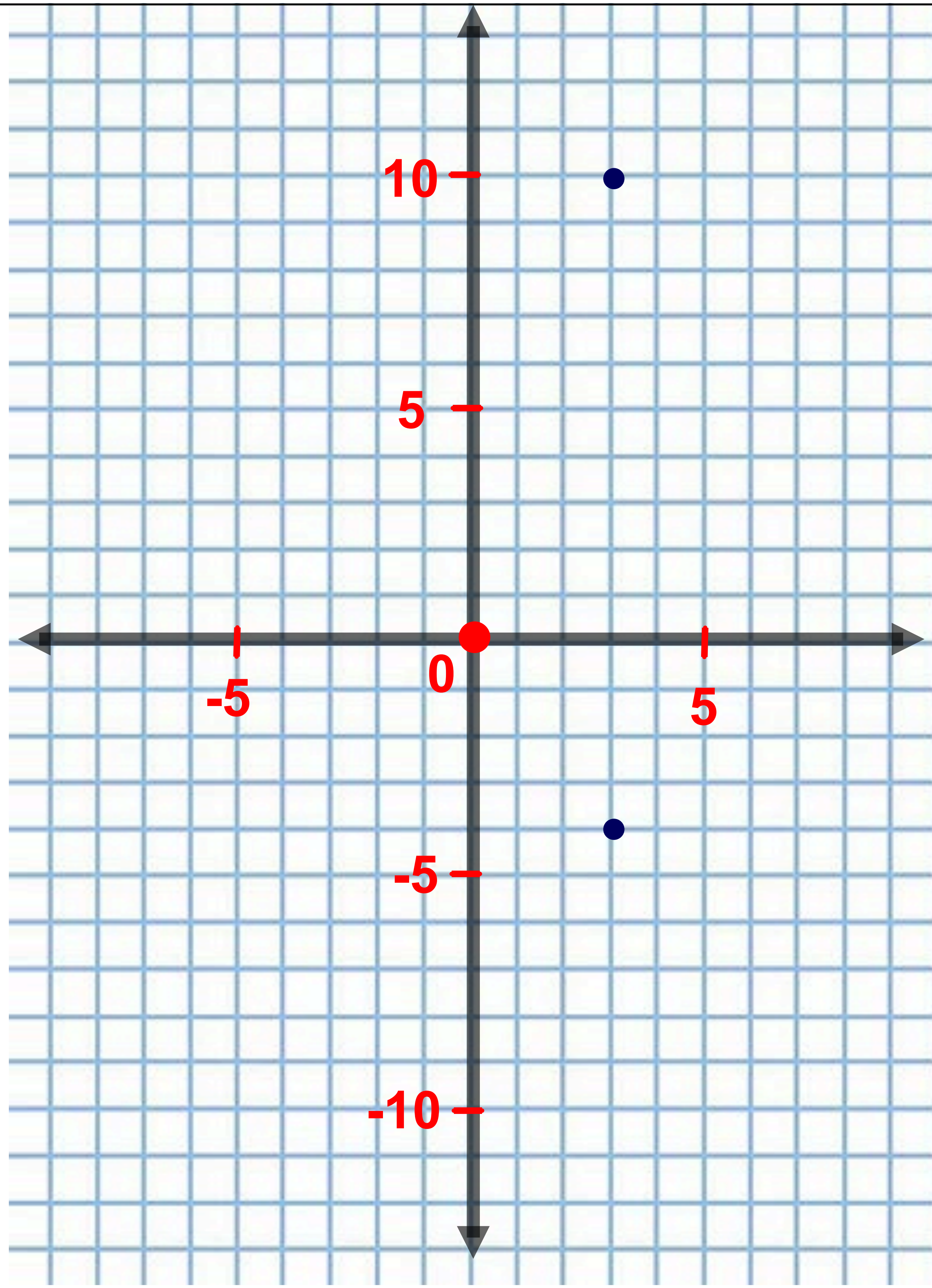
C (5, 5)

D (10, 10)



Answer

9 What is the midpoint between the indicated points?



A (3, 3)

B (3, 4)

C (4, 3)

D (5, 3)

Answer

10 What is the midpoint between the points: (4, 8) and (7, 3)?

A (8, 2)

B (4, 7)

C (5.5, 5.5)

D (6, 5)

Answer

11 What is the midpoint between the points: (-4, 8) and (4, -8)?

A (8, 2)

B (0, 0)

C (12, 12)

D (4, 4)

Answer

12 What is the midpoint between the points: (-4, -8) and (-6, -4)?

A (-5, -6)

B (-10, -12)

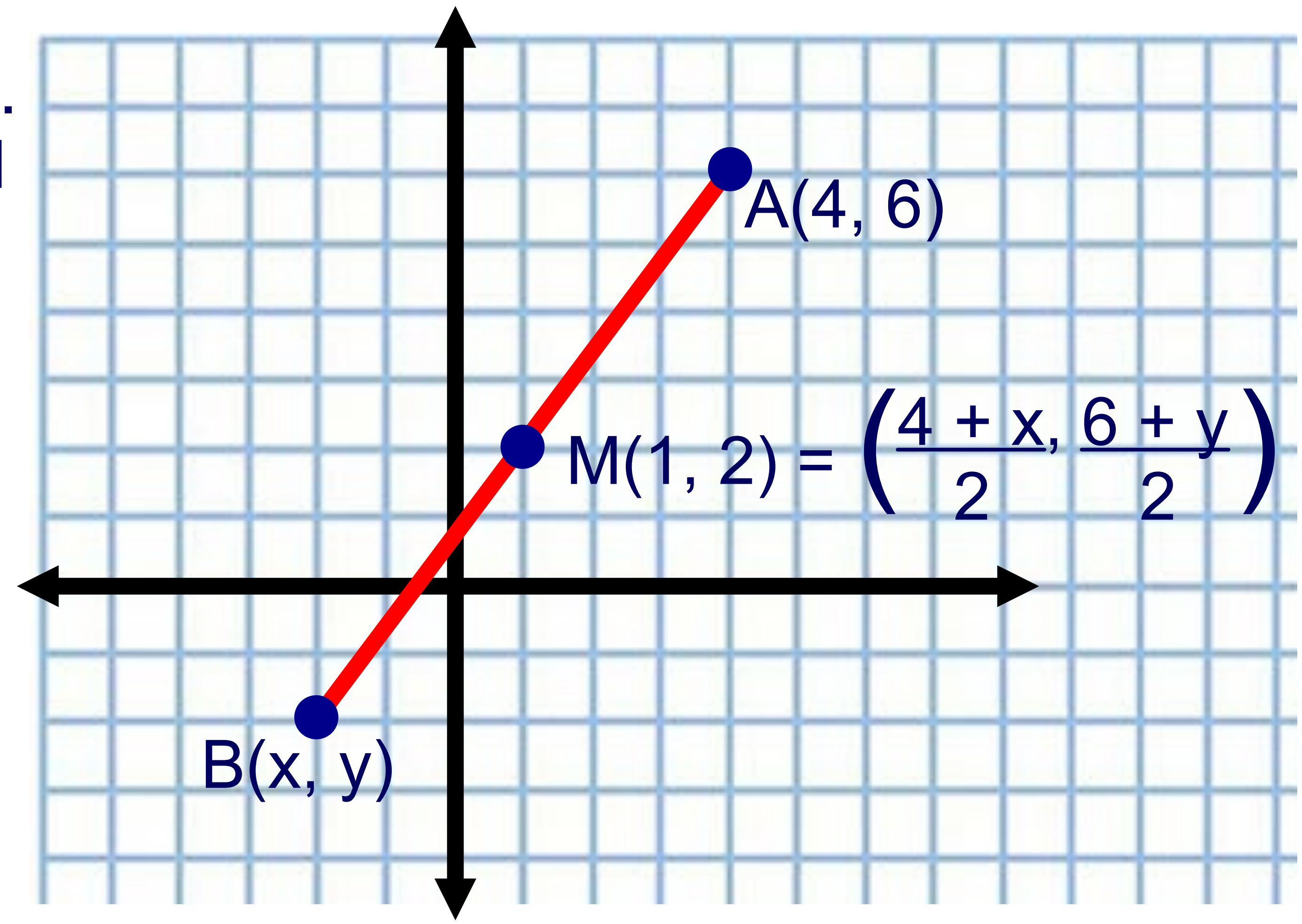
C (2, 4)

D (5, 6)

Answer

Finding the Coordinates of an Endpoint of a Segment

The midpoint of $\overline{AB}$ is M(1, 2). One endpoint is A(4, 6). Find the coordinates of the other endpoint B(x, y).



You can use the midpoint formula to write equations using x and y. Then, set up 2 equations and solve each one.

Finding the Coordinates of an Endpoint of a Segment

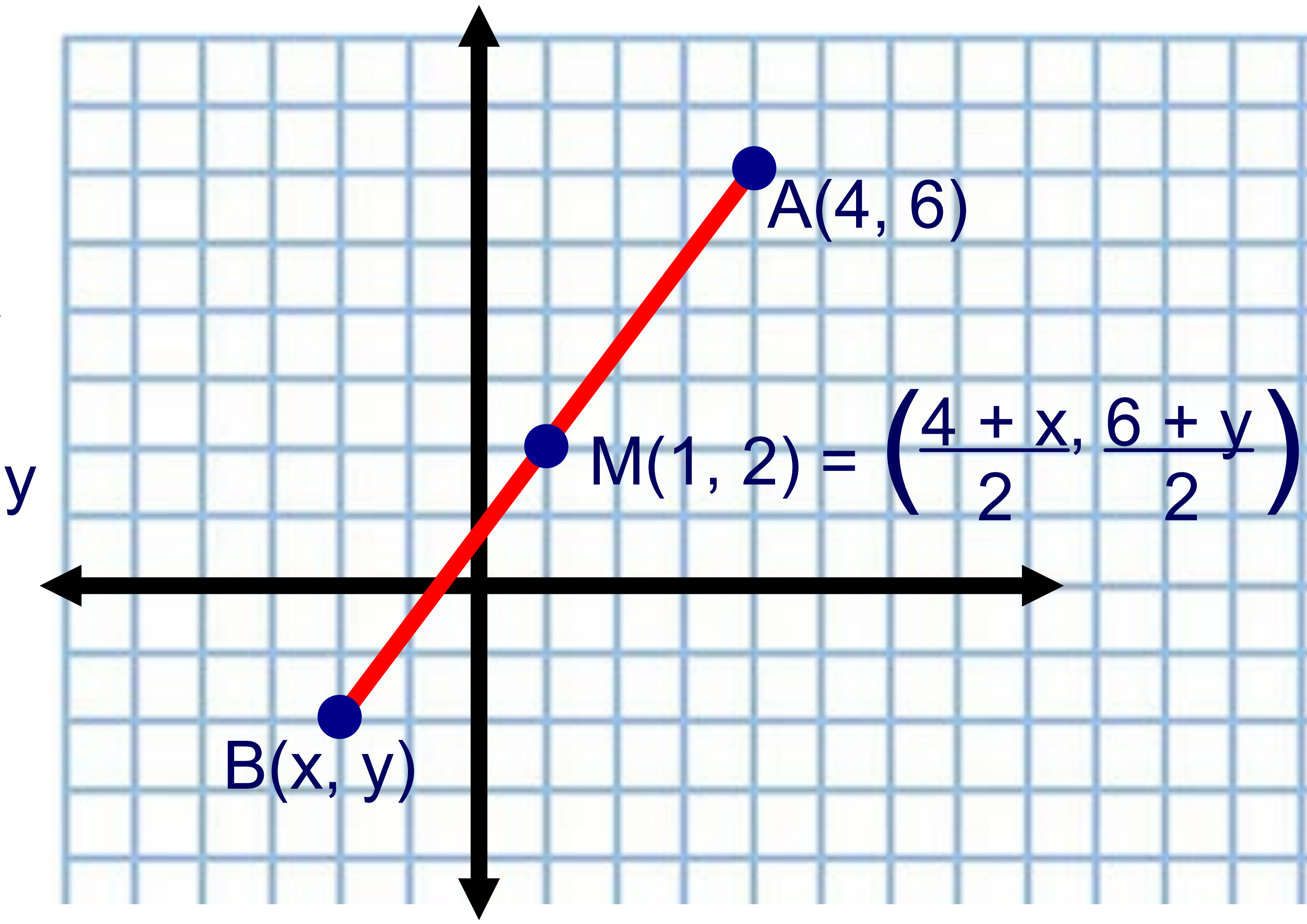
$$M(1, 2) = \left(\frac{4 + x}{2}, \frac{6 + y}{2}\right)$$

$$1 = \frac{4 + x}{2} \qquad 2 = \frac{6 + y}{2}$$

$$2 = 4 + x \qquad 4 = 6 + y$$

$$-2 = x \qquad -2 = y$$

Therefore, the coordinates of B(-2, -2).

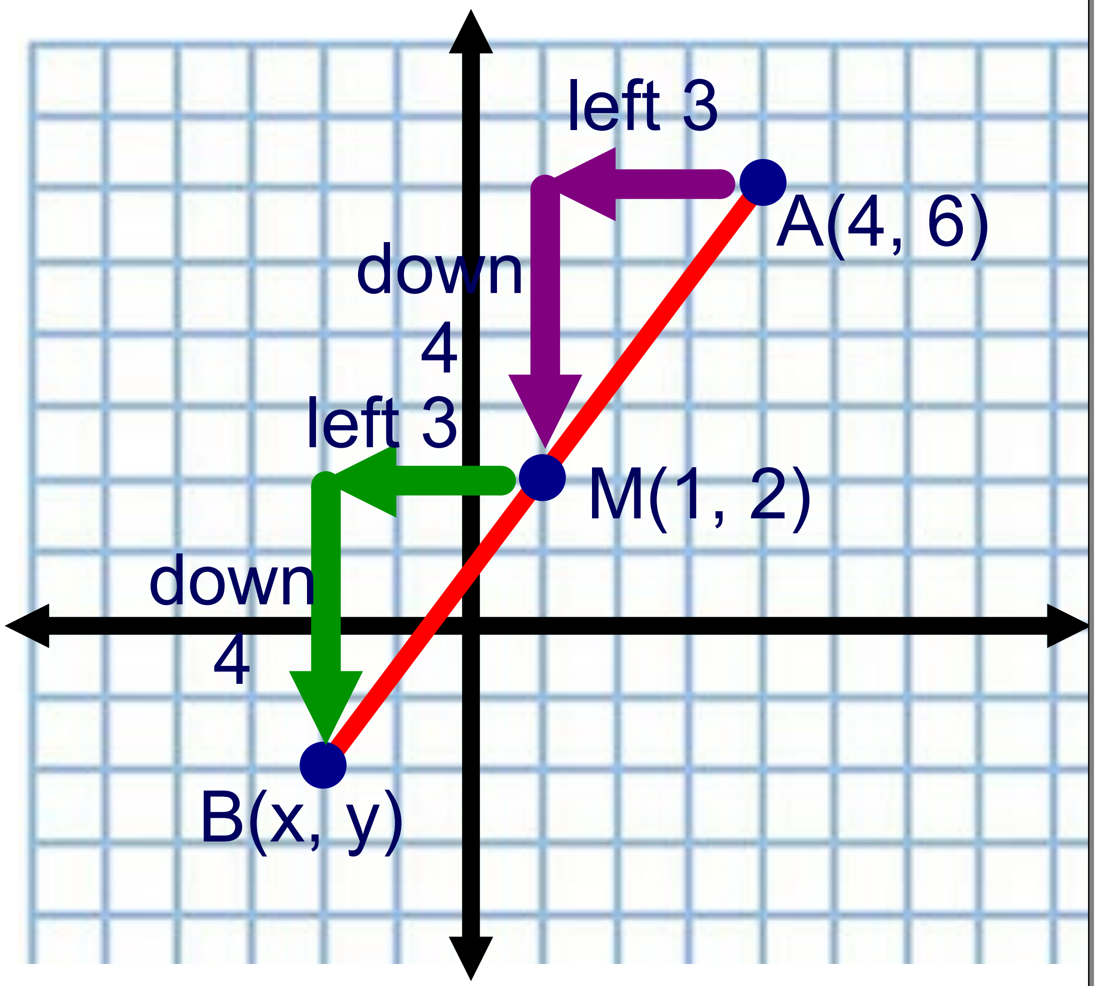


Can you find a shortcut to solve this problem? How would your shortcut make the problem easier?

Finding the Coordinates of an Endpoint of a Segment

Another way of approaching this problem is to look for the pattern that occurs between the endpoint A (4, 6) and midpoint M (1, 2).

Looking only at our points, we can determine that we traveled left 3 units and down 4 units to get from A to M. If we travel the same units in the same direction starting at M, we will get to B(-2, -2).



Finding the Coordinates of an Endpoint of a Segment

Similarly, if we line up the points vertically and determine the pattern of the numbers, without a graph, we can calculate the coordinates for our missing endpoint.

A (4, 6)

-3 -4

M (1, 2)

-3 -4

B (-2, -2)

If you use this method, always determine the operation required to get from the given endpoint to the midpoint. The reverse will not work.

13 Find the other endpoint of the segment with the endpoint (7, 2) and midpoint (3, 0)

A (-1, -2)

B (-2, -1)

C (4, 2)

D (2, 4)

Answer

14 Find the other endpoint of the segment with the endpoint (1, 4) and midpoint (5, -2)

A (11, -8)

B (9, 0)

C (9, -8)

D (3, 1)

Answer

15 Find the other endpoint of the segment with the endpoint (-4, -1) and midpoint (-2, 3).

A (-6, -5)

B (-3, -2)

C (0, 7)

D (1, 9)

Answer

16 Find the other endpoint of the segment with the endpoint (-2, 5) and midpoint (0, 2).

A (-1, -3.5)

B (-4, 8)

C (1, 0.5)

D (2, -1)

Answer

Partitions of a Line Segment

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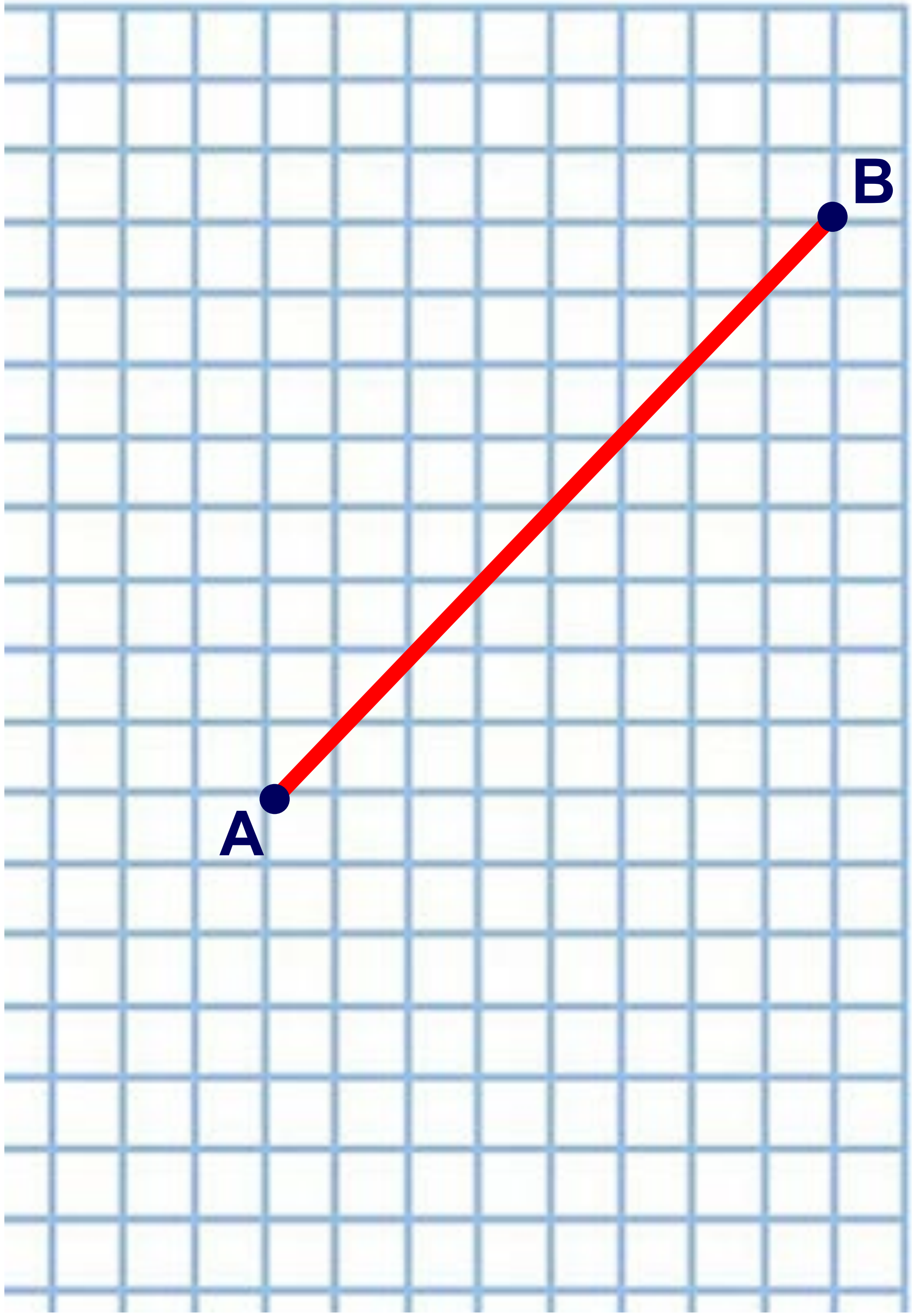
Partitions of a Line Segment



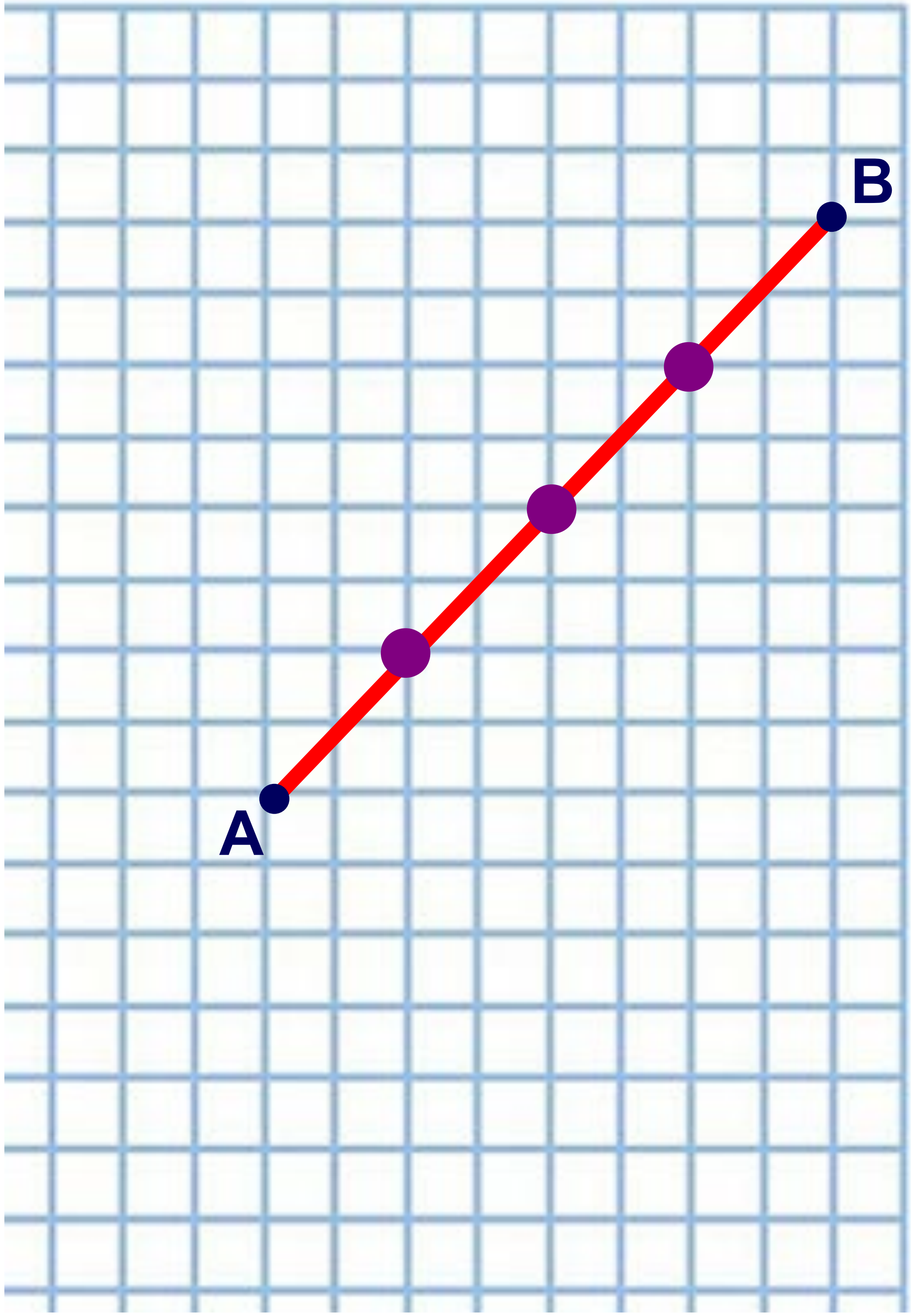
Partitioning a line segment simply means to divide the segment into 2 or more parts, based on a given ratio.

The midpoint partitions a segment into 2 congruent segments, forming a ratio of 1:1.

But if you need to partition a segment so that its ratio is something different, for example 3:1, how can it be done?

Math Practice

Partitions of a Line Segment

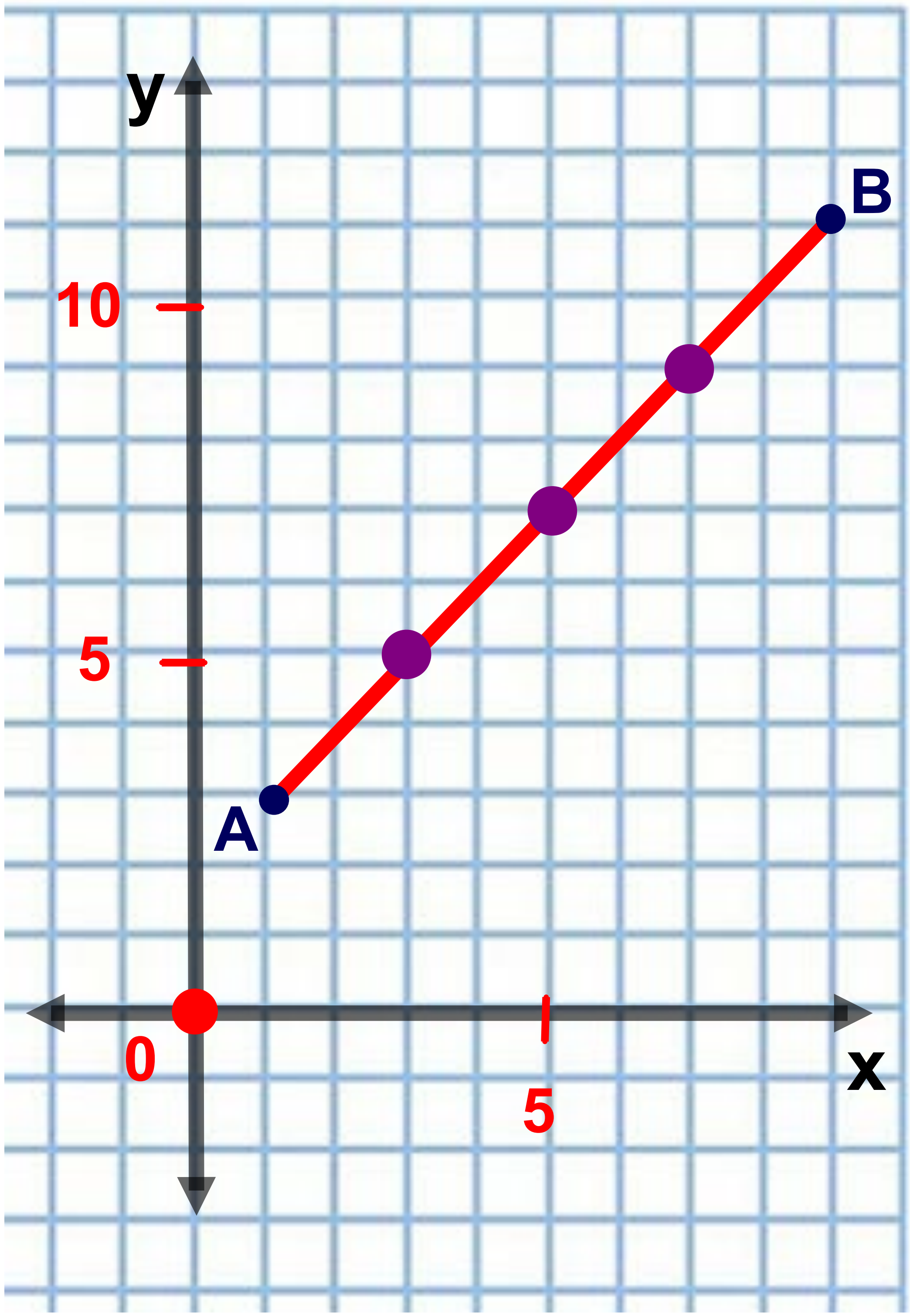


In order to divide the segment in the ratio of 3:1, think of dividing the segment into 3 + 1, or 4 congruent pieces.

Plot the points that would divide $\overline{AB}$ into 4 congruent pieces.

- Click on one of the points in the grid to show them all.

Partitions of a Line Segment



If we add a coordinate plane to our segment, could we also determine the coordinates of our points?

Yes, we can. We can also eliminate one of our points that does not divide our segment into the ratio 3:1. In our case, the midpoint.

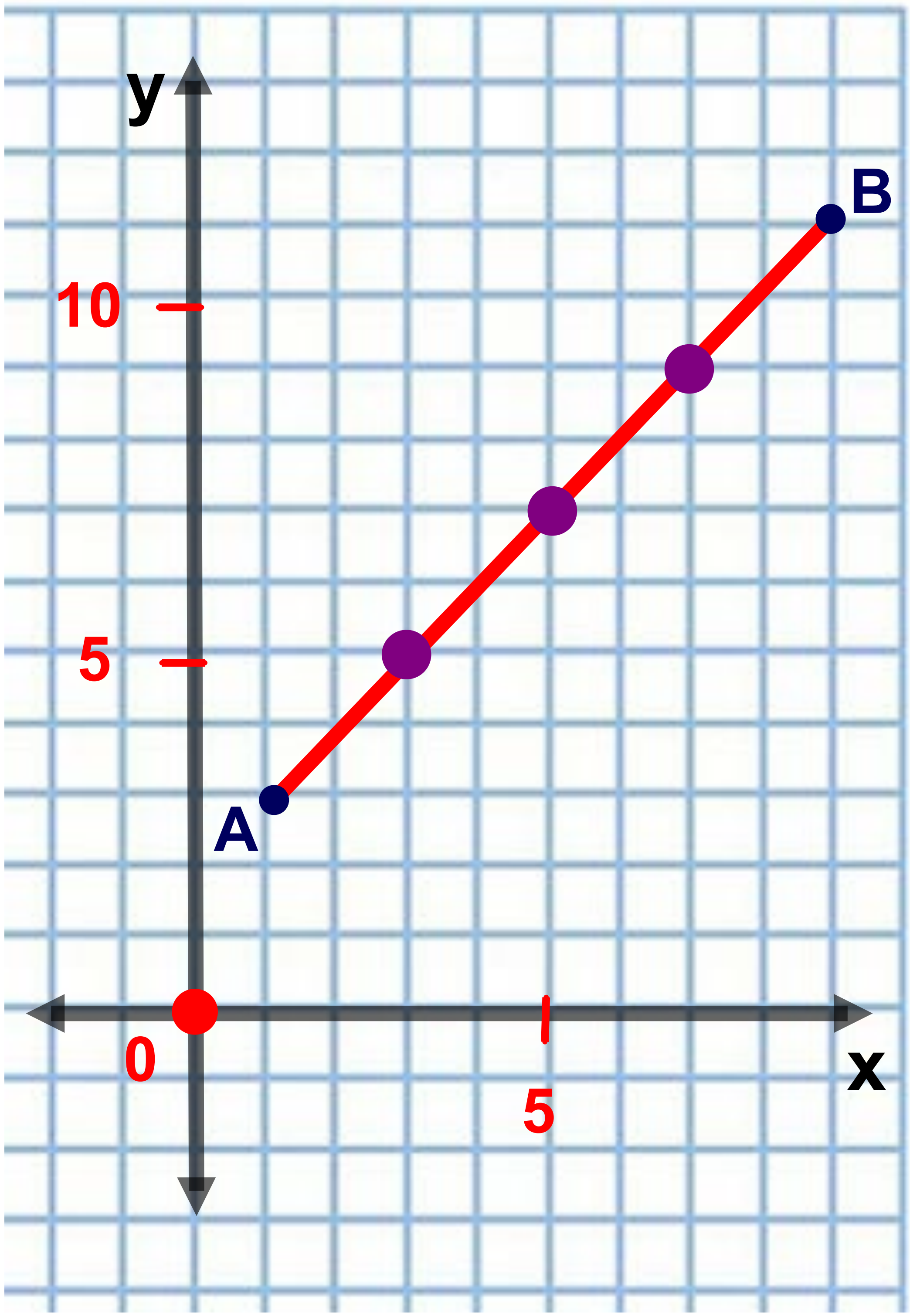
- Click on midpoint to hide it

Let's say that the ratio of the two segments that we're looking for from left to right is 3:1. Which other point should be eliminated?

- Click on that point to hide it

Answer

Partitions of a Line Segment



Could we also determine the coordinates of our point if the ratio was reversed (1:3 instead of 3:1)?

Yes, we can. Again, our midpoint can be eliminated.

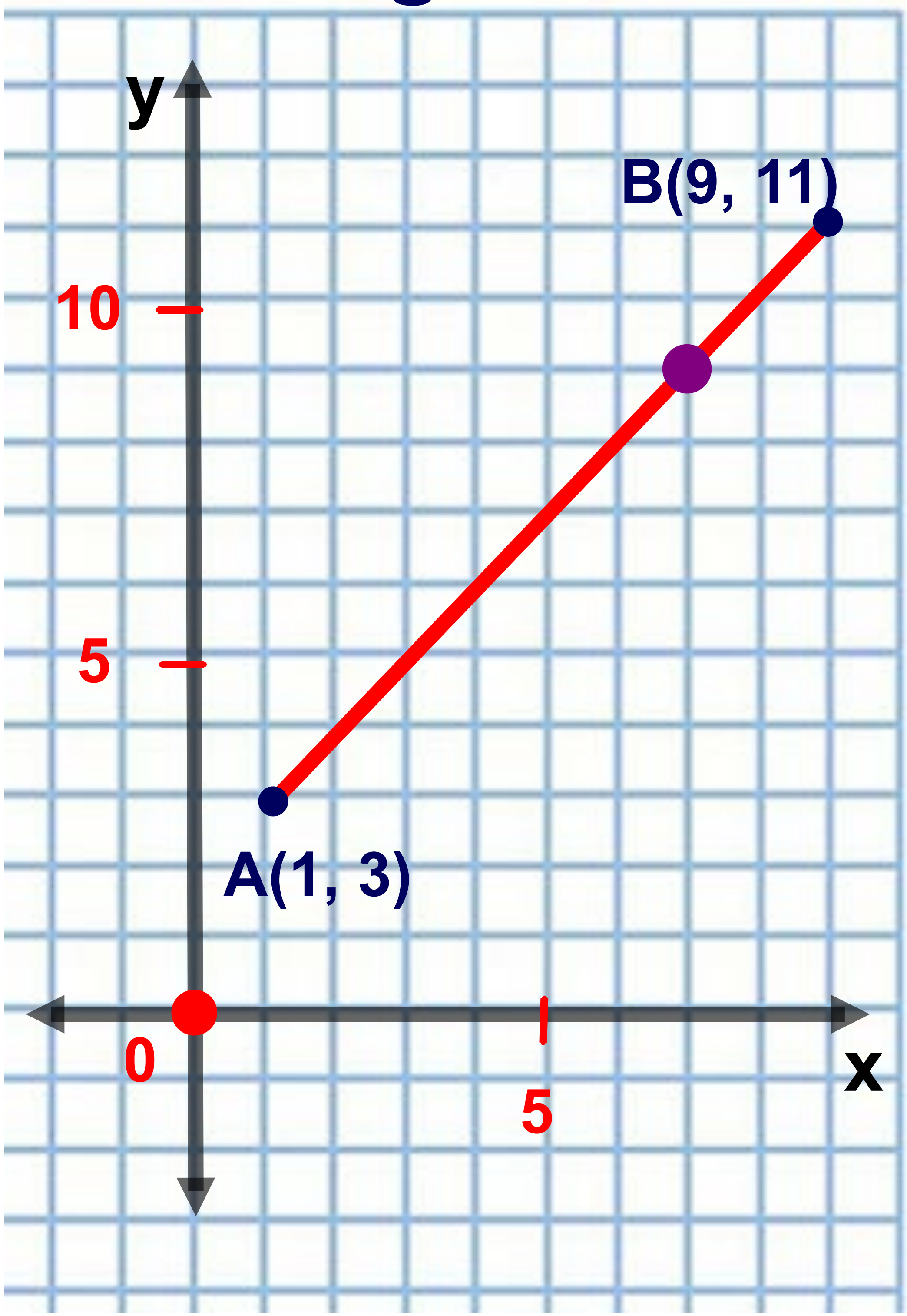
- Click on midpoint to hide it

Now, the ratio of the two segments that we're looking for from left to right is 1:3. Which other point should be eliminated?

- Click on that point to hide it

Answer

Partitions of a Line Segment



We can also calculate both the x and y coordinates between the two given points by using a formula that is similar to the midpoint formula.

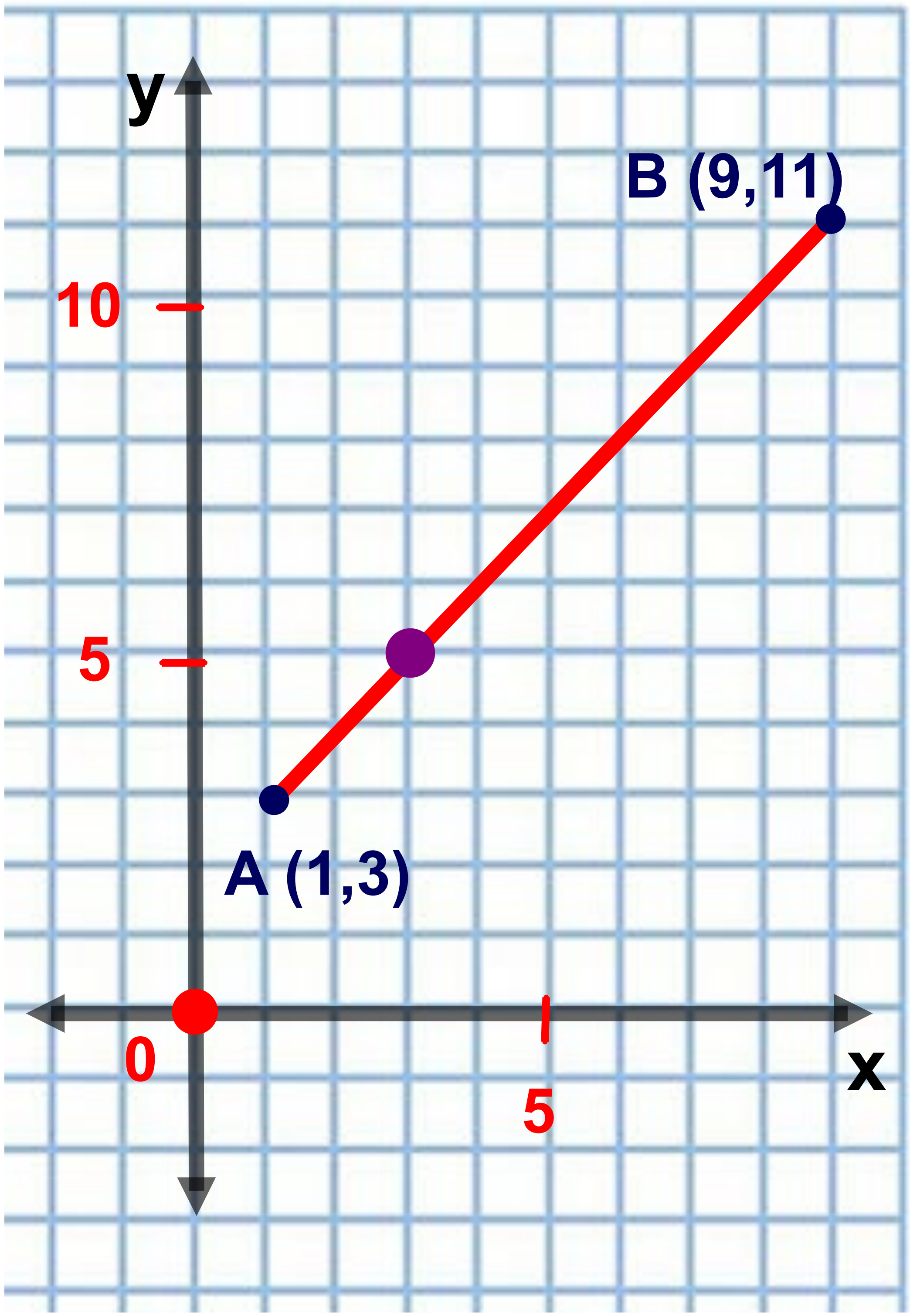
But instead of having a common ratio (1:1) and dividing by 2, what the midpoint formula has us do, we need to multiply the one set of coordinates by the first number in the ratio and the other set of coordinates by the second number in the ratio and divide by the number of segments that are required for our ratio 3:1, or 3 + 1 = 4.

$$\left(\frac{3(9) + 1(1)}{3 + 1}, \frac{3(11) + 1(3)}{3 + 1}\right)$$

$$\left(\frac{27 + 1}{4}, \frac{33 + 3}{4}\right) = (7, 9)$$

Math Practice

Partitions of a Line Segment



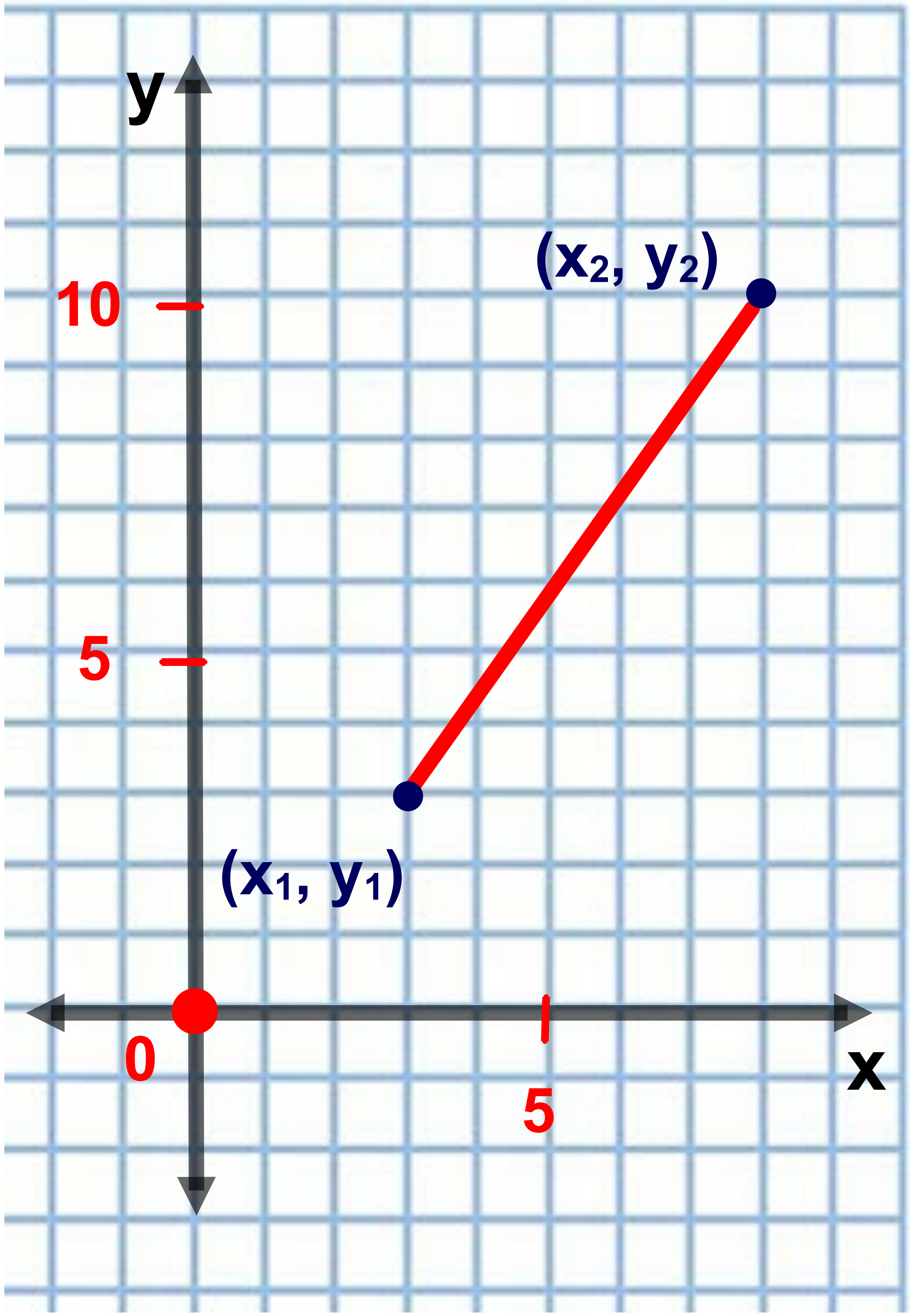
If we use the same formula, but switch to the other ratio 1:3, we will get the other point that partitions $\overline{AB}$.

$$\left(\frac{1(9) + 3(1)}{3 + 1}, \frac{1(11) + 3(3)}{3 + 1}\right)$$

$$\left(\frac{9 + 3}{4}, \frac{11 + 9}{4}\right) = (3, 5)$$

Math Practice

Partitions of a Line Segment



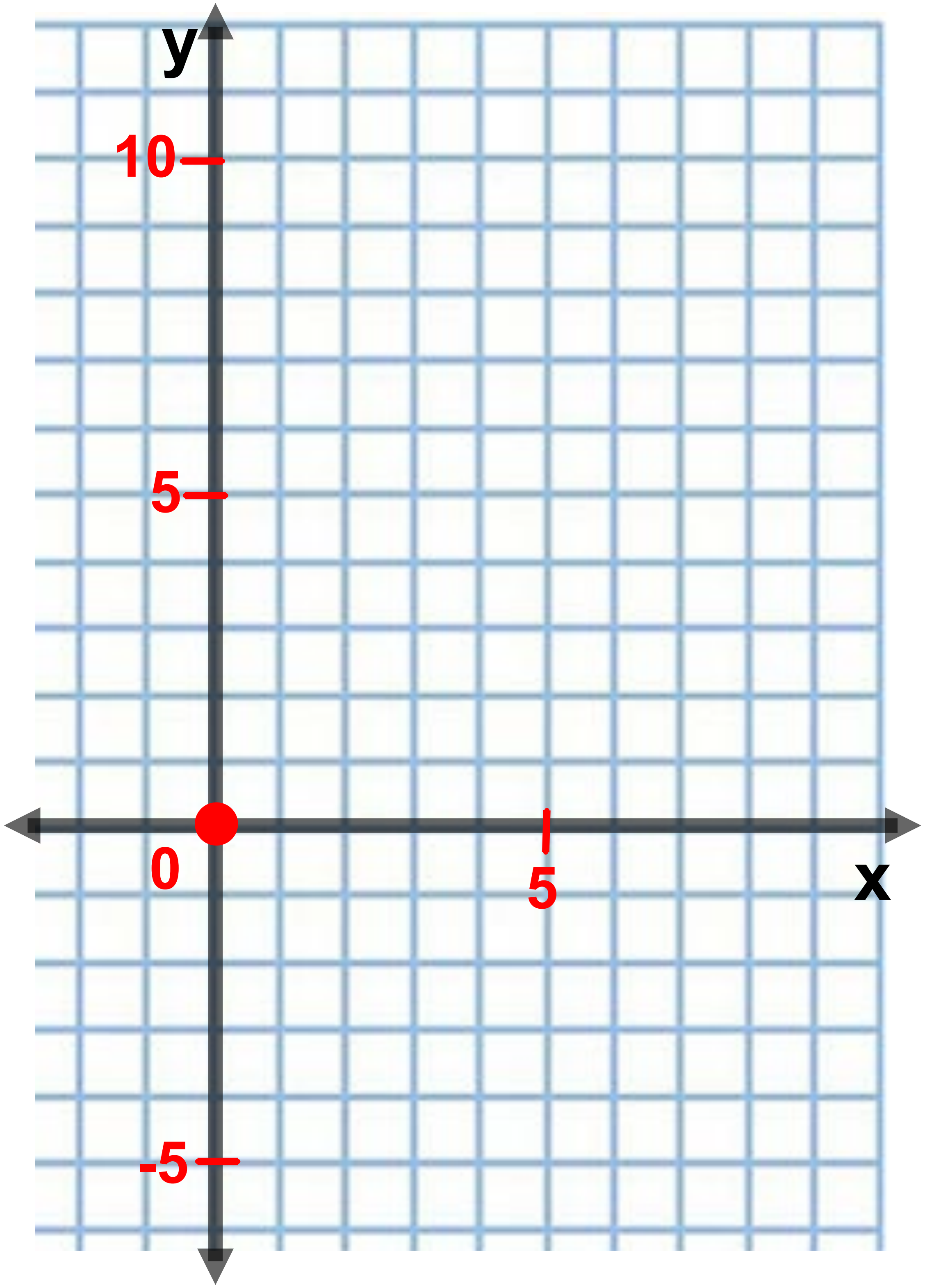
The more general solution is given below for any points: (x_1, y_1) and (x_2, y_2).

If a point P partitions a segment in a ratio of m:n, then the coordinates of P are

$$P = \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

Remember to also use the formula twice switching the m:n ratio to n:m.

Partitions of a Line Segment



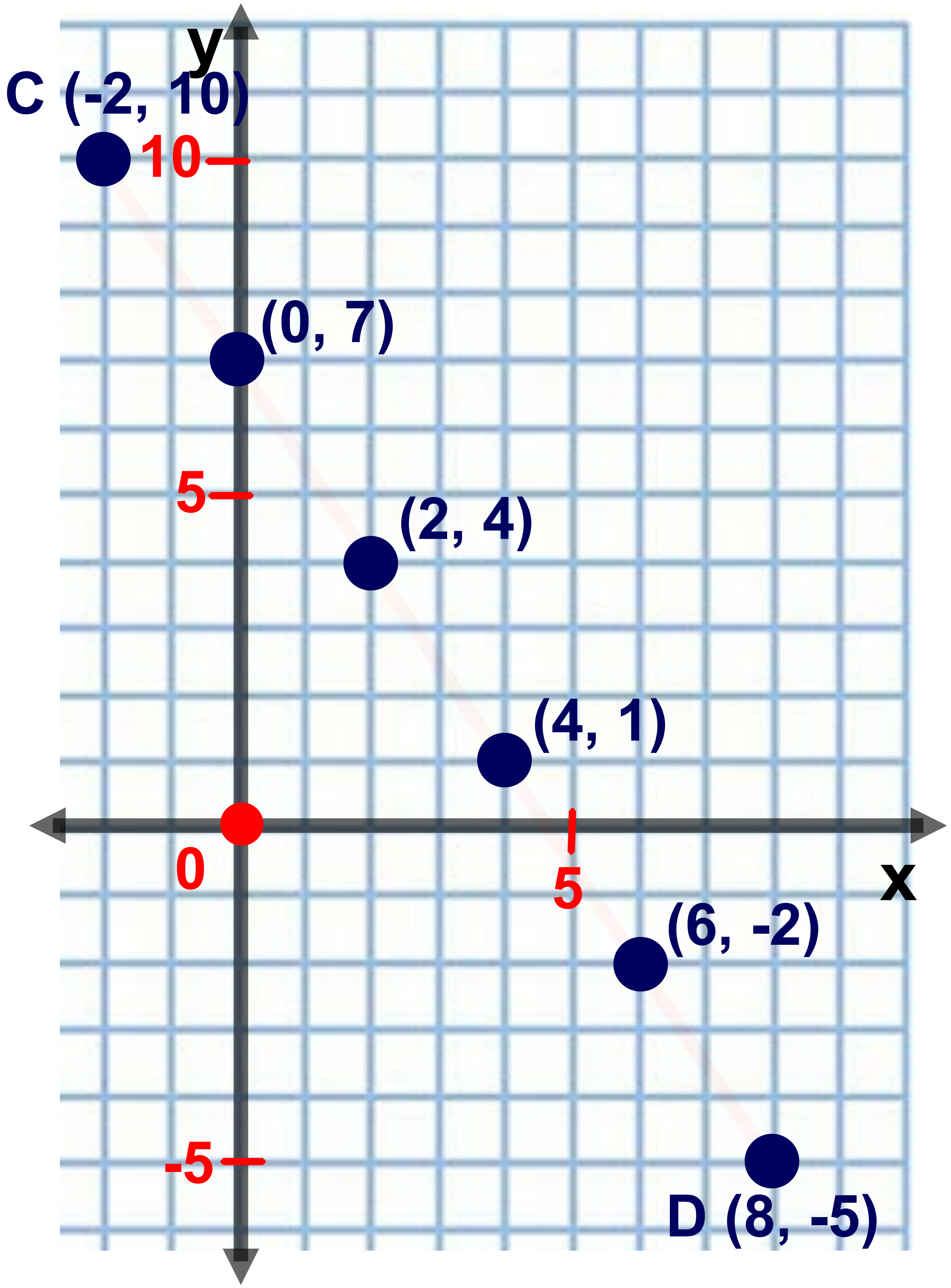
Example with a graph:

Line segment CD in the coordinate plane has endpoints with coordinates (-2, 10) and (8, -5).

Graph $\overline{CD}$ and find two possible locations for a point P that divides $\overline{CD}$ into two parts with lengths in a ratio of 2:3.

Math Practice

Partitions of a Line Segment



Step #1: Graph the points. C(-2, 10) and D(8, -5)

- Click on the points to show.

Step #2: Connect them w/ the segment $\overline{CD}$.

- Click on the segment to show.

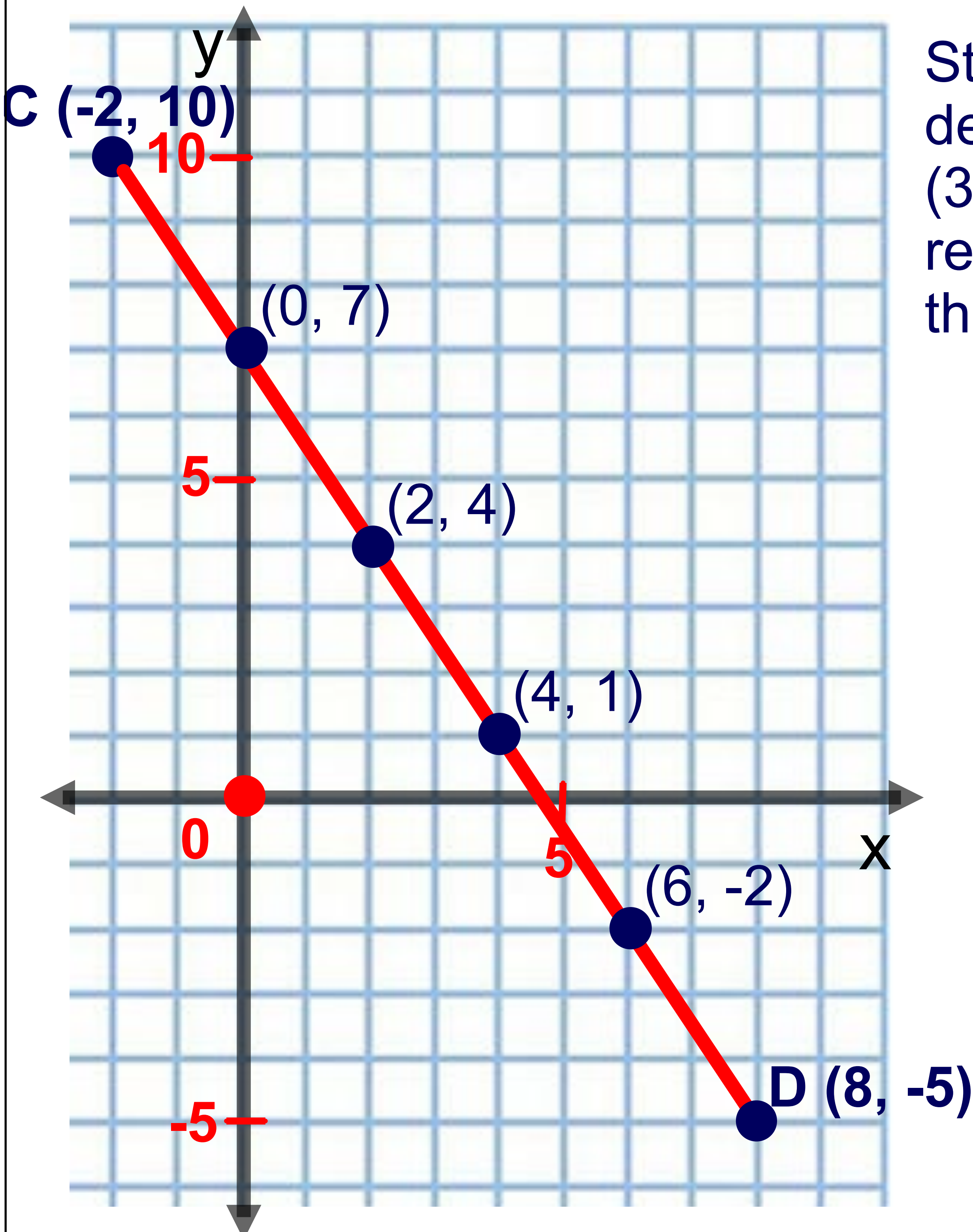
Step #3: Determine the total number of ways required to divide the segment using the ratio 2:3.

Click

Step #4: Plot the points on the graph that divide $\overline{CD}$ into the desired number of sections.

- Click on the points to show.

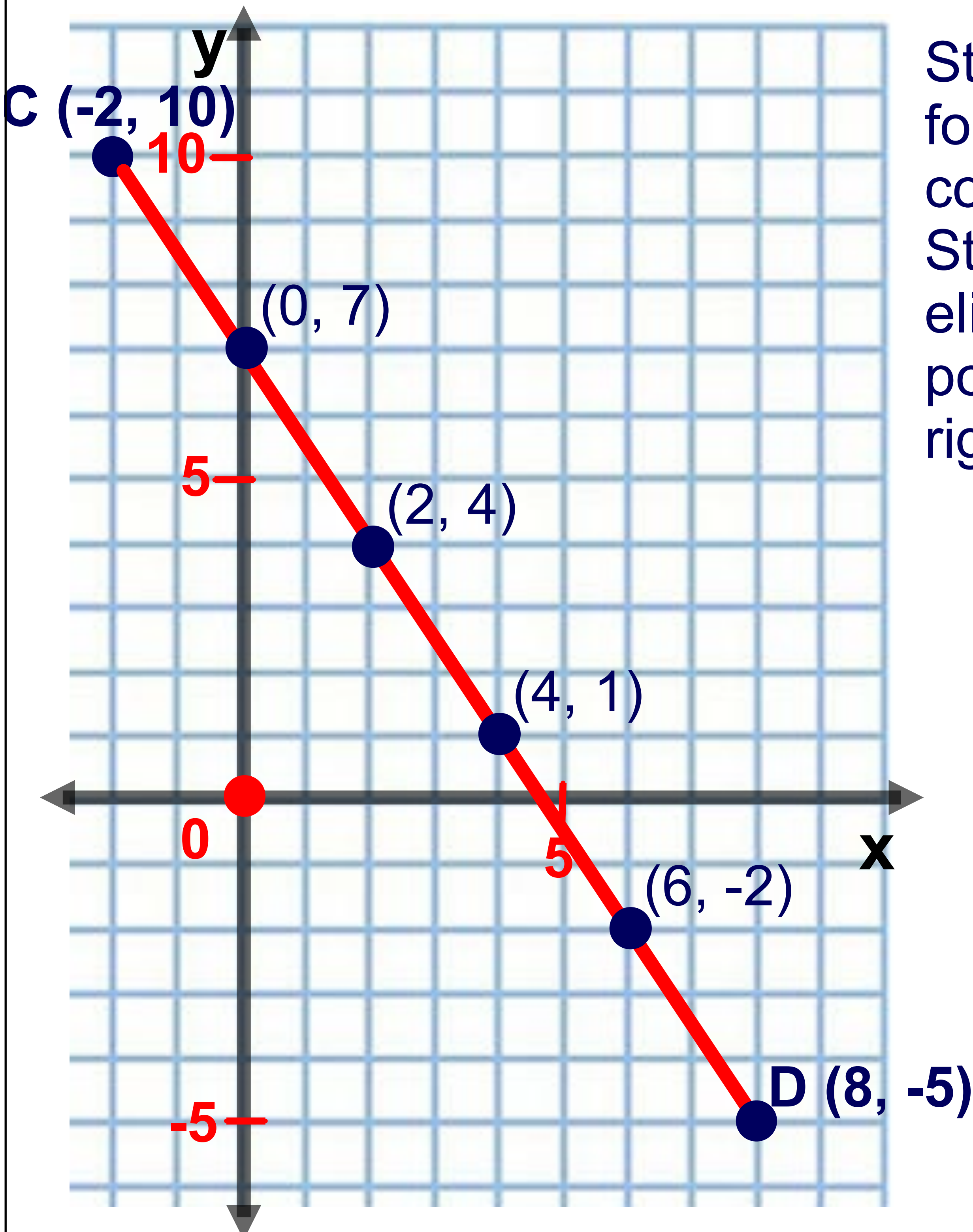
Partitions of a Line Segment



Step #5: Since the ratio is 2:3, determine which points to eliminate (3 of them) leaving only 1 point remaining so that, from left to right, the ratio is 2:3.

Answer

Partitions of a Line Segment



Step #6: Because the question asks for 2 points, we also need to consider the ratio 3:2. Similar to Step #5, determine which points to eliminate (3 of them) leaving only 1 point remaining so that, from left to right, the ratio is 3:2.

Answer

Partitions of a Line Segment

Example without a graph:

Line segment EF in the coordinate plane has endpoints with coordinates (10, -11) and (-4, 10).

Find two possible locations for a point P that divides $\overline{EF}$ into two parts with lengths in a ratio of 5:2.

Math Practice

Partitions of a Line Segment

(10, -11) and (-4, 10)

Step #1: Determine the total number of ways required to divide the segment using the ratio 5:2.

Click

Step #2: Use the formula to determine the coordinates of our first point P.

click

Partitions of a Line Segment

(10, -11) and (-4, 10)

Step #3: Reverse the ratio 2:5 and reuse the formula to determine the coordinates of our second point P.

click

17 Line segment GH in the coordinate plane has endpoints with coordinates (-7, -9) & (8, 3), shown in the graph below. Find 2 possible locations for a point P that divides GH into two parts with lengths in a ratio of 2:1.

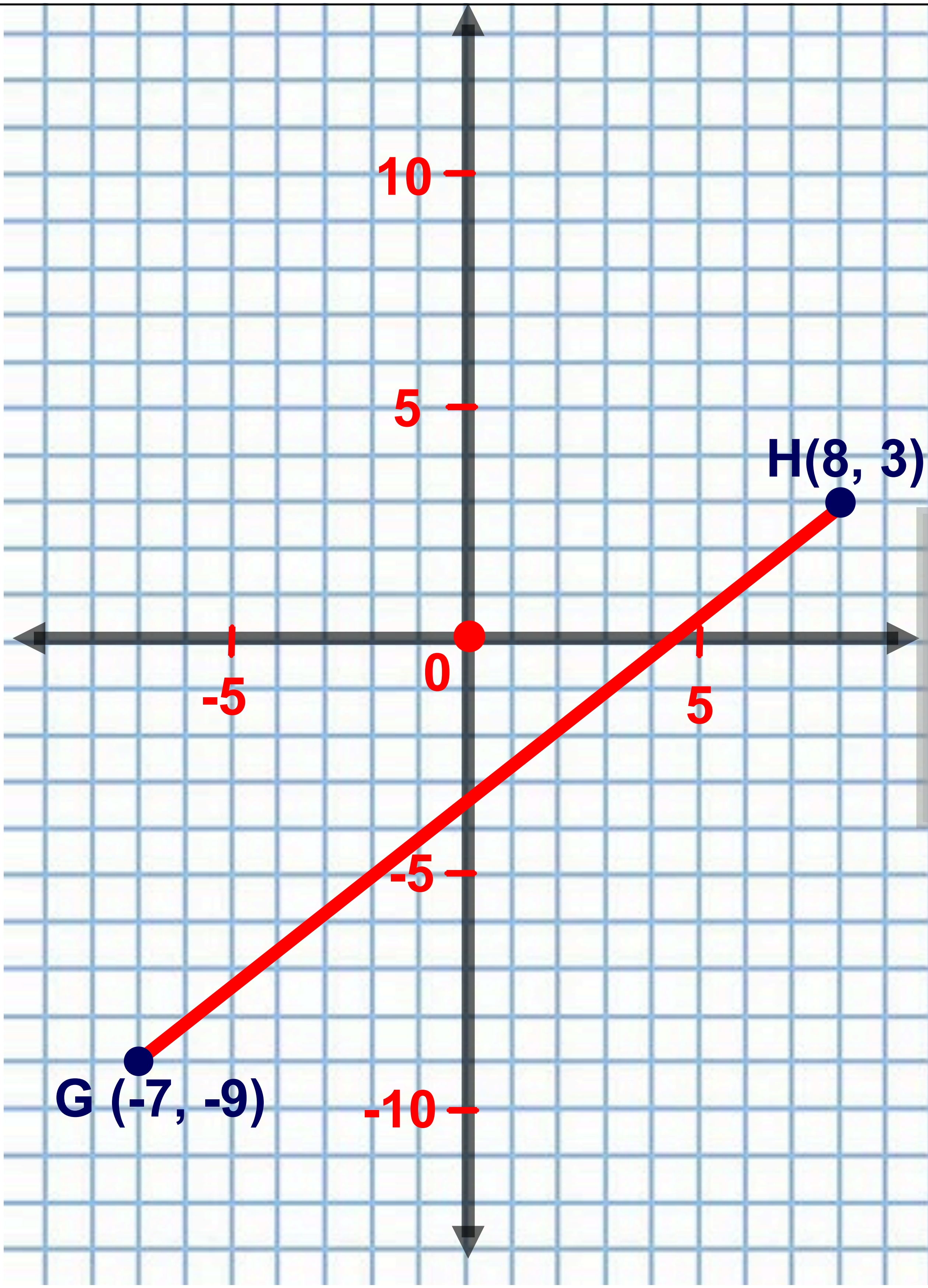
A (0.5, -3)

B (3, -1)

C (6, 1)

D (-2, -5)

E (-5, -7)



Answer

18 Line segment JK in the coordinate plane has endpoints with coordinates (10, 12) & (-10, -13), shown in the graph below. Find 2 possible locations for a point P that divides $\overline{JK}$ into two parts with lengths in a ratio of 4:1.

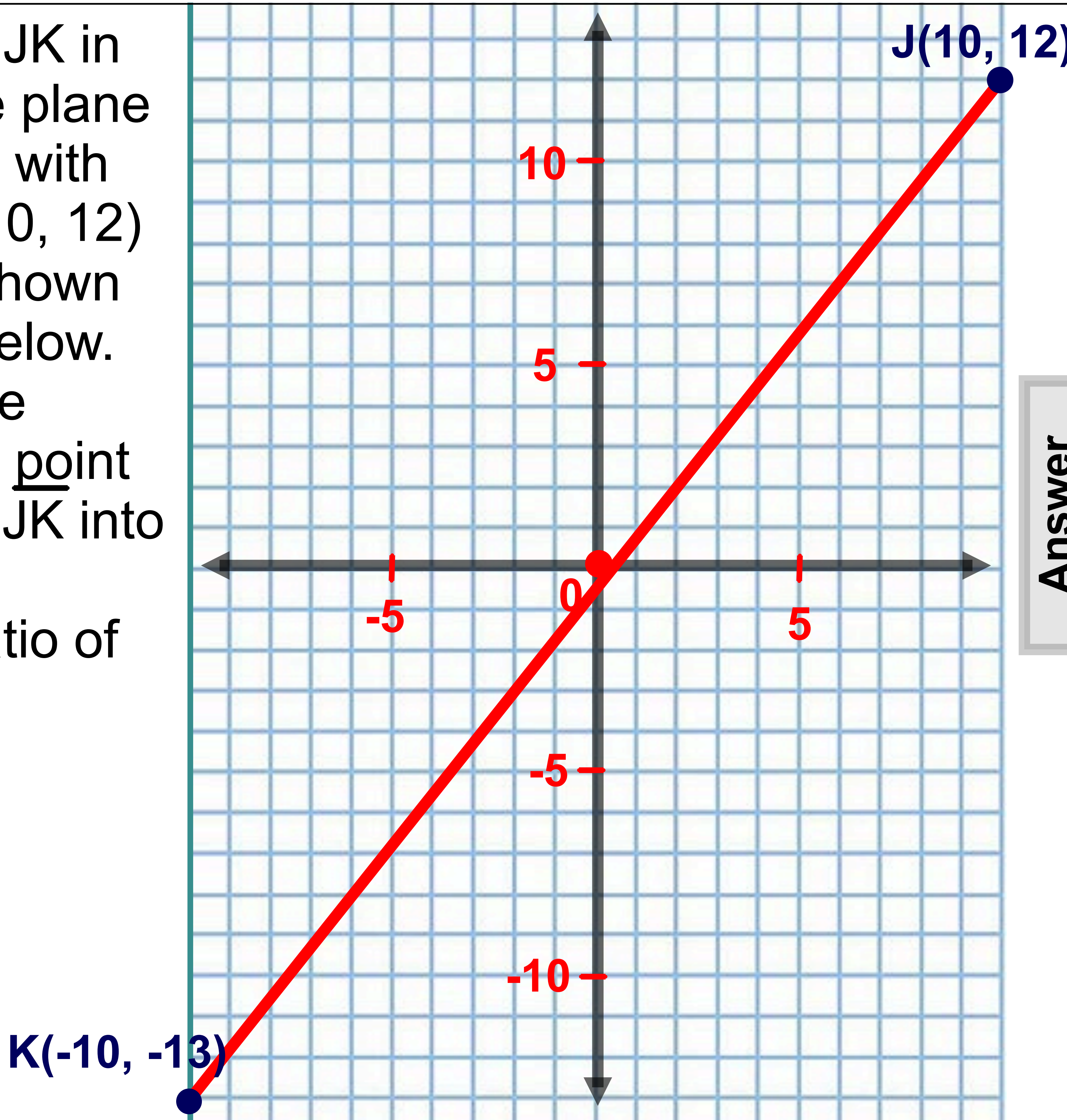
A (-6, -8)

B (-2, -3)

C (0, -0.5)

D (2, 2)

E (6, 7)



Answer

19 Line segment LM in the coordinate plane has endpoints with coordinates (-8, 13) & (6, -8), shown in the graph below. Find 2 possible locations for a point P that divides $\overline{LM}$ into two parts with lengths in a ratio of 3:4.

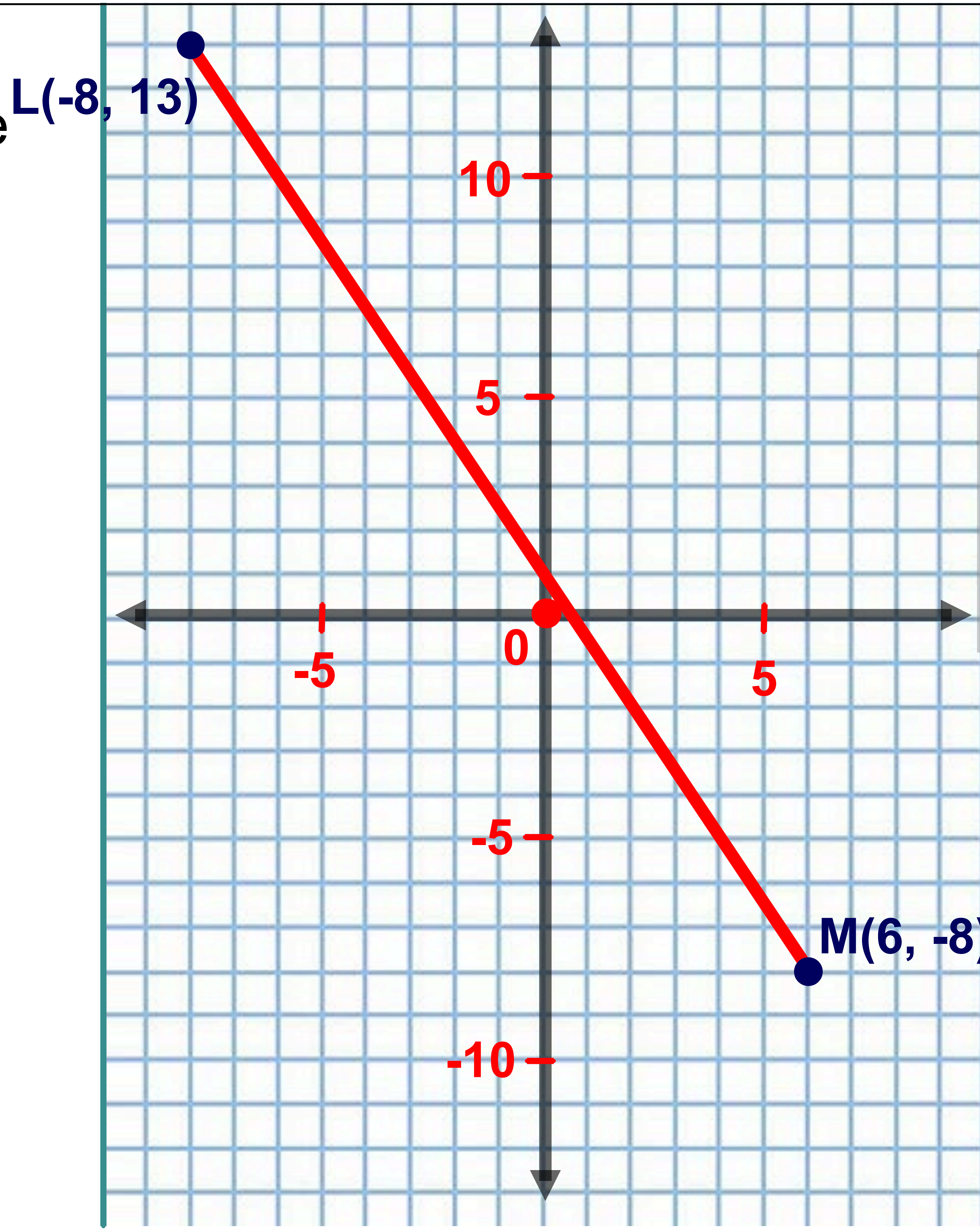
A (-6, 10)

B (-4, 7)

C (-2, 4)

D (0, 1)

E (2, -3)



Answer

20 Line segment LM in the coordinate plane has endpoints with coordinates (-8, 13) & (6, -8), shown in the graph below. Find 2 possible locations for a point P that divides $\overline{LM}$ into two parts with lengths in a ratio of 6:1.

A (-6, 10)

B (-4, 7)

C (-2, 4)

D (0, 1)

E (4, -6)

Answer

21 Line segment NO in the coordinate plane has endpoints with coordinates (10, 12) & (-10, -13), shown in the graph below. Find 2 possible locations for a point P that divides $\overline{NO}$ into two parts with lengths in a ratio of 3:2.

A (-6, -8)

B (-2, -3)

C (0, -0.5)

D (2, 2)

E (6, 7)

Answer

22 Line segment QR in the coordinate plane has endpoints with coordinates (-12, 11) & (12, -13), shown in the graph below. Find 2 possible locations for a point P that divides $\overline{QR}$ into two parts with lengths in a ratio of 5:3.

A (3, -4)

B (0, -1)

C (-3, 2)

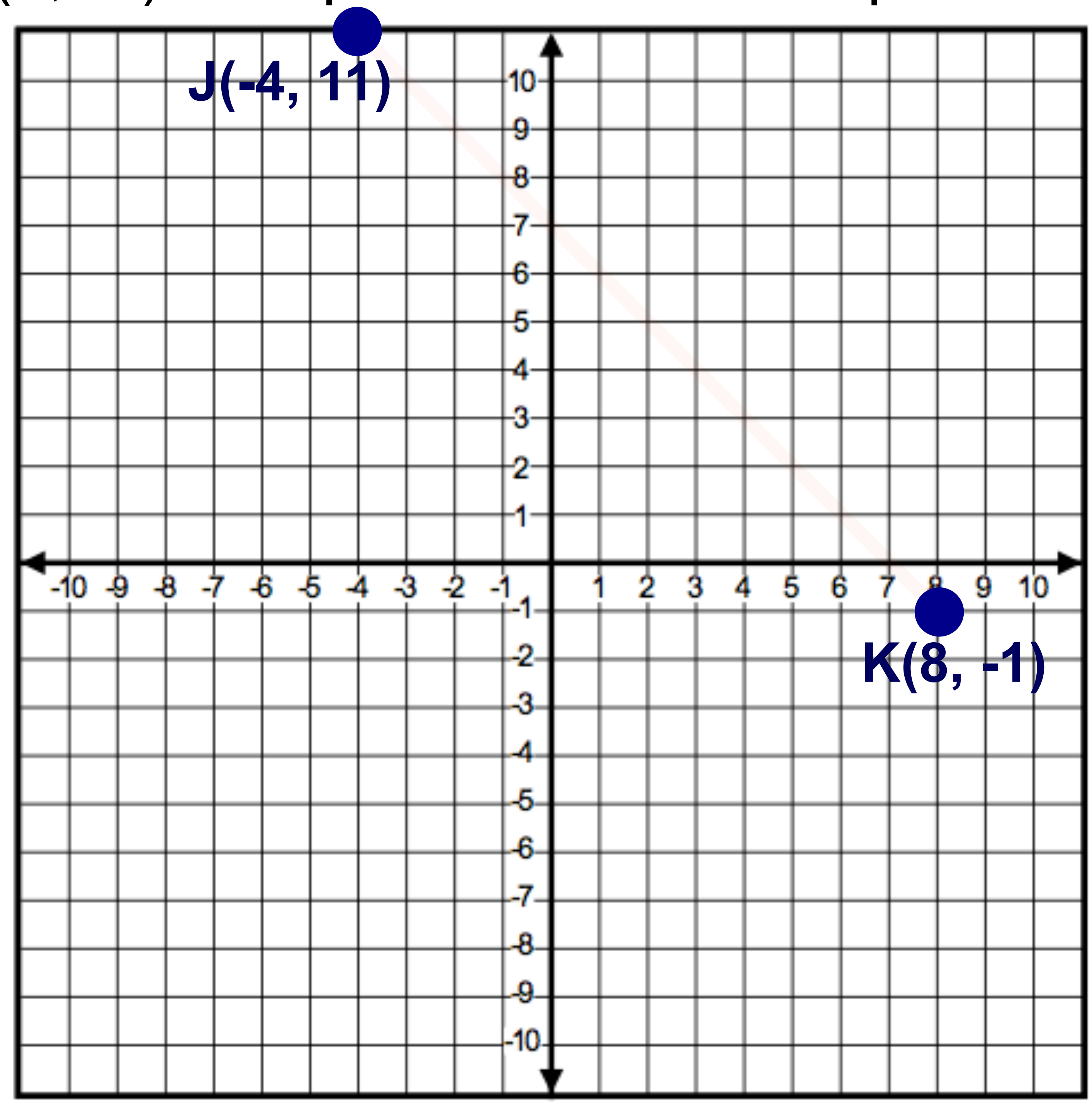
D (-6, 5)

E (-9, 8)

Answer

Line segment JK in the coordinate plane has endpoints with coordinates (-4, 11) & (8, -1). Graph $\overline{JK}$ and find two possible locations for point M so that M divides $\overline{JK}$ into two parts with lengths in a ratio of 1:3.

To graph a line segment, click the locations for the two points. Then click in between the two points to make the segment.



23 Find two possible locations for point M so that M divides JK into two parts with lengths in a ratio of 1:3.

A (-2, 9)

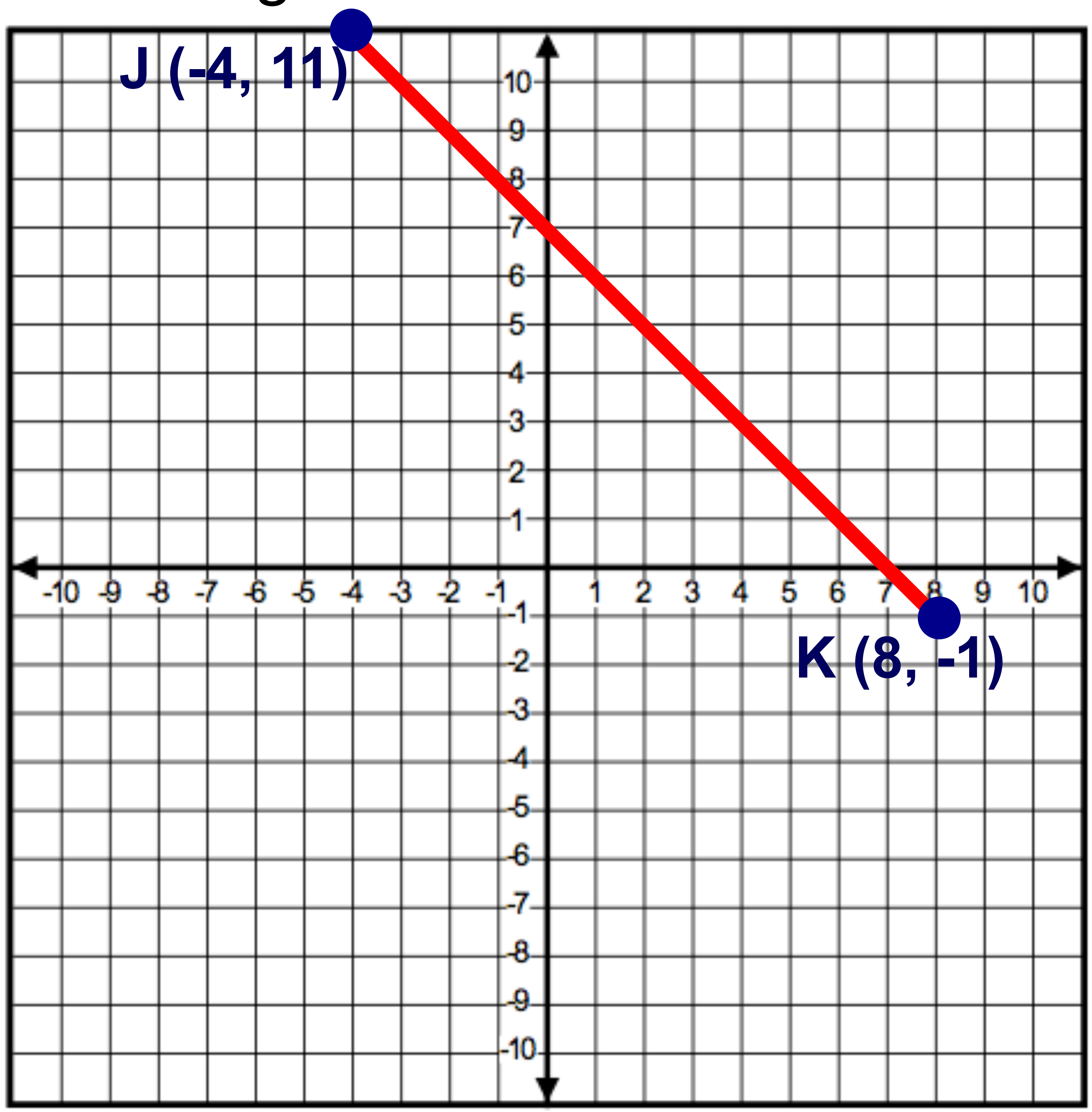
B (-1, 8)

C (0, 7)

D (4, 3)

E (5, 2)

F (6, 1)



Answer

PARCC Released Question (EOY) - Non-Calculator Section - Problem 5 - Part 2 - Response Format

24 Point Q lie on ST, where point S is located at (-2, -6) and point T is located at (5, 8). If SQ:QT = 5:2, where is point Q on ST?

A (-1, -4)

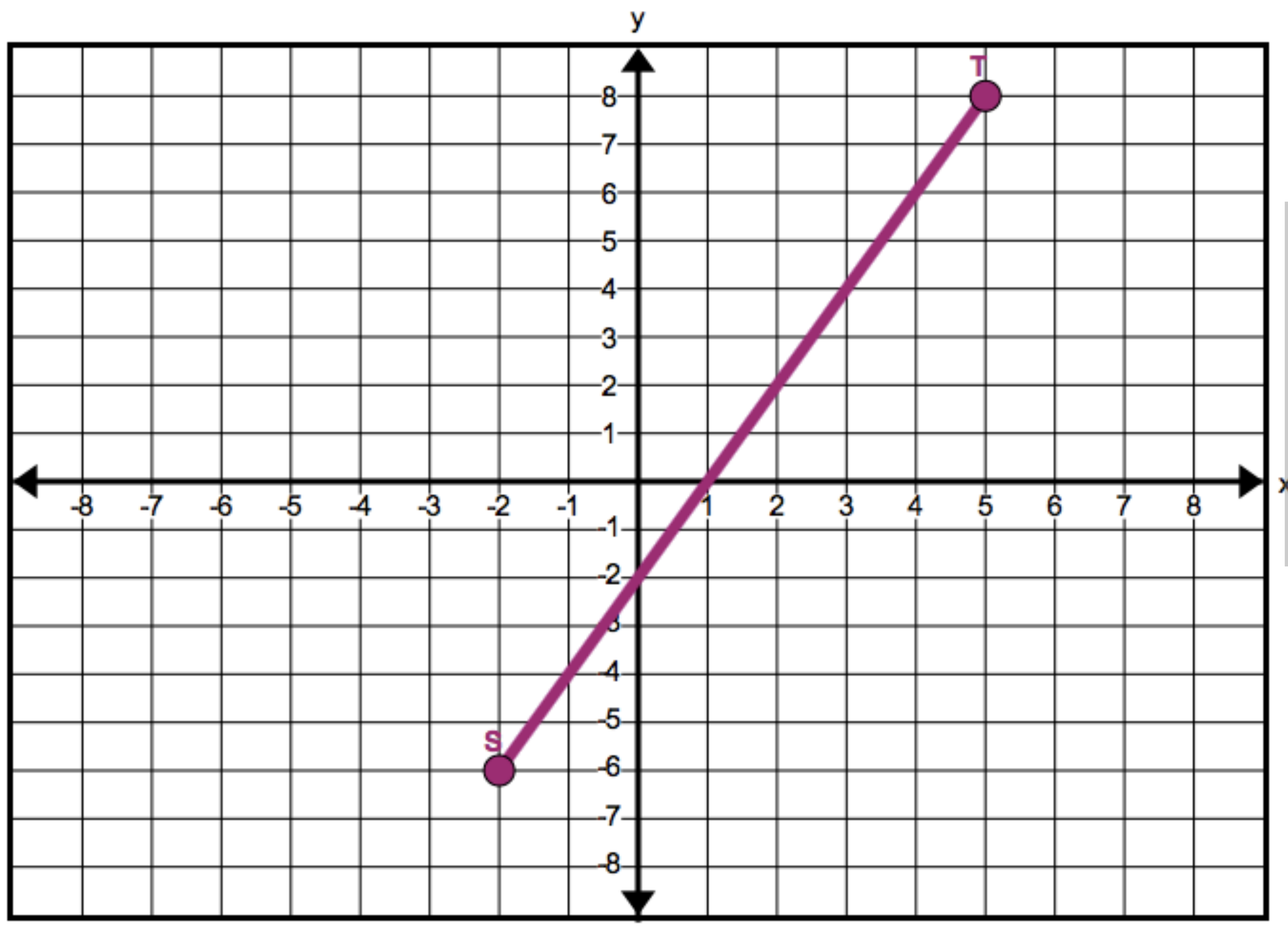
B (0, -2)

C (1, 0)

D (2, 2)

E (3, 4)

F (4, 6)



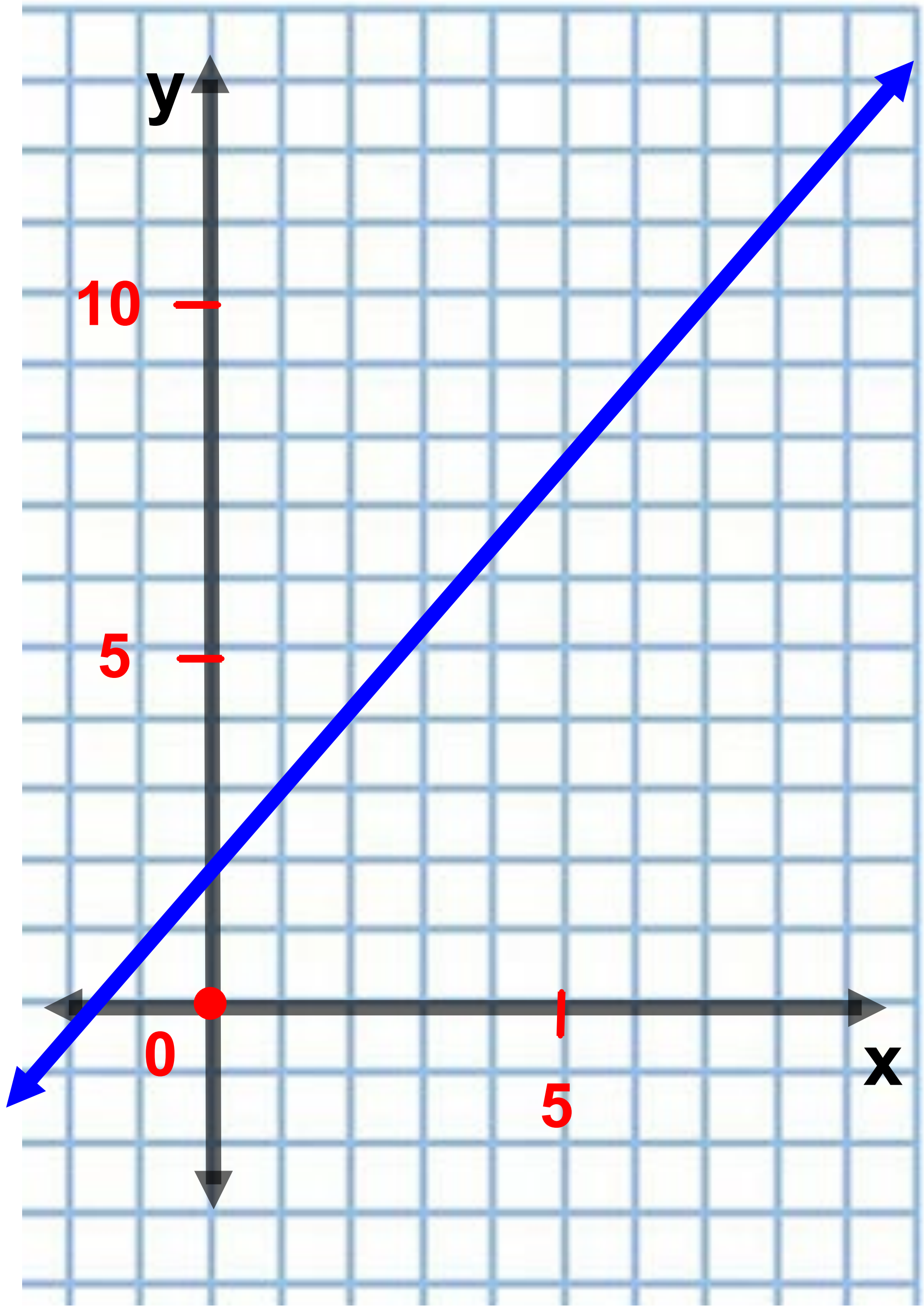
Answer

PARCC Released Question (PBA) - Non-Calculator Section - Problem 2

Slopes of Parallel & Perpendicular Lines

Return to Table of Contents

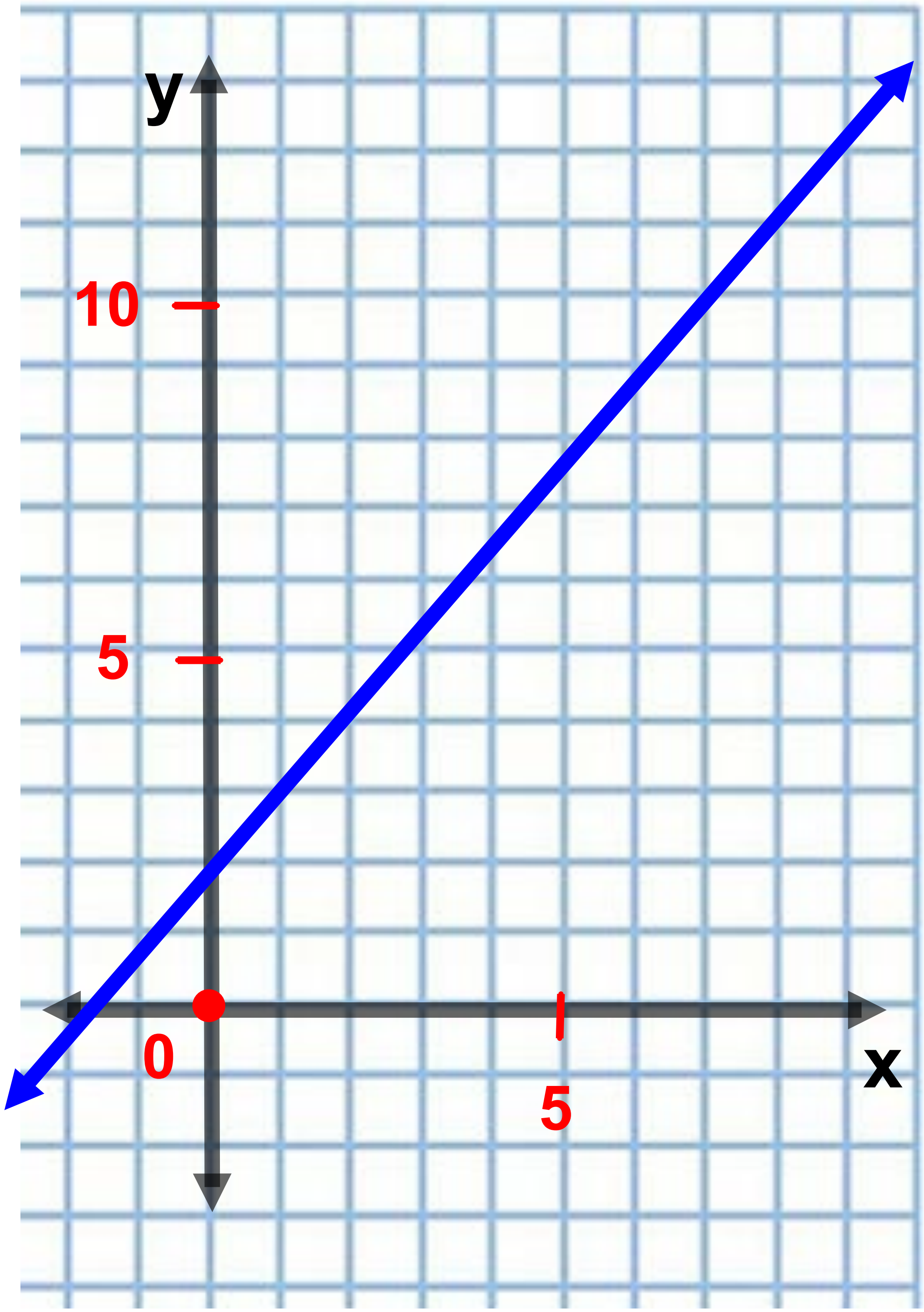
Slope



The slope of a line indicates the angle it makes with the x-axis.

The symbol for slope is "m".

Slope



A horizontal line has a slope of zero.

A vertical line has an undefined slope.

A line which rises as you move from left to right has a positive slope.

A line which falls as you move from left to right has a negative slope.

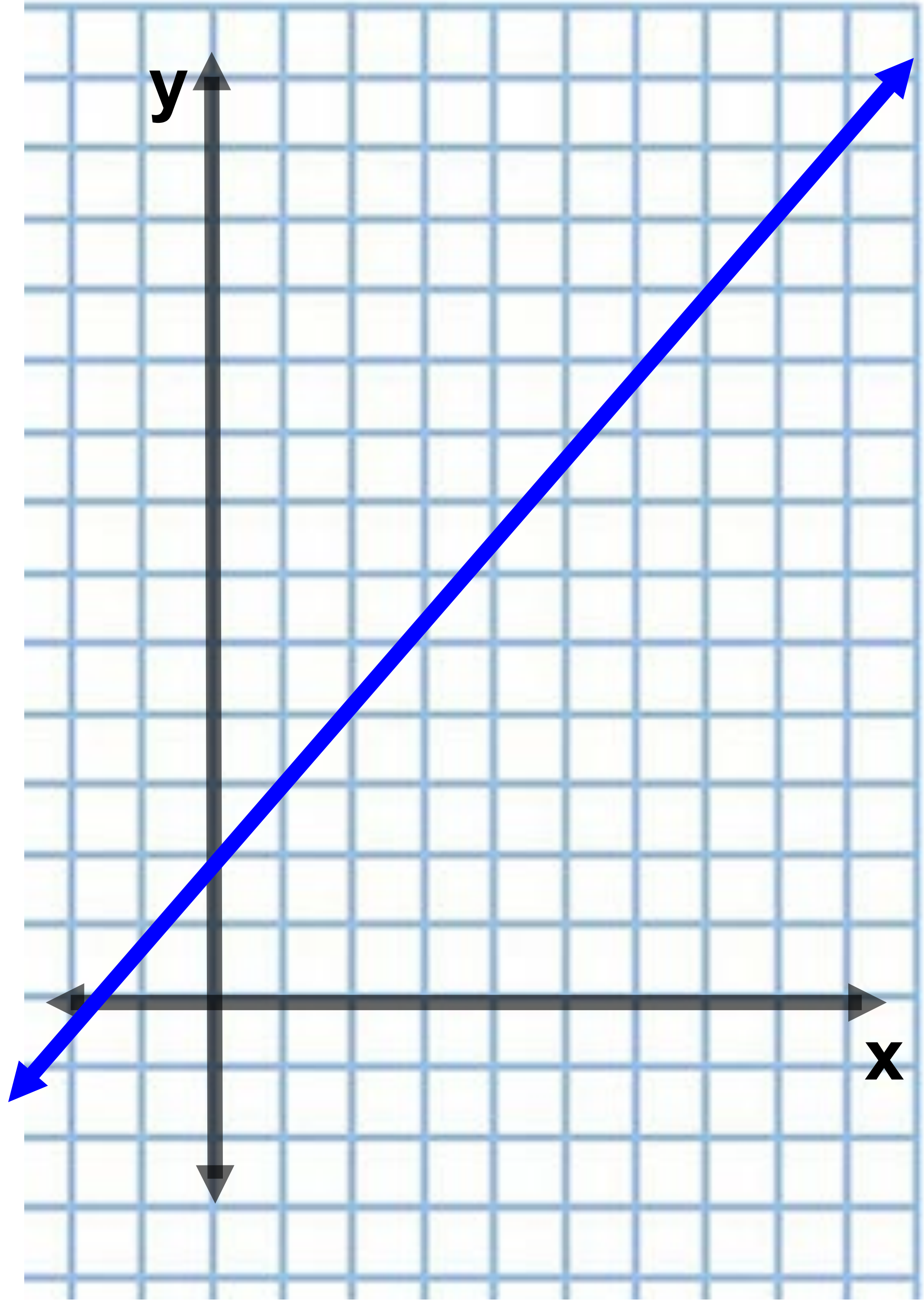
25 The slope of the indicated line is:

A negative

B positive

C zero

D undefined



Answer

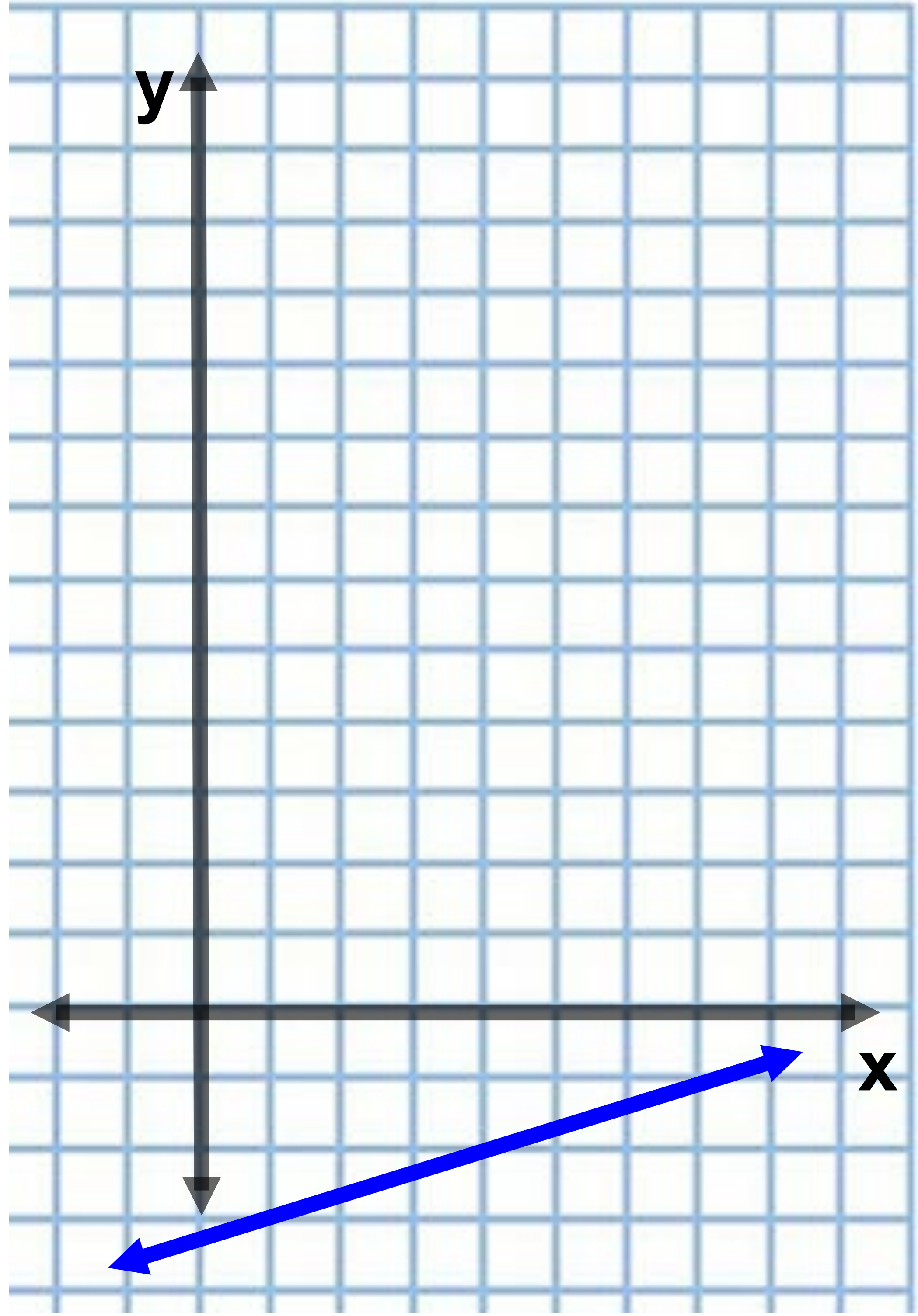
26 The slope of the indicated line is:

A negative

B positive

C zero

D undefined



Answer

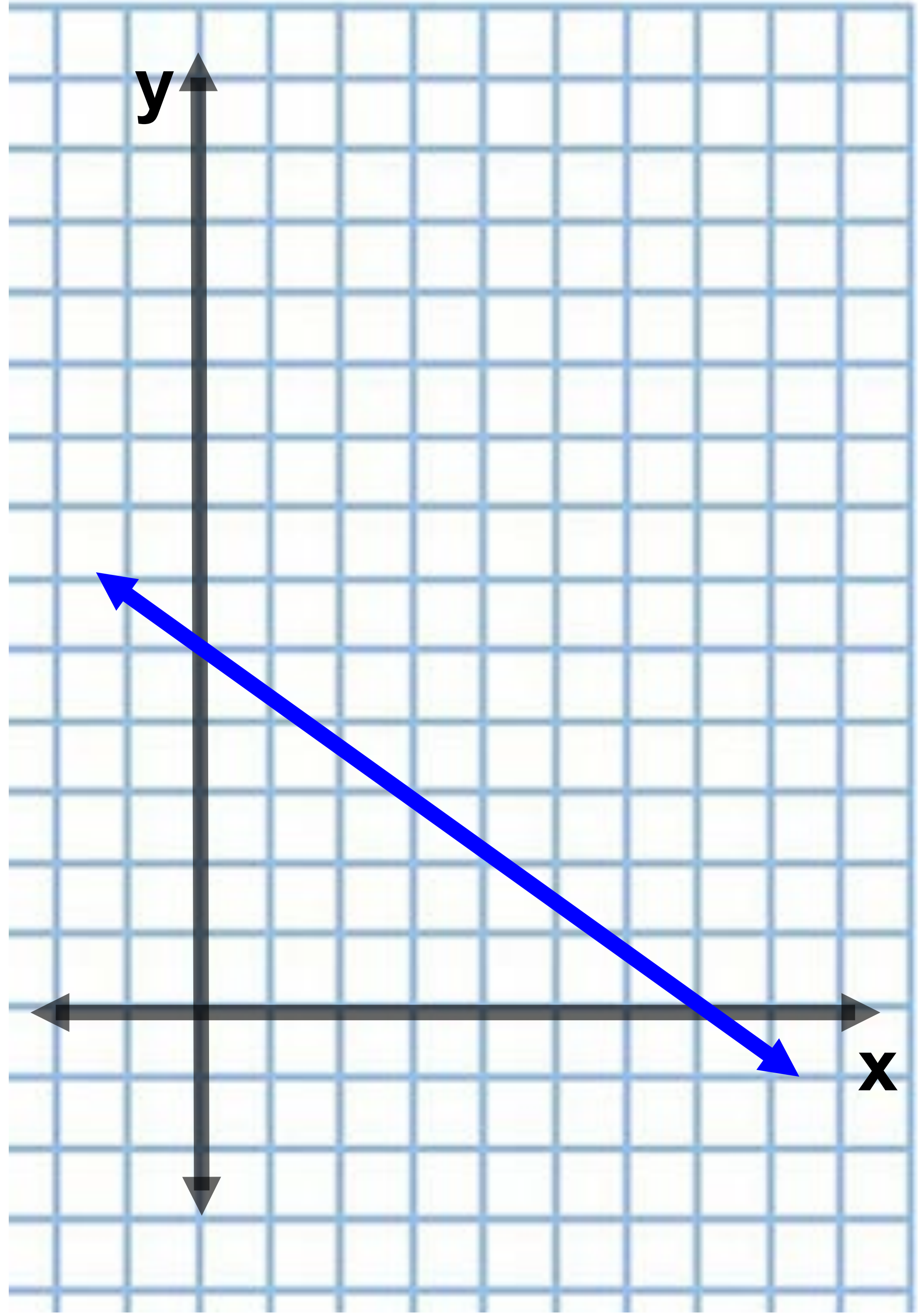
27 The slope of the indicated line is:

A negative

B positive

C zero

D undefined



Answer

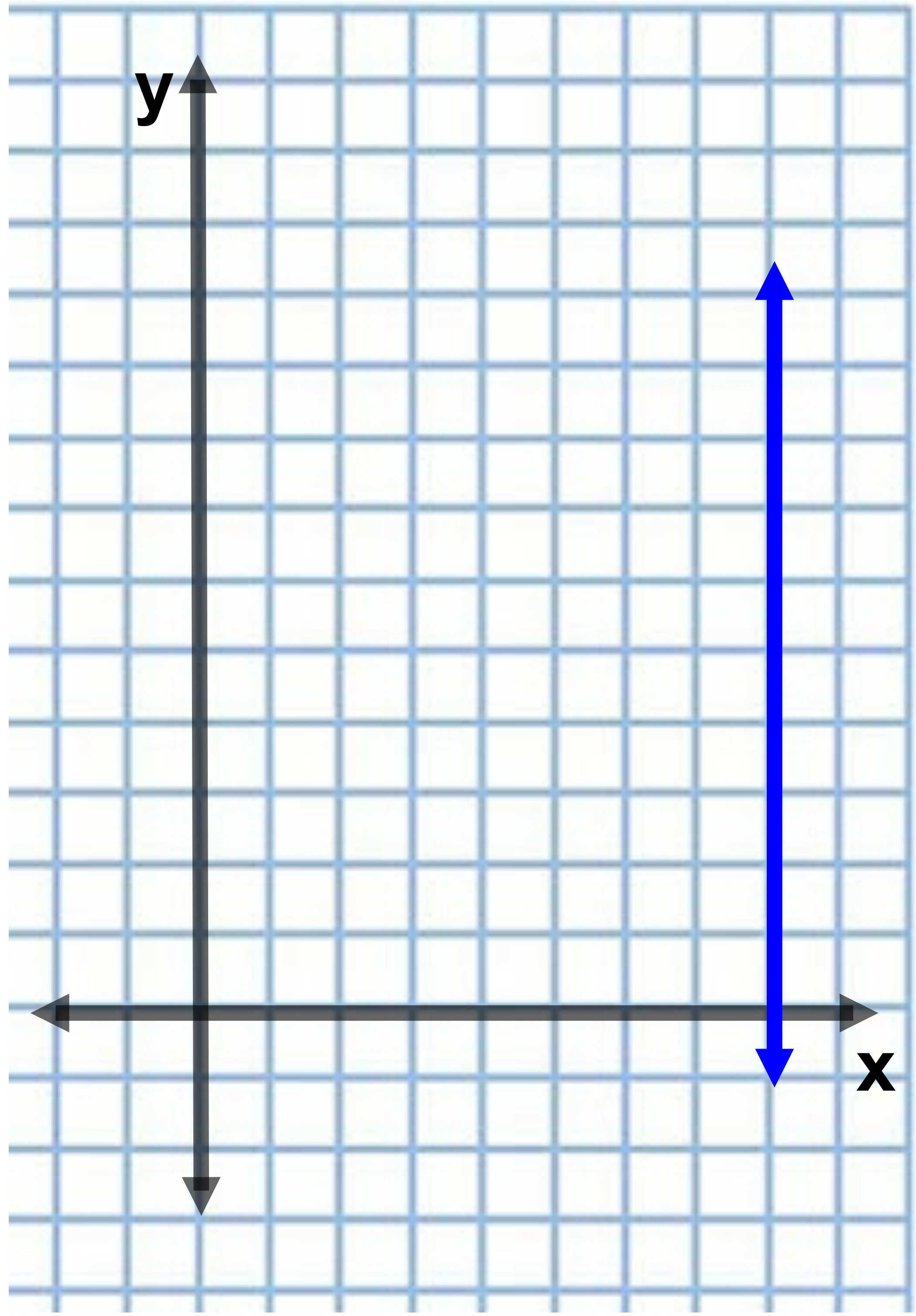
28 The slope of the indicated line is:

A negative

B positive

C zero

D undefined



Answer

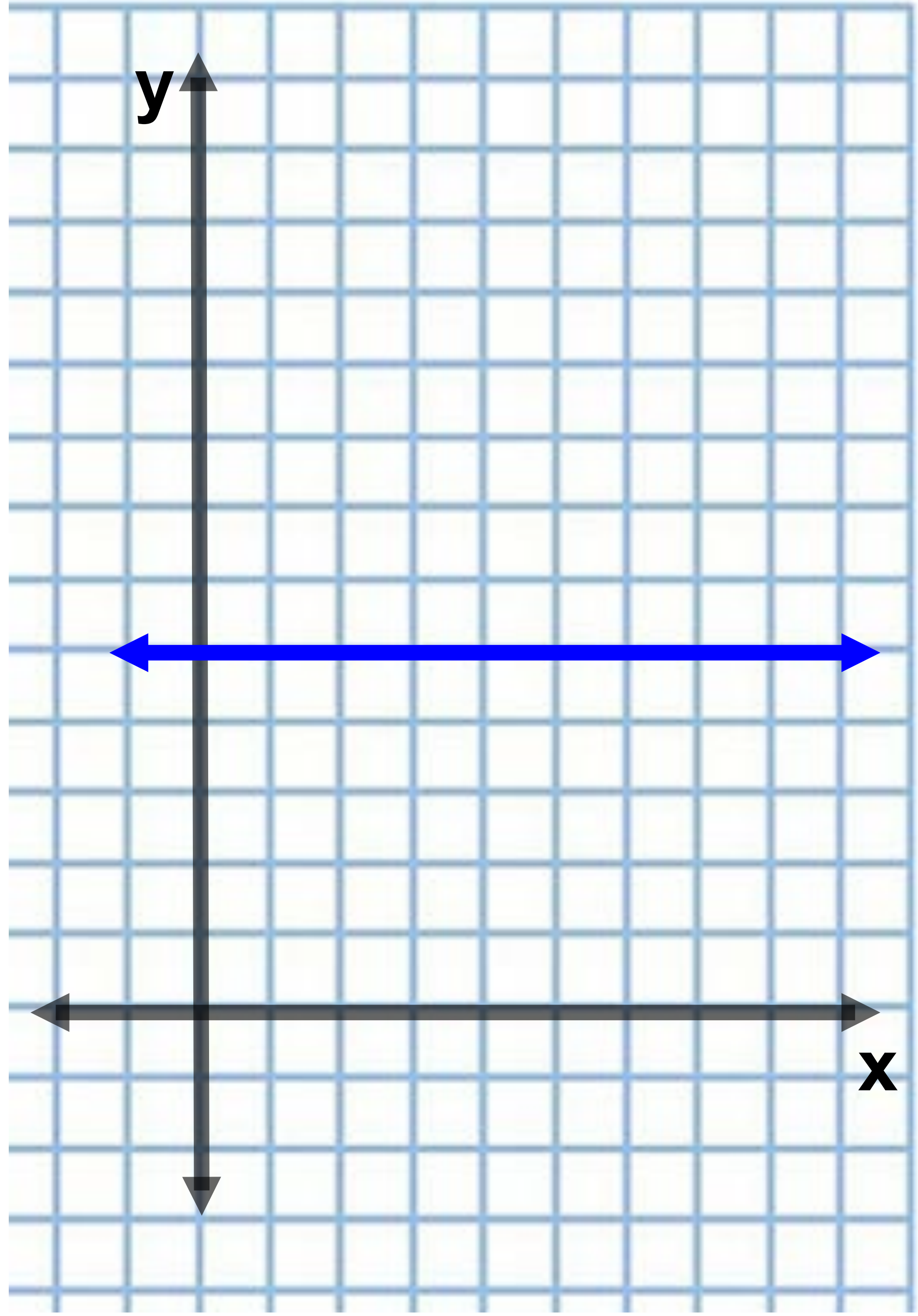
29 The slope of the indicated line is:

A negative

B positive

C zero

D undefined



Answer

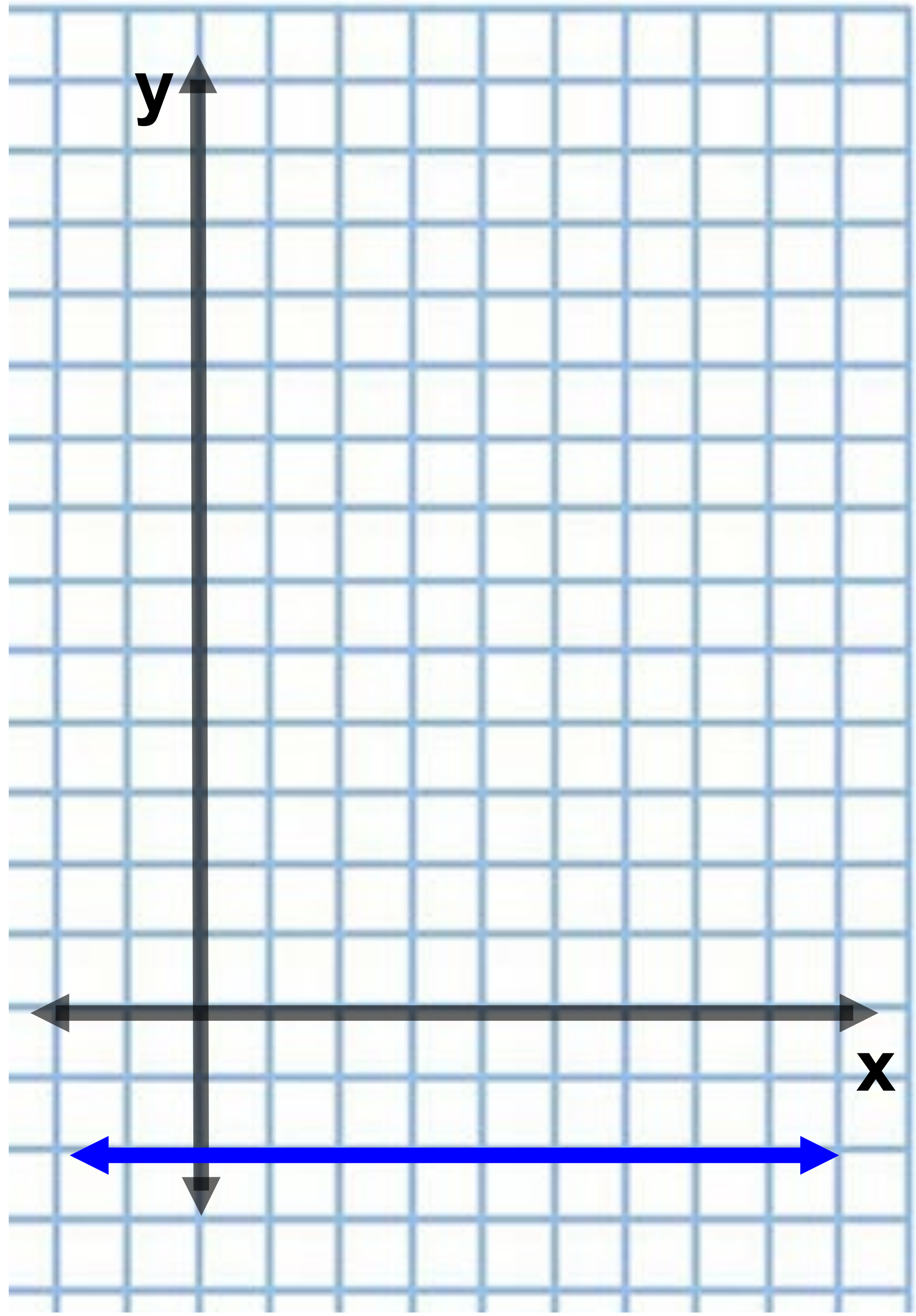
30 The slope of the indicated line is:

A negative

B positive

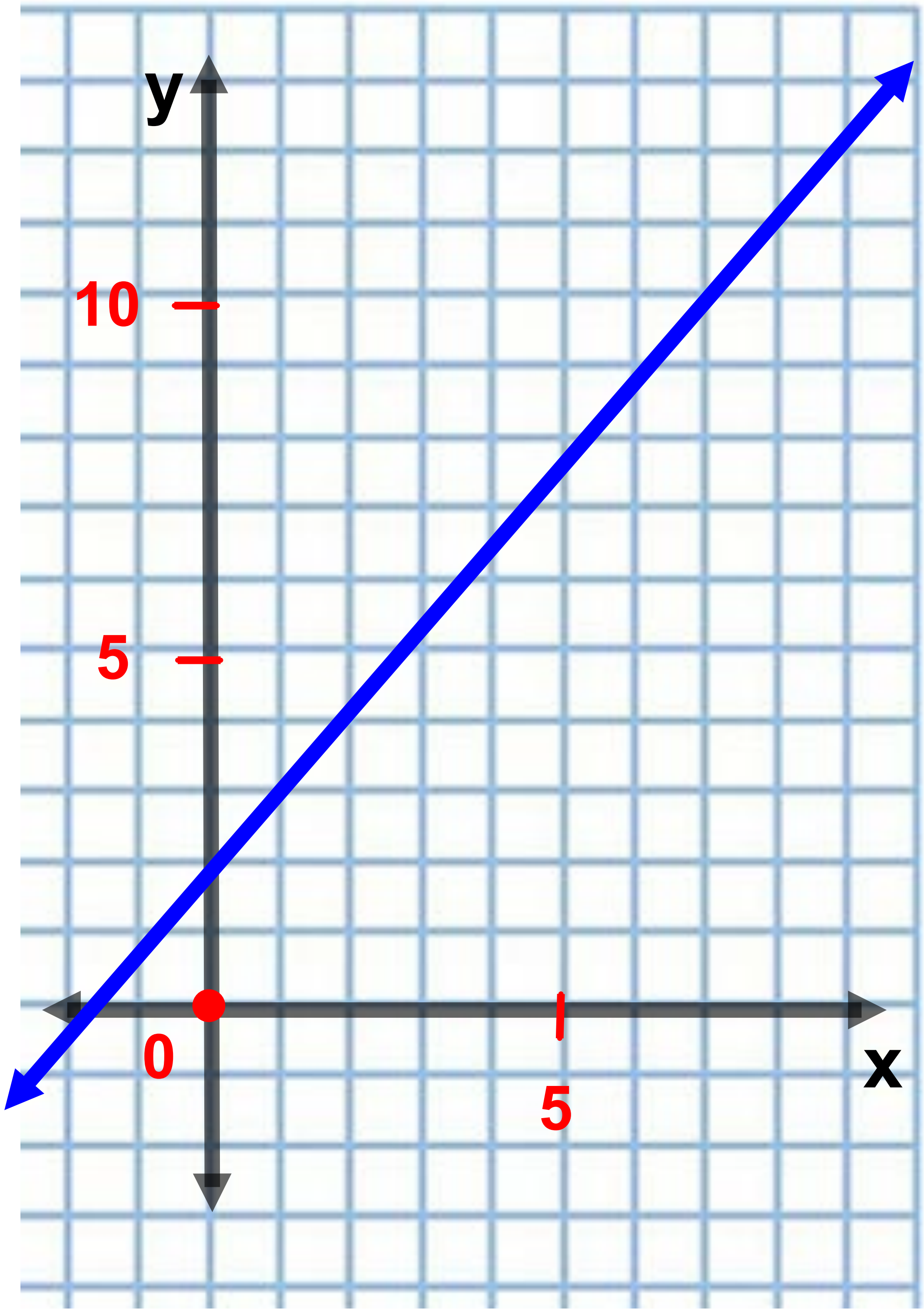
C zero

D undefined



Answer

Slope

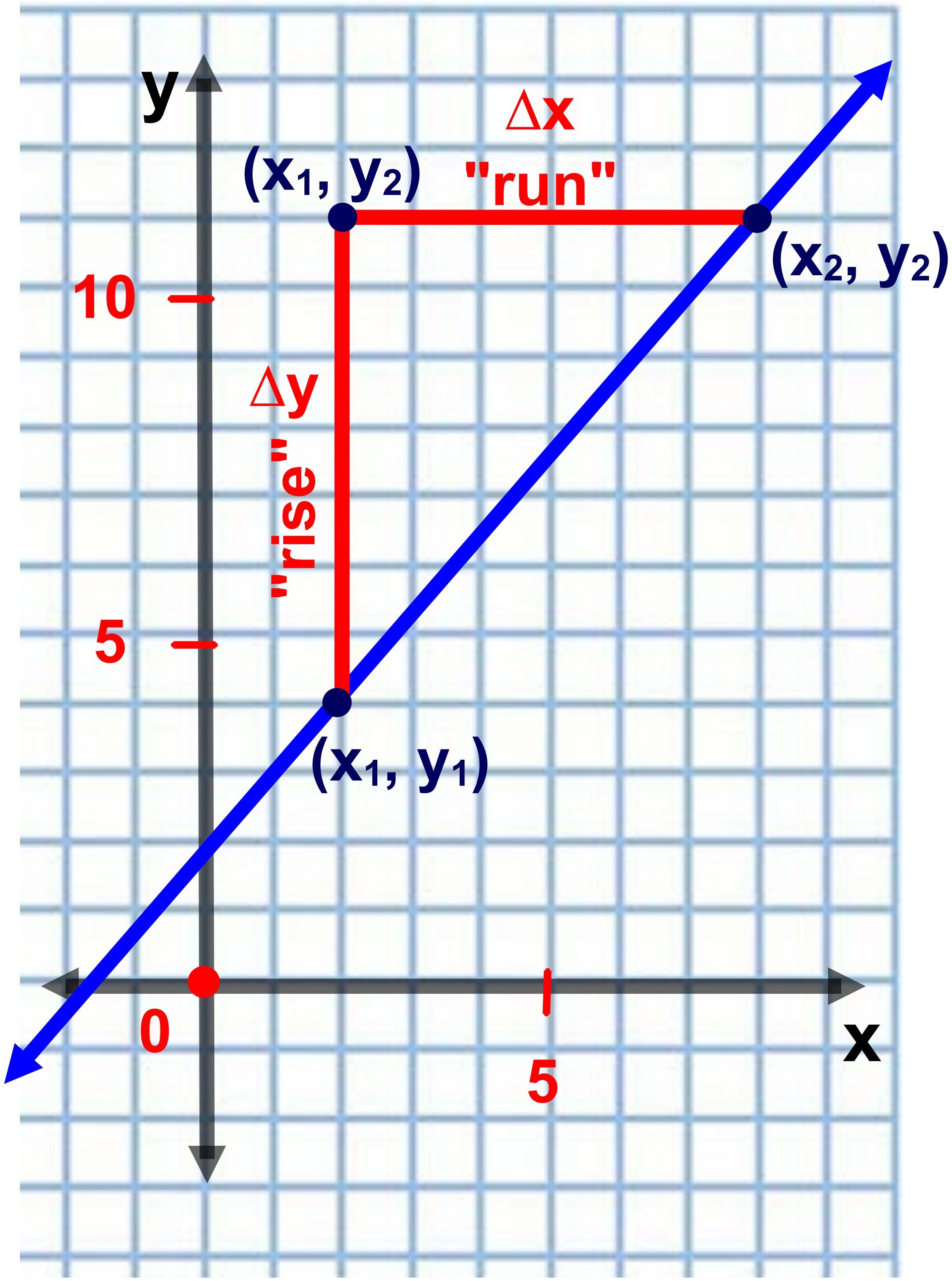


The slope of a line is not given in degrees.

Rather, it is given as the ratio of "rise" over "run".

The slope of a line is the same anywhere along the line, so any two points on the line can be used to calculate the slope.

Slope



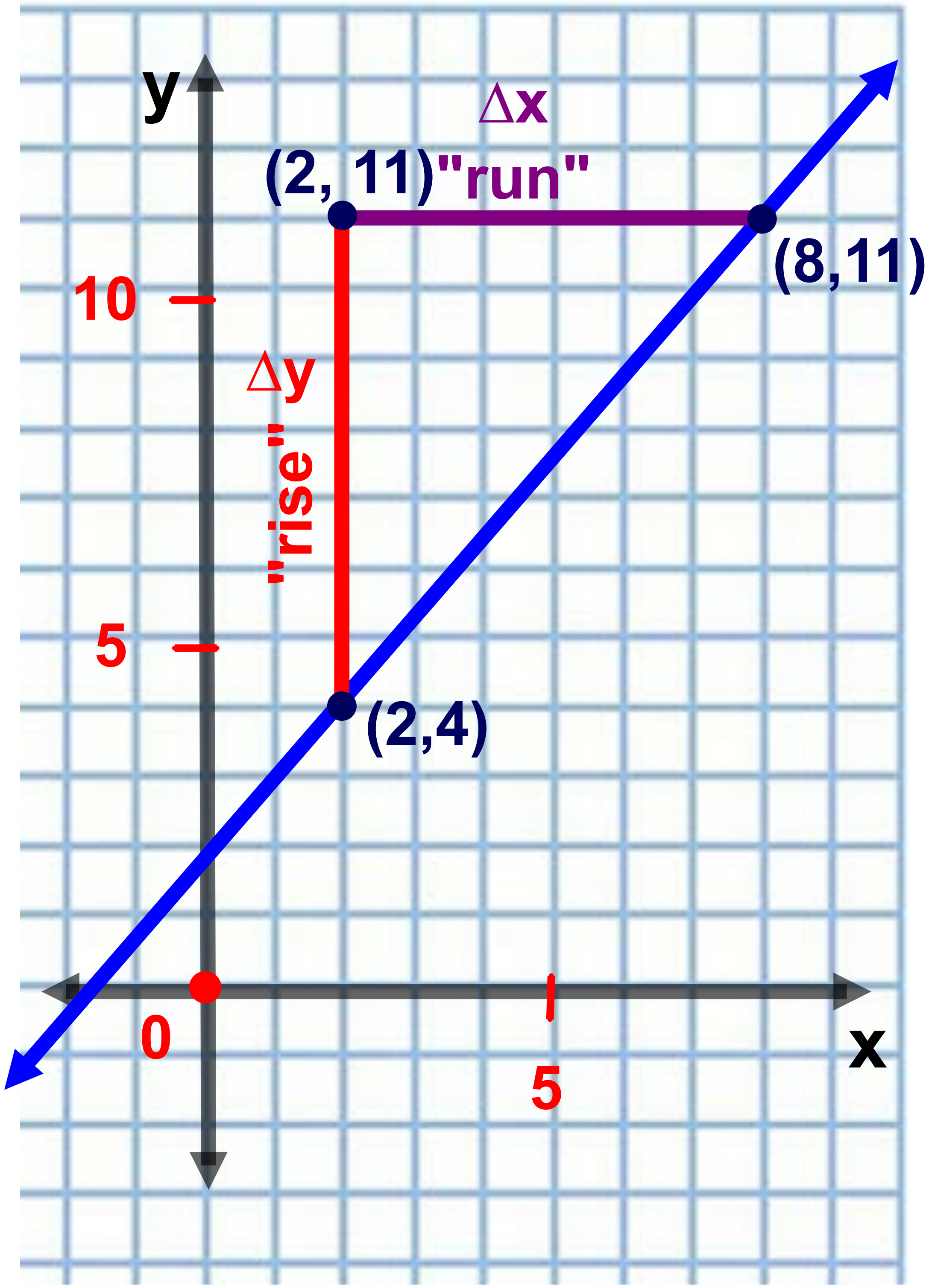
The "rise" is the change in the value of the y-coordinate while the "run" is the change in the value of the x-coordinate.

The symbol for change is the Greek letter delta, "Δ", which just means "change in".

So the slope is equal to the change in y divided by the change in x, or Δy divided by Δx...delta y over delta x.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope



$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

In this case:

The rise is from 4 to 11

$\Delta y = 11 - 4 = 7$

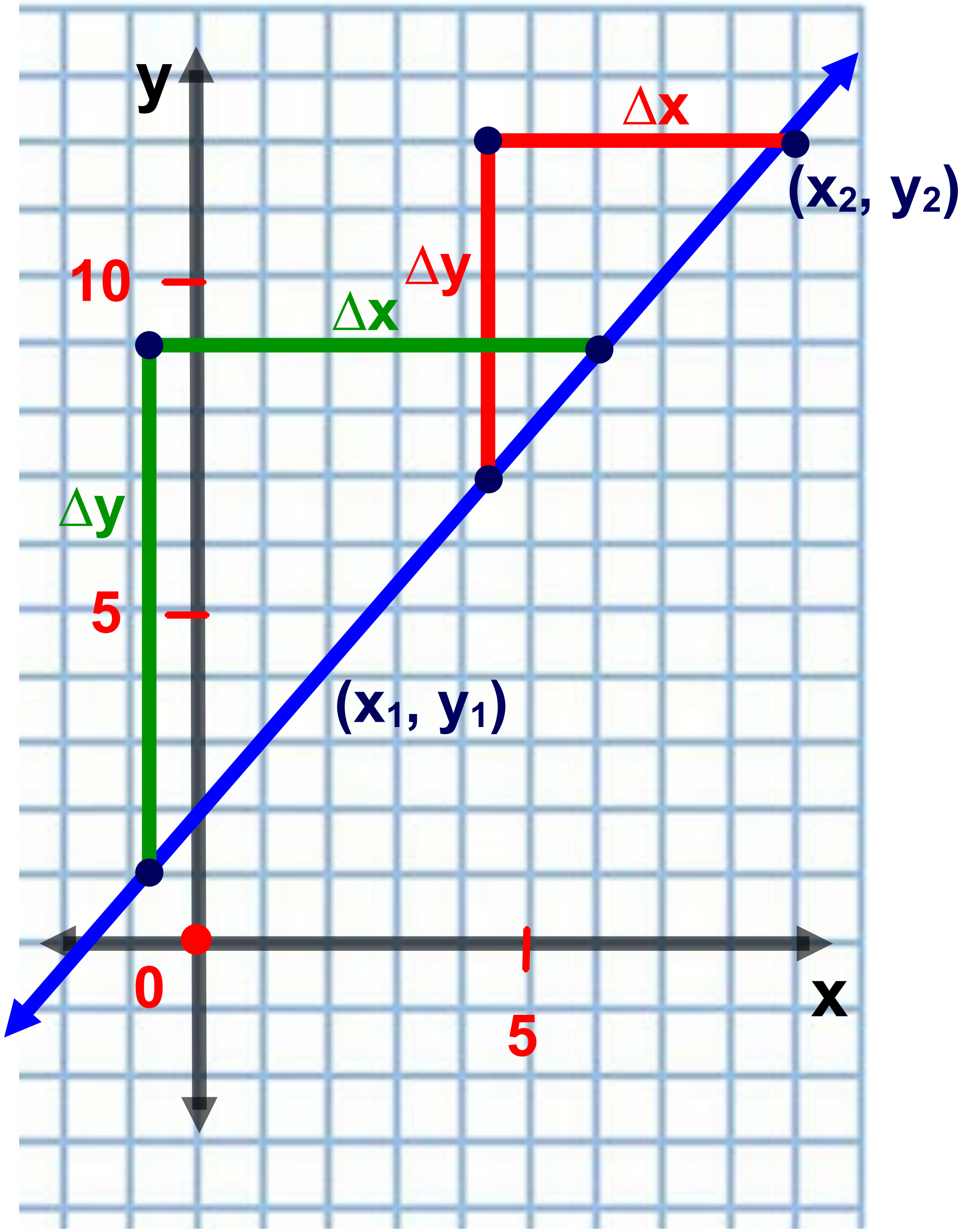
And the run is from 2 to 8,

$\Delta x = 8 - 2 = 6$

So the slope is

$$m = \frac{\Delta y}{\Delta x} = \frac{7}{6}$$

Slope



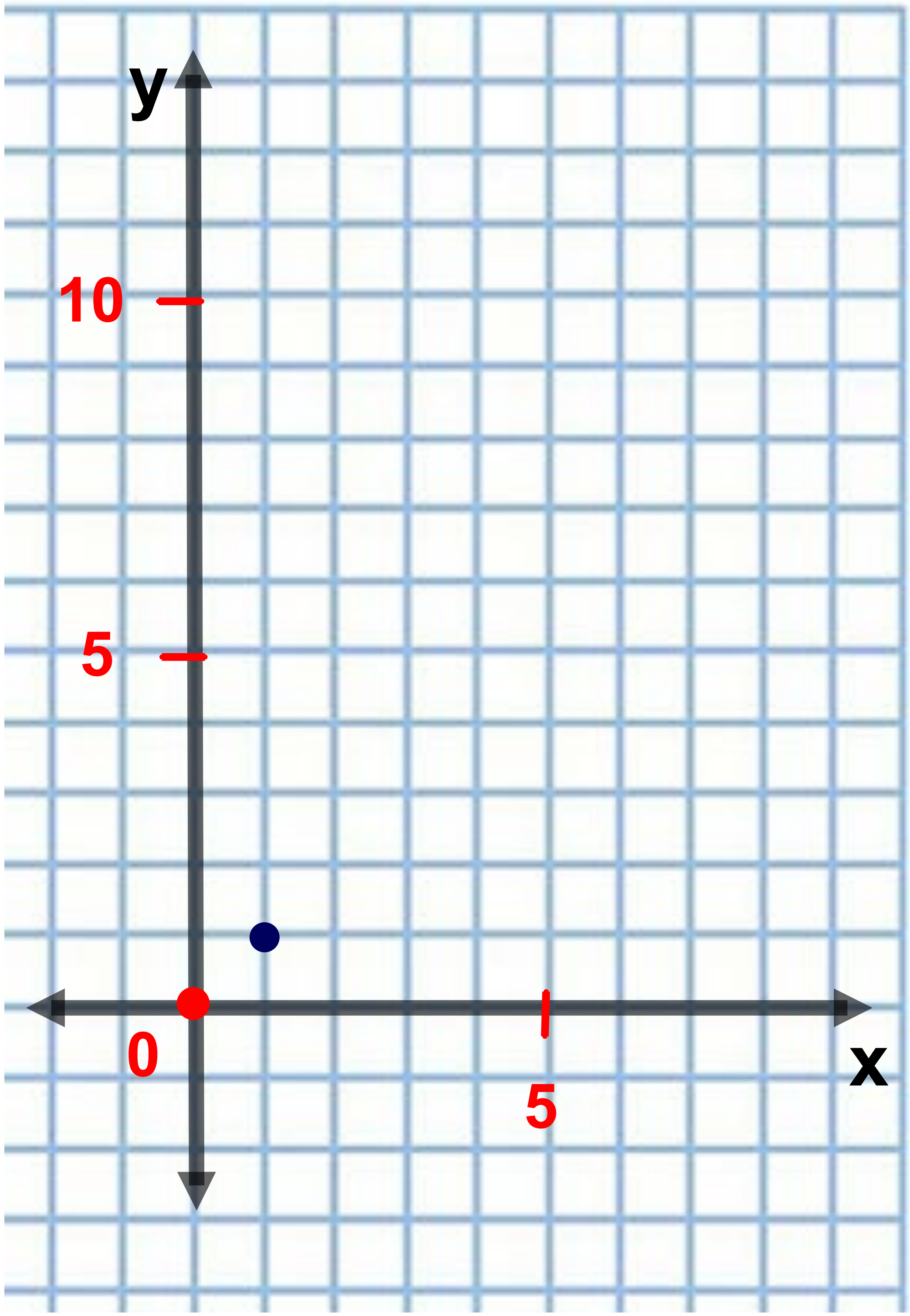
Any points on the line can be used to calculate its slope, since the slope of a line is the same everywhere.

The values of Δy and Δx may be different for other points, but their ratio will be the same.

You can check that with the red and green triangles shown here.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Using Slope to Draw a Line

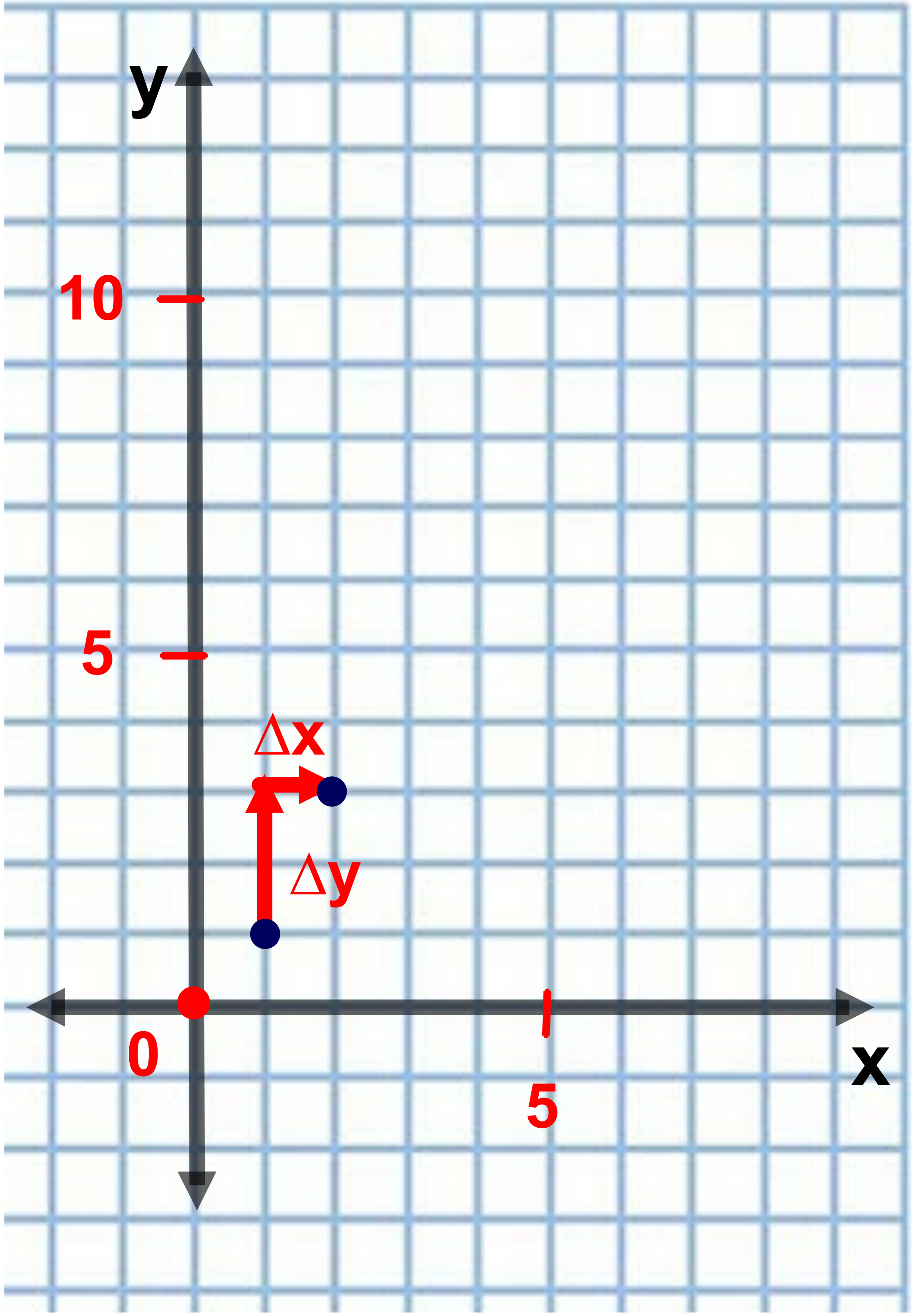


The slope also allows us to quickly graph a line, given one point on the line.

For instance, if I know one point on a line is (1, 1) and that the slope of the line is 2, I can find a second point, and then draw the line.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

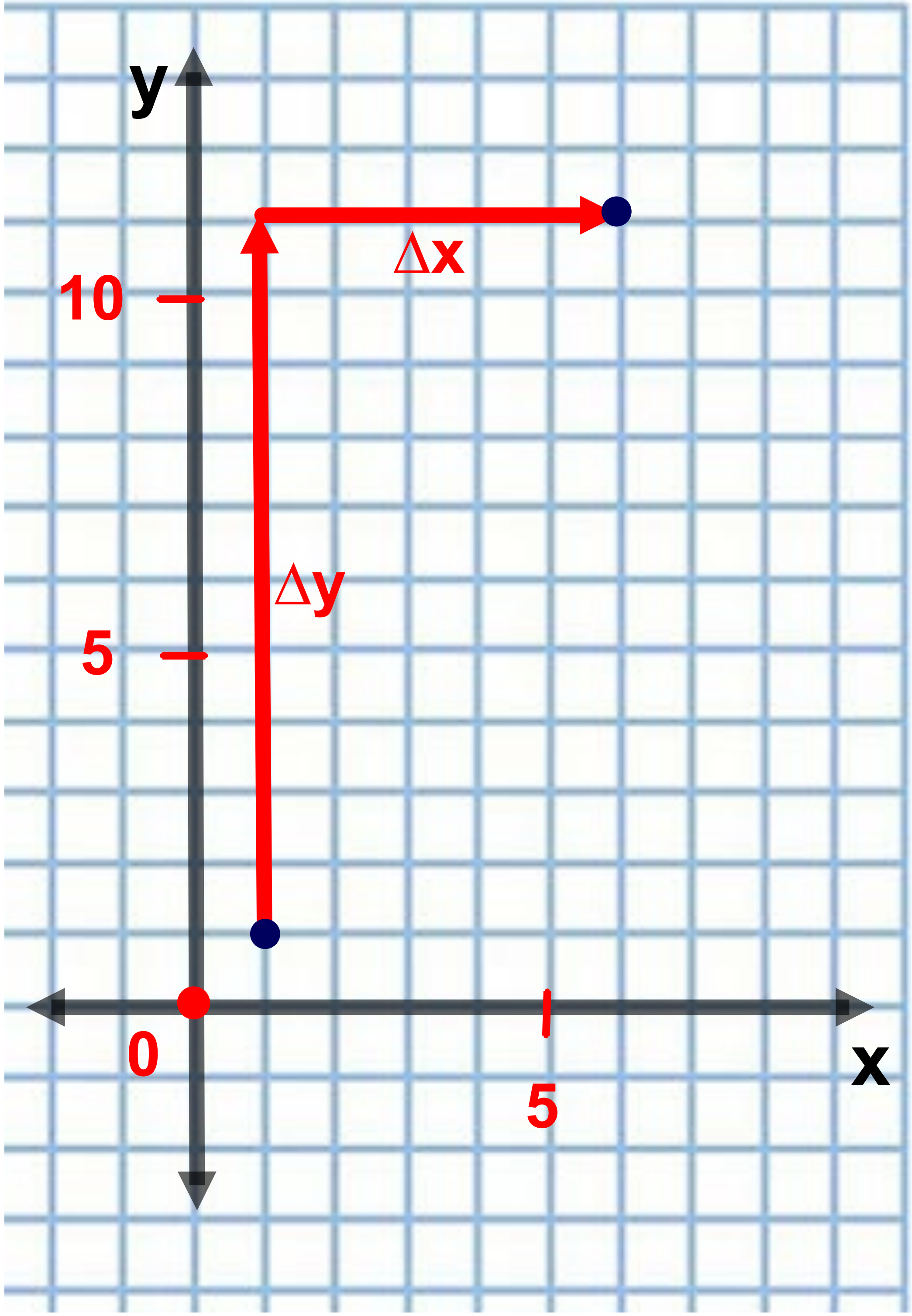
Using Slope to Draw a Line



I do this by recognizing that the slope of 2 means that if I go up 2 units on the y-axis I have to go 1 unit to the right on the x-axis .

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Using Slope to Draw a Line

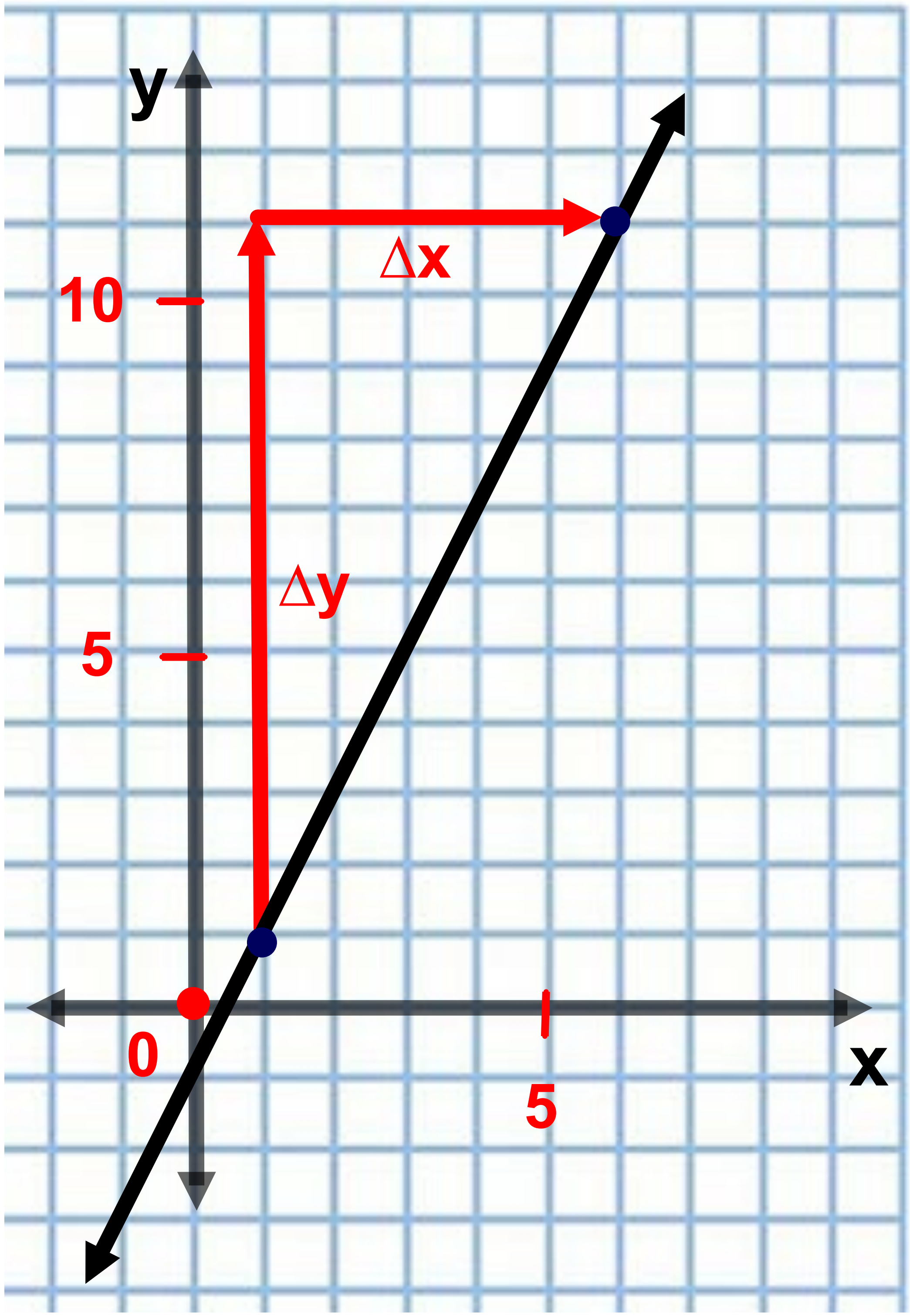


I do this by recognizing that the slope of 2 means that if I go up 2 units on the y-axis I have to go 1 unit to the right on the x-axis .

Or if I go up 10, I have to go over 5 units, etc.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Using Slope to Draw a Line



Then I draw the line through any two of those points.

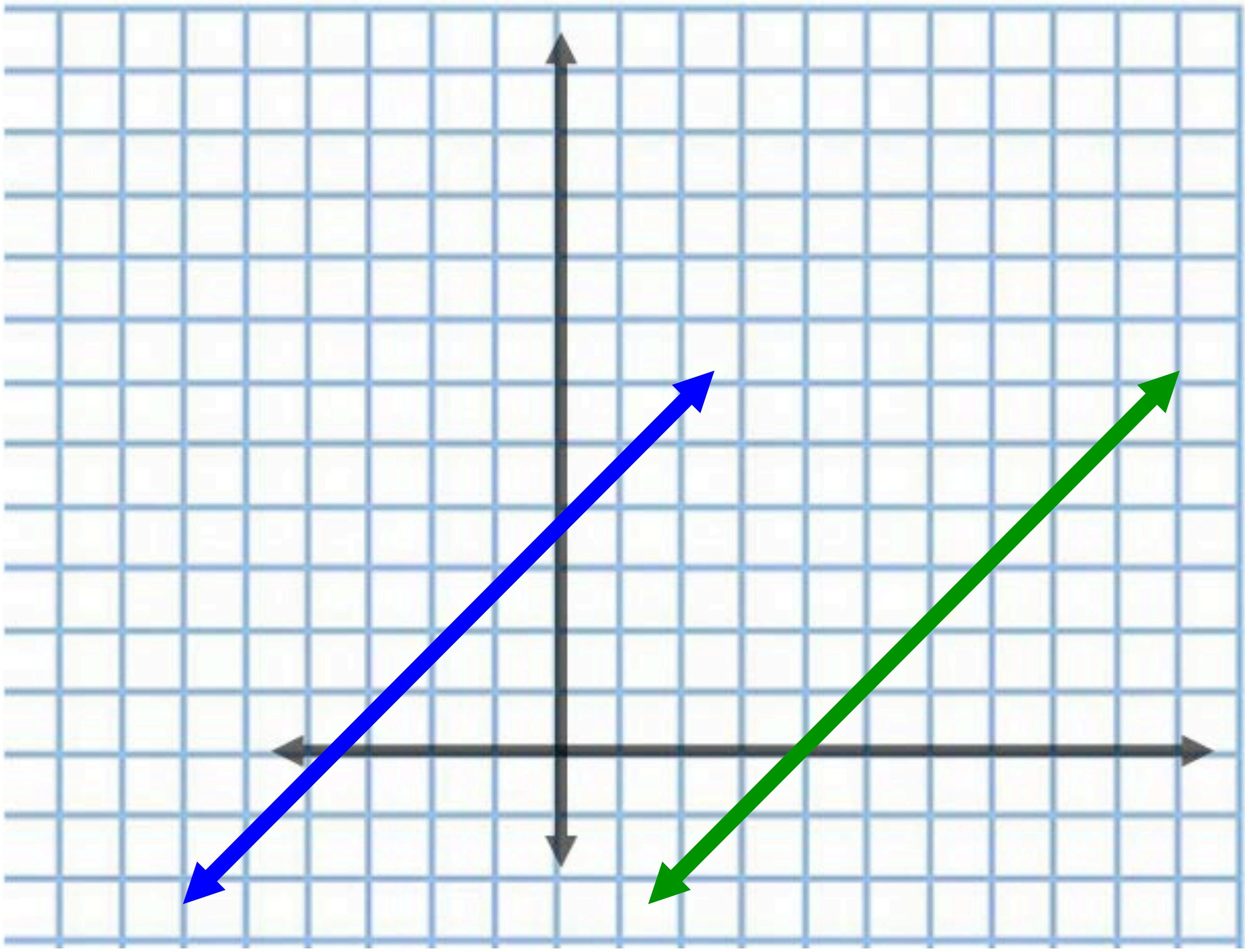
This method is the easiest to use if you just have to draw a line given a point and slope.

The same approach works for writing the equation of a line.

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

Slopes of Parallel Lines

Parallel lines have the same slope.



With what we've learned about parallel lines this is easy to understand.

The slope of a line is related to the angle it makes with the x-axis.

Now, think of the x-axis as a transversal.

Math Practice

31 What is the name of this pair of angles, formed by a transversal intersecting two lines.

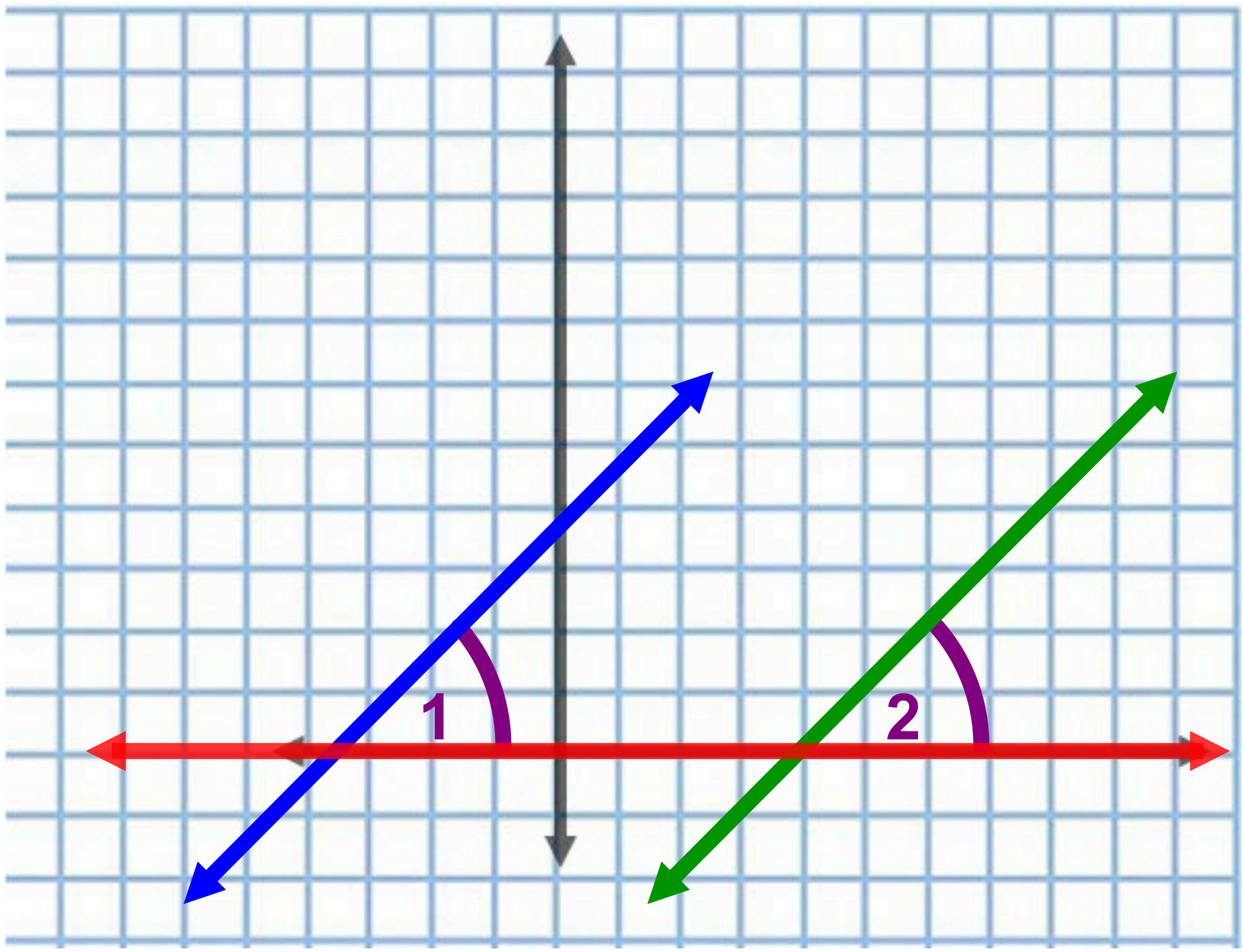
A Alternate Interior Angles

B Corresponding Angles

C Alternate Exterior Angles

D Same Side Interior Angles

Answer



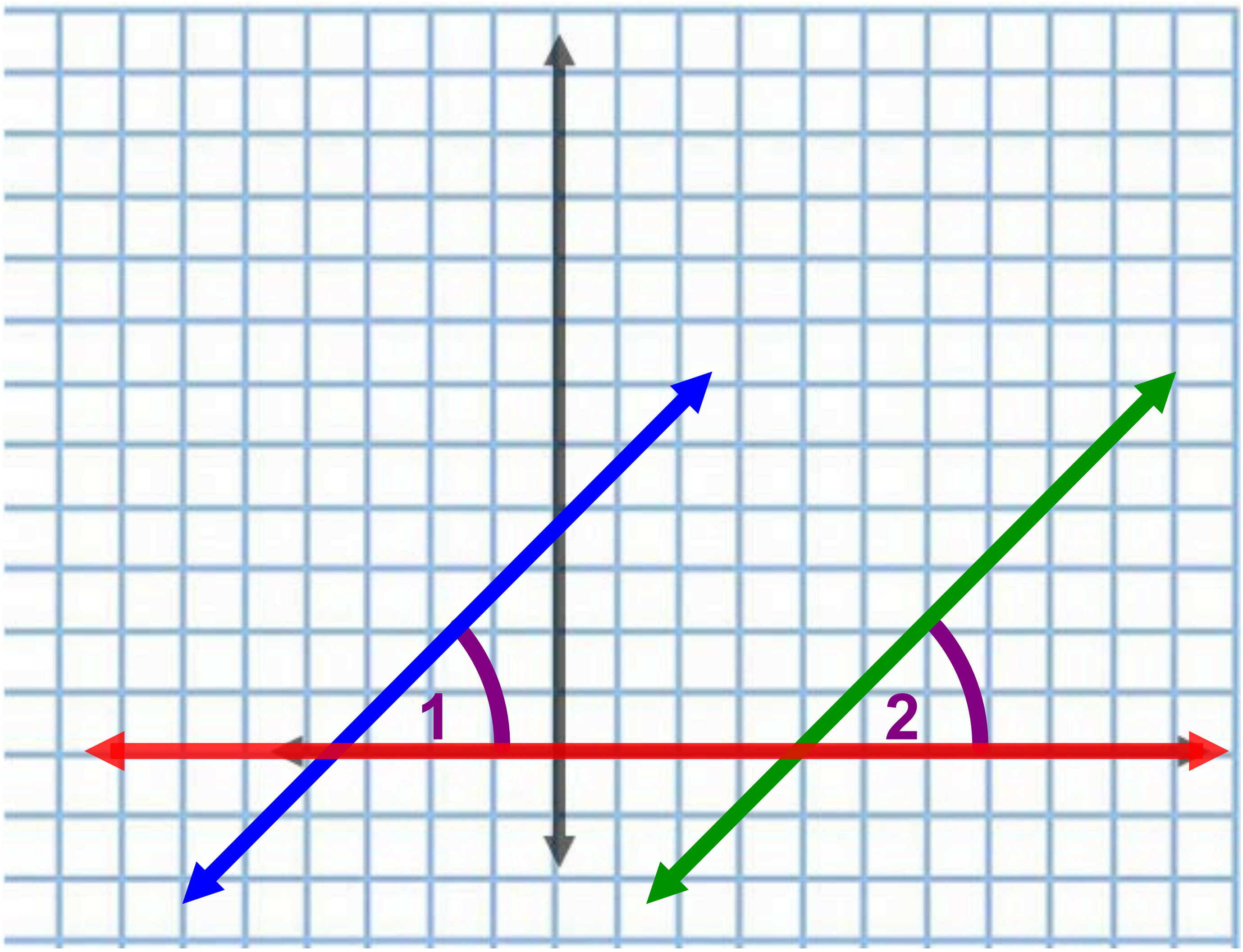
32 We know those angles must then be:

A Supplementary

B Complementary

C Equal

D Adjacent



Answer

33 That means that the slopes of parallel lines must be:

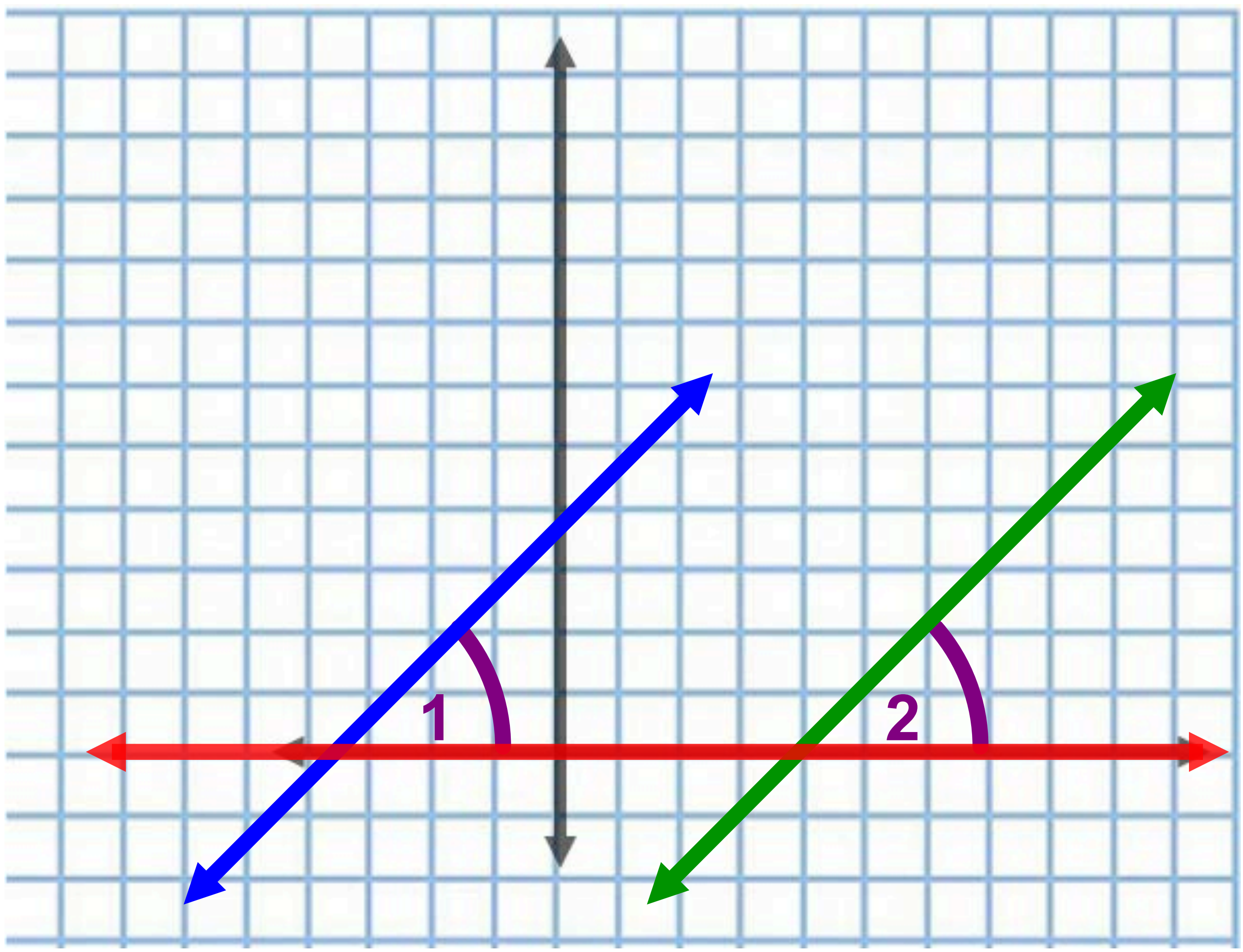
A Reciprocals

B Inverses

C Equal

D Nothing special

Answer



34 If one line has a slope of 4, what must be the slope of any line parallel to it?

Answer

35 If one line passes through the points (0, 0) and (2, 2) what must be the slope of any line parallel to that first line?

Answer

36 If one line passes through the points (-5, 9) and (5, 8) what must be the slope of any line parallel to that first line?

Answer

37 If one line passes through the points (2, 2) and (5, 5) and a parallel line passes through the point (1, 5) which of these points could lie on that second line?

A (2, 2)

B (4, 4)

C (5, 6)

D (-1, 3)

Answer

38 If one line passes through the points (-3, 4) and (0, 10) and a parallel line passes through the point (-1, -4) which of these points could lie on that second line?

A (0, -2)

B (2, -5)

C (3, 1)

D (-1, 3)

Answer

39 If one line passes through the points (3, 5) and (5, -1) and a parallel line passes through the point (-1, -1) which of these points could lie on that second line?

A (0, 1)

B (-2, 2)

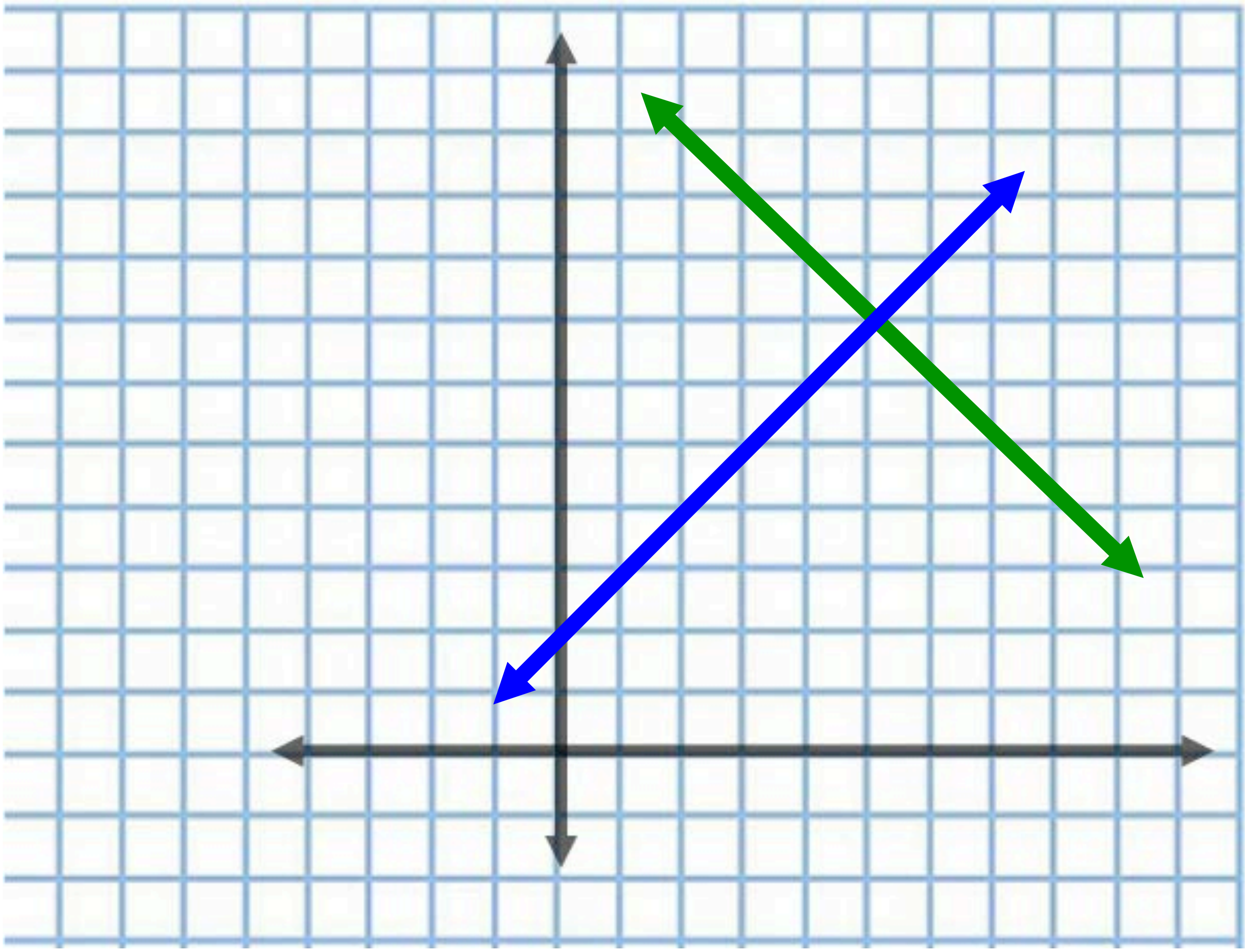
C (4, 8)

D (-4, -4)

Answer

Slopes of Perpendicular Lines

The slopes of perpendicular lines are negative (or opposite) reciprocals.



There are three ways of expressing this symbolically:

$$m_1 m_2 = -1$$

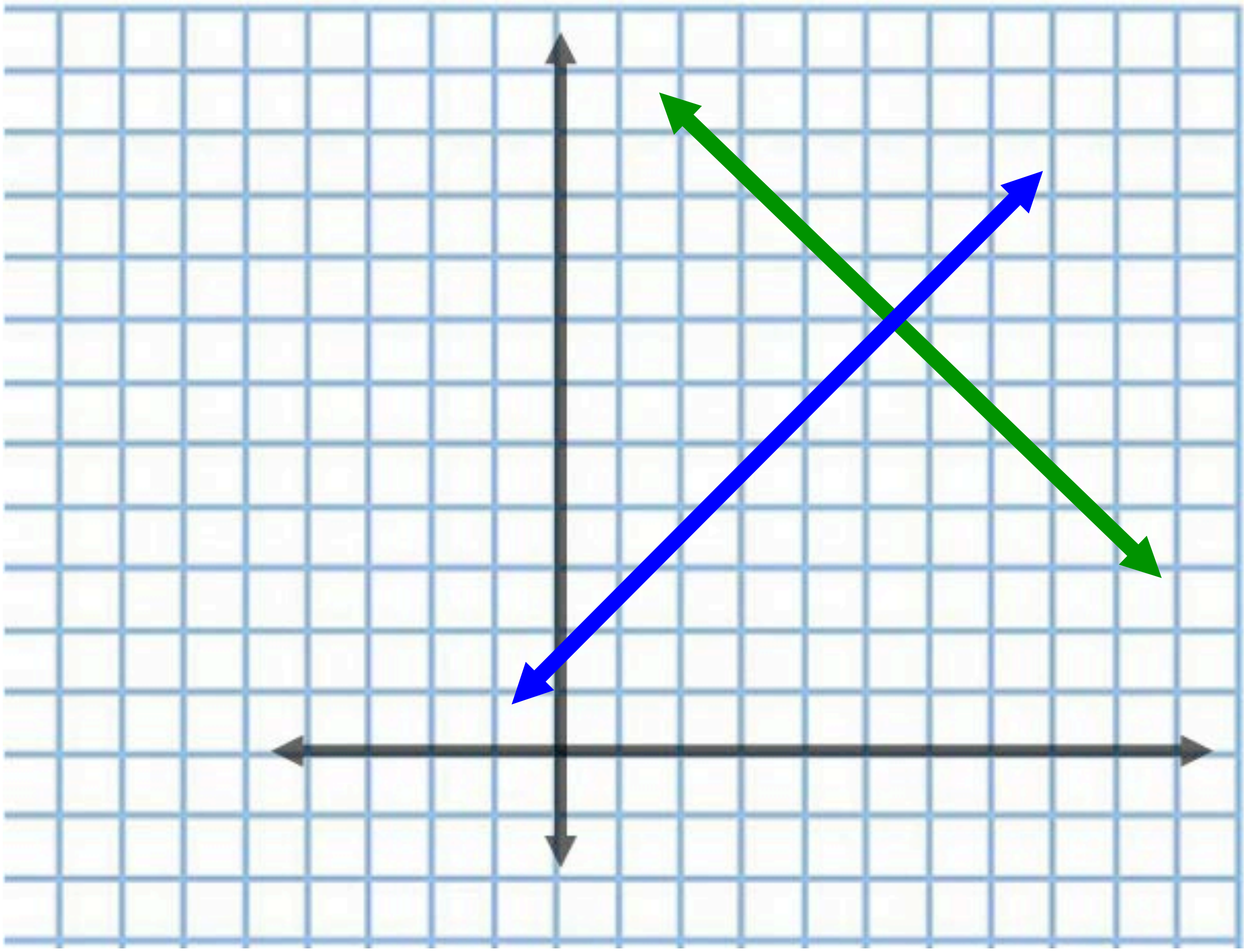
$$m_1 = \frac{-1}{m_2}$$

$$m_2 = \frac{-1}{m_1}$$

These all have the same meaning.

Slopes of Perpendicular Lines

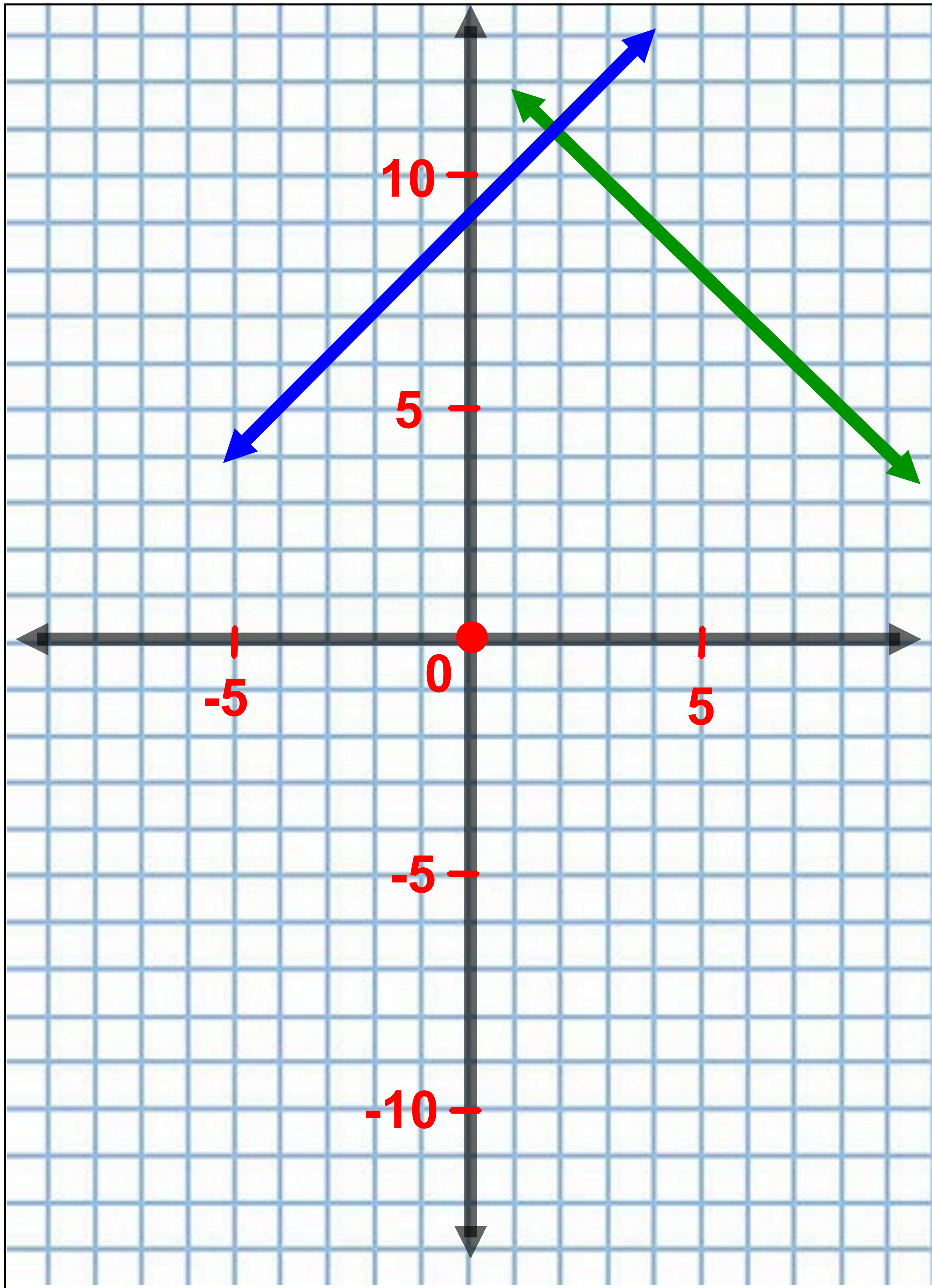
The slopes of perpendicular lines are negative (or opposite) reciprocals.



This will be useful in cases where you need to prove lines perpendicular, including proving that triangles are right triangles.

First, let's prove this is true.

Math Practice



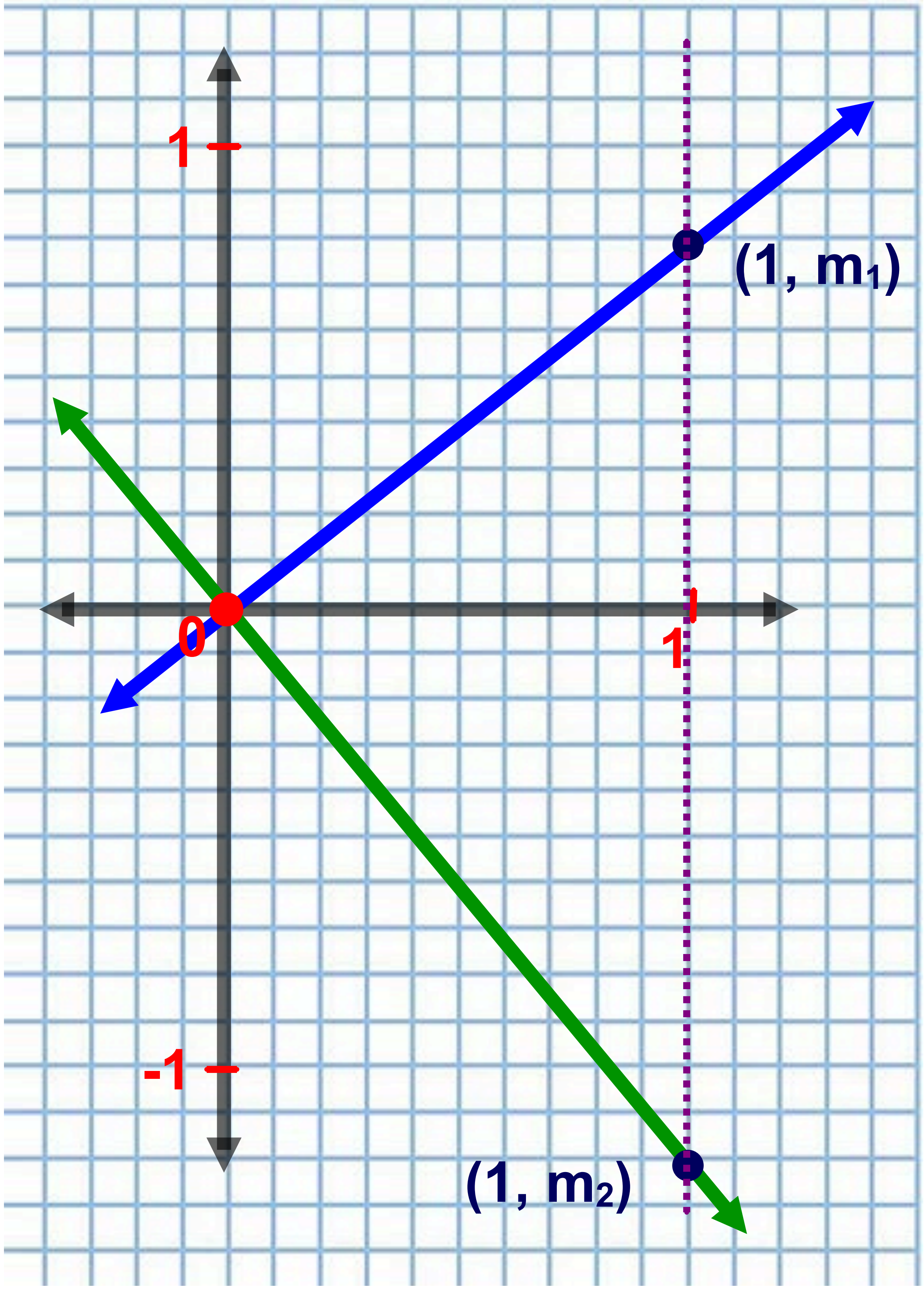
Slopes of Perpendicular Lines

First, let's move our lines to the origin so our work gets easier and we can focus on the important parts.

We can move them since we can just think of drawing new parallel lines through the origin that have the same slopes as these lines.

We earlier showed that parallel lines have the same slope, so the slopes of these new lines will be the same as that of the original lines.

Slopes of Perpendicular Lines



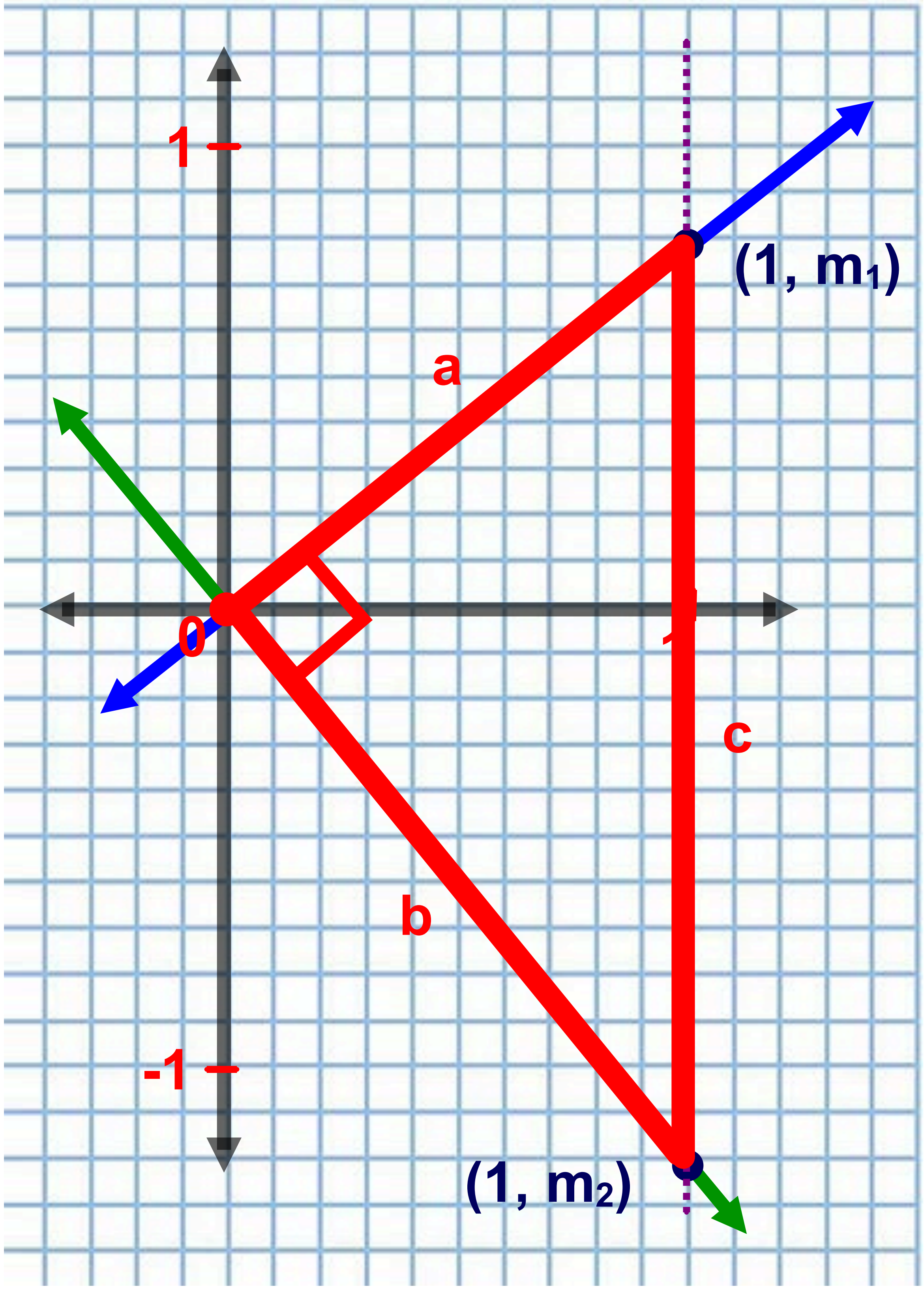
Now, let's zoom in and focus on the lines between x = 0 and x = 1.

When the lines are at x = 1, their y-coordinates are m_1 for the first line and m_2 for the second line.

That's because if the slope of the first line is m_1, then when we move +1 along the x-axis, the y-value must increase by the amount of the slope, m_1.

The same for the second line, whose slope is m_2.

Slopes of Perpendicular Lines



Perpendicular lines obey the Pythagorean Theorem:

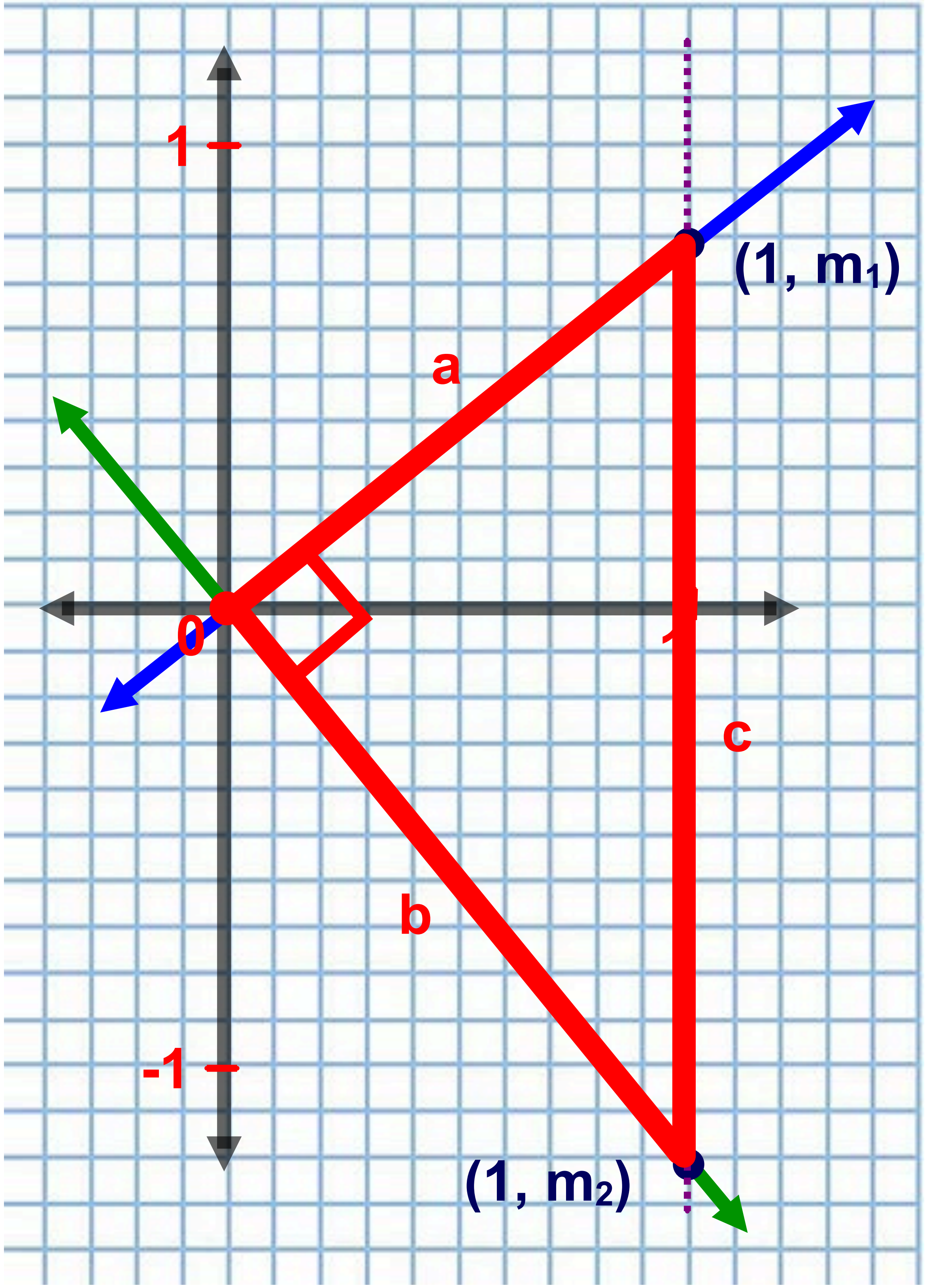
$$c^2 = a^2 + b^2$$

Let's find expressions for those three terms so we can substitute them in.

Side "c" is hypotenuse and is the distance between the points along the vertical line x=1. It has length m_1-m_2 (in this case, that effectively adds their magnitudes since m_2 is negative).

We can use the distance formula to find the length of each leg.

Slopes of Perpendicular Lines



The lengths of legs "a" and "b" can be found using the distance formula.

$$\mathbf{d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2}$$

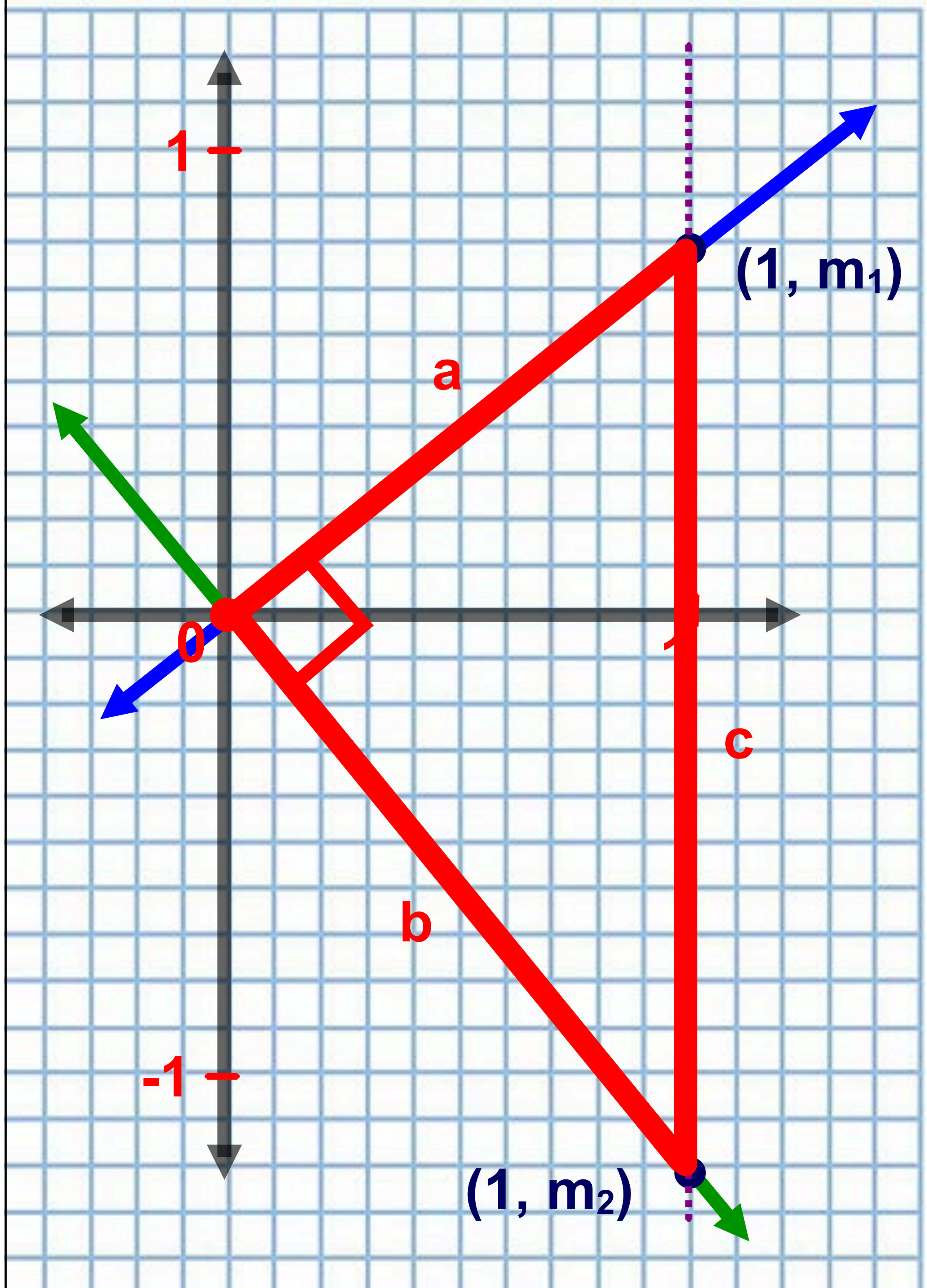
$$a^2 = (1-0)^2 + (m_1-0)^2$$
$$= 1 + m_1^2$$

$$b^2 = (1-0)^2 + (m_2-0)^2$$
$$= 1 + m_2^2$$

And, we get c^2 by squaring our result for c from the prior slide.

$$c^2 = (m_1-m_2)^2$$
$$= m_1^2 - 2m_1m_2 + m_2^2$$

Slopes of Perpendicular Lines



We're going to substitute the expressions from the prior slide into the Pythagorean Theorem.

We'll color code them so we can keep track.

$c^2 = m_1^2 - 2m_1m_1 + m_2^2$

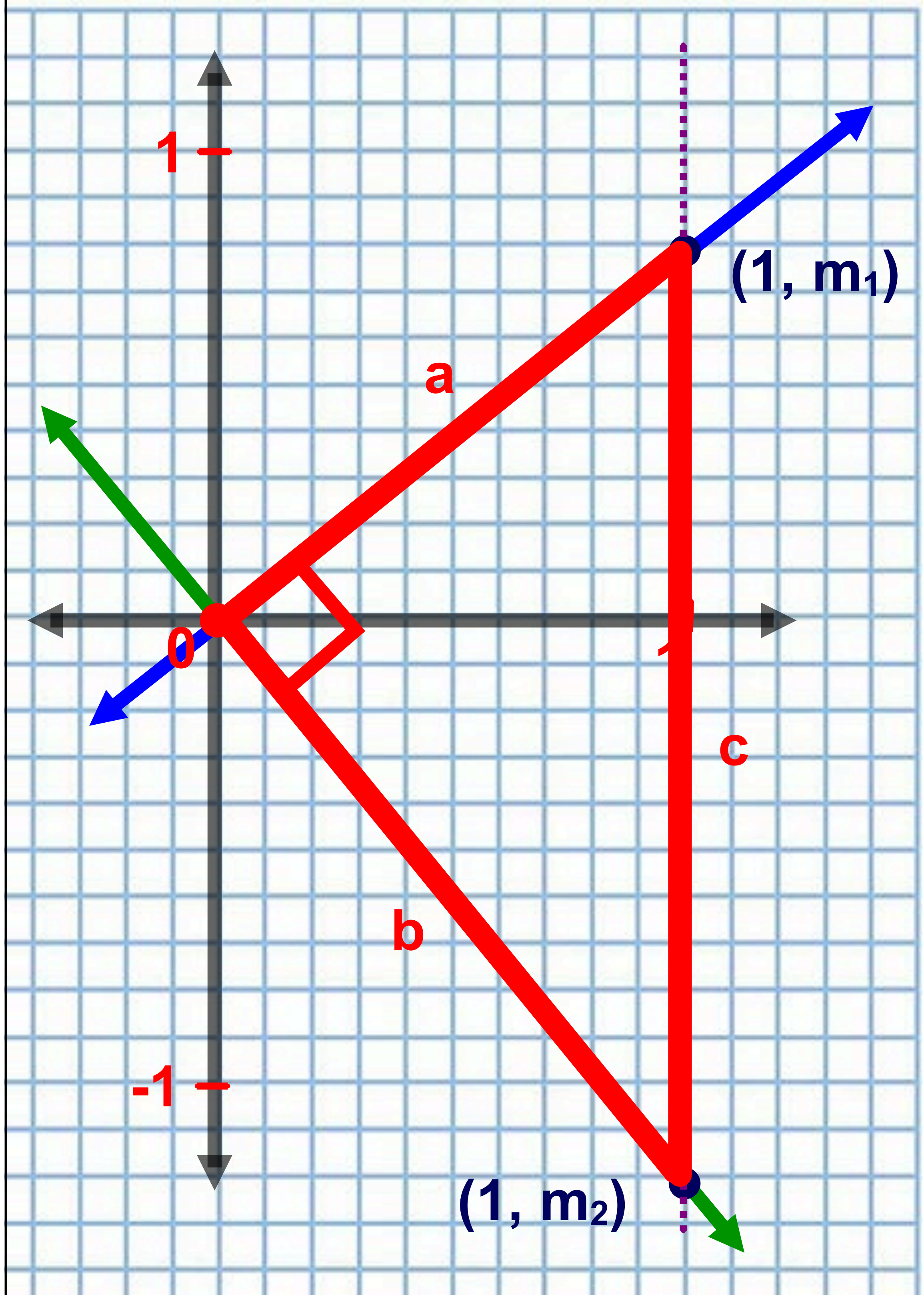
$a^2 = 1 + m_1^2$

$b^2 = 1 + m_2^2$

$$\mathbf{c^2 = a^2 + b^2}$$

$m_1^2 - 2m_1m_2 + m_2^2 = 1 + m_1^2 + 1 + m_2^2$

Slopes of Perpendicular Lines



$m_1^2 - 2m_1m_2 + m_2^2 = 1 + m_1^2 + 1 + m_2^2$

Now, let's identify like terms

$m_1^2 - 2m_1m_2 + m_2^2 = 1 + m_1^2 + 1 + m_2^2$

Notice that we have m_1^2 and m_2^2 on both sides so they can be canceled, and that $1 + 1 = 2$. So,

$-2m_1m_2 = 2$

$m_1m_2 = -1$

which is what we set out to prove
Notice that this also be written as:

$$m_1 = \frac{-1}{m_2}$$

$$m_2 = \frac{-1}{m_1}$$

40 If one line has a slope of 4, what must be the slope of any line perpendicular to it?

Answer

41 If one line has a slope of -1/2, what must be the slope of any line perpendicular to it?

Answer

42 If one line passes through the points (0, 0) and (4, 2) what must be the slope of any line perpendicular to that first line?

Answer

43 If one line passes through the points (-5, 9) and (5, 8) what must be the slope of any line perpendicular to that first line?

Answer

44 If one line passes through the points (1, 2) and (5, 6) and a perpendicular line passes through the point (1, 5) which of these points could lie on that second line?

A (2, 2)

B (4, 4)

C (2, 4)

D (-1, 3)

Answer

45 If one line passes through the points (-3, 4) and (0, 10) and a perpendicular line passes through the point (-1, -4) which of these points could lie on that second line?

A (0, -2)

B (2, -5)

C (3, 1)

D (-3, -5)

Answer

46 If one line passes through the points (3, 5) and (5, -1) and a perpendicular line passes through the point (-1, -1) which of these points could lie on that second line?

A (5, 0)

B (2, 0)

C (4, 8)

D (-4, -4)

Answer

Equations of Parallel & Perpendicular Lines

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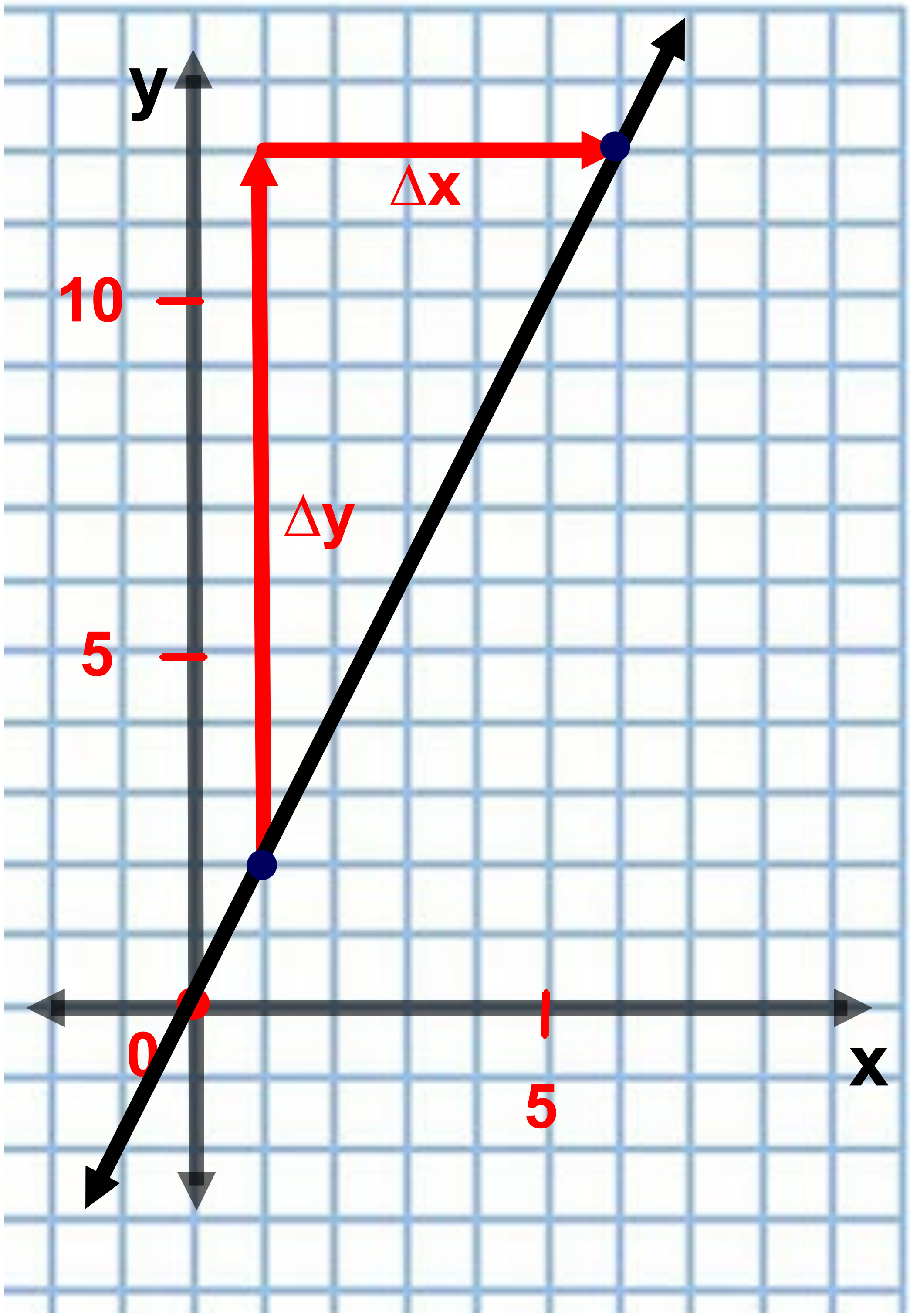
Writing the Equation of a Line



$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Let's start with the above definition of slope, multiply both sides by (x_2-x_1), and rearrange to get:

$$(y_2-y_1) = m(x_2-x_1)$$

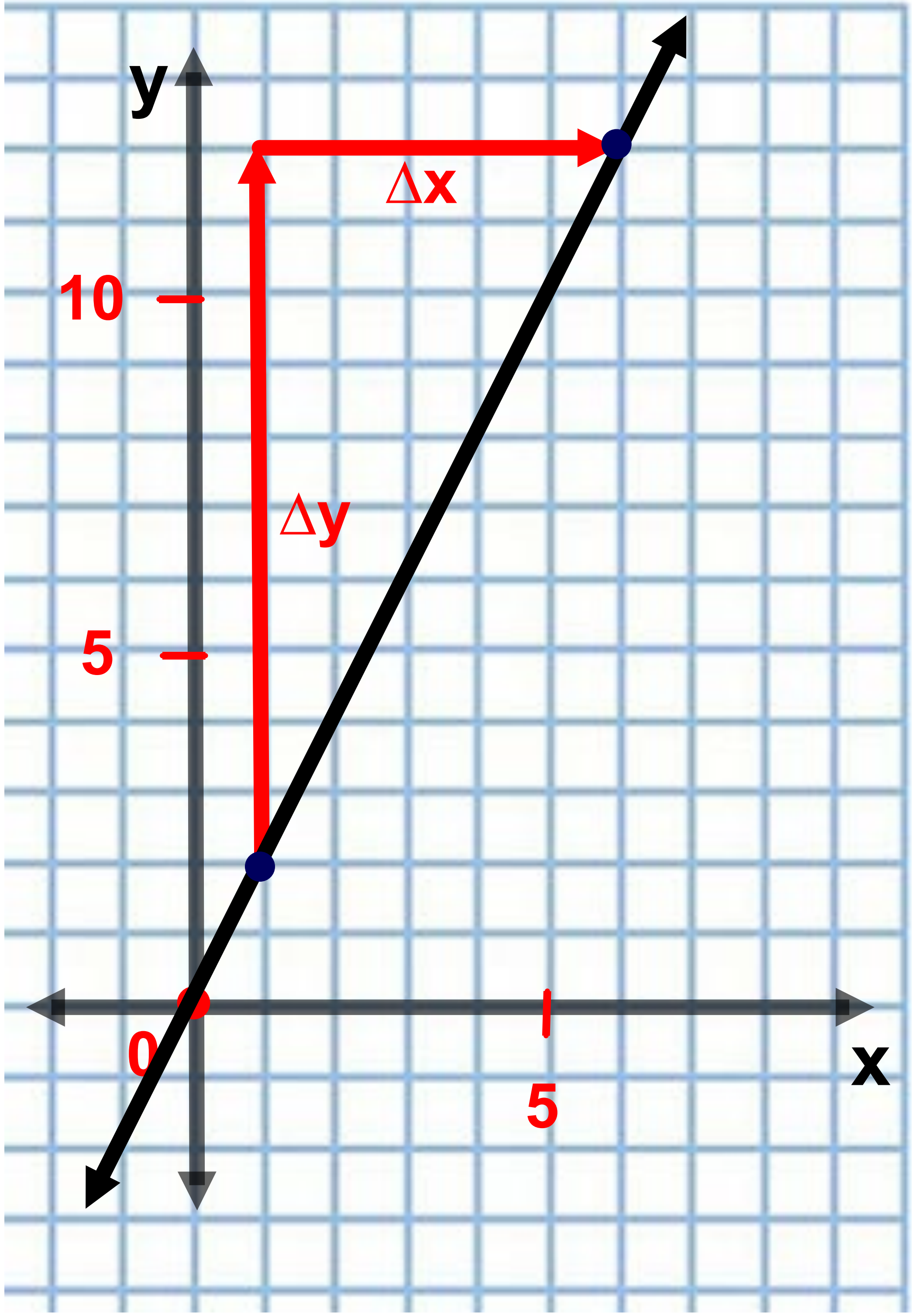
Now, if I enter one point (x_1,y_1) this defines the infinite locus of other points on the line.

$y-y_1 = m(x-x_1)$ ← Point-slope form

Then, I can add y_1 to both sides to isolate the variable y.

$$y = m(x-x_1)+y_1$$

Writing the Equation of a Line



$y = m(x-x_1)+y_1$

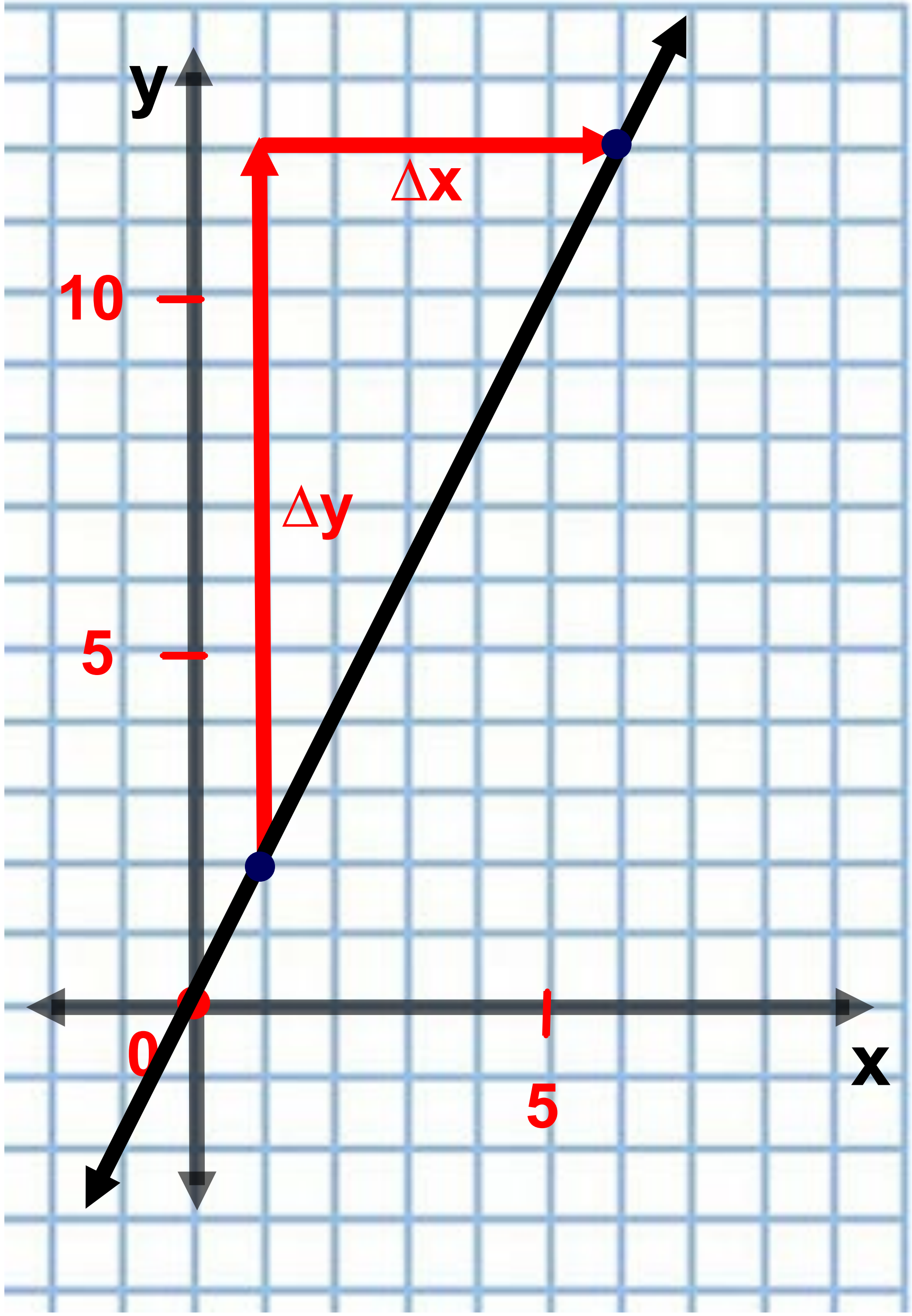
The term in red is just your steps to the right along the x-axis, your "run".

Multiplying by slope tells you how many steps up you must take along the y-axis, your "rise".

Those steps are added to the term in green, your original position on the y-axis to find your final y-coordinate.

That tells you the y-coordinate on the line for the given x-coordinate.

Slope Intercept Equation of a Line



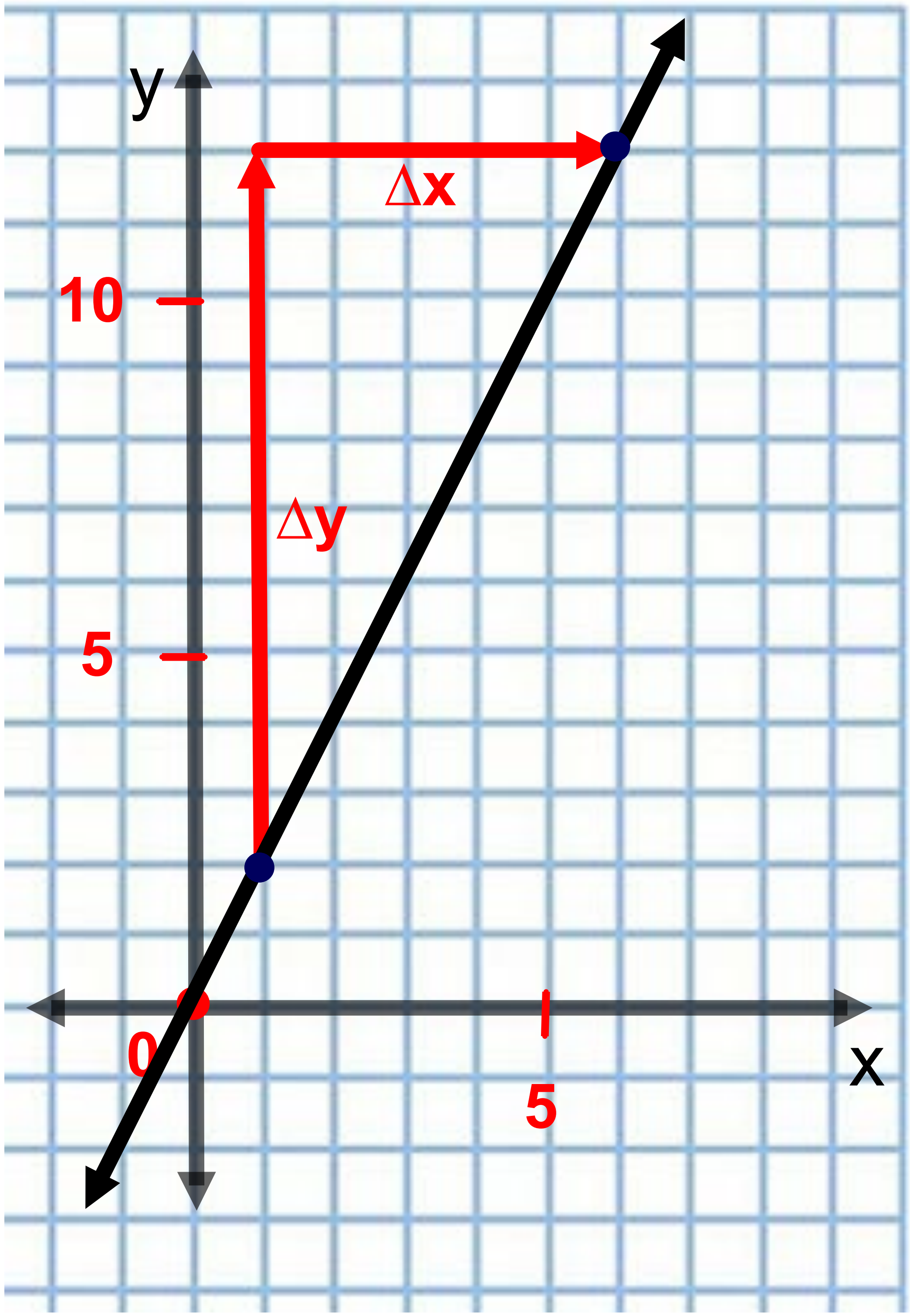
$y = m(x-x_1)+y_1$

A common simplification of this is to use the y-intercept for (x_1, y_1).

The y-intercept is the point where the line crosses the y-axis. Its symbol is "b" so the coordinates of that point are (0, b), since x = 0 on the y-axis.

Just substitute (0, b) in the above equation for (x_1, y_1).

Slope Intercept Equation of a Line



$y = m(x-x_1)+y_1$

$y = m(x-0) + b$

$y = mx + b$

This is very useful since both the slope and the y-intercepts often have important meaning when we are solving real problems.

In the graph to the left, b = 0, so the equation is y = 2x.

Teacher Note

Writing Equations of Parallel Lines

1. Find an equation of the line passing through the point (-4, 5) and parallel to the line whose equation is $-3x + 2y = -1$.

Step 1: Identify the information given in the problem.

Contains the point (-4, 5) & is parallel to the line $-3x + 2y = -1$

Step 2: Identify what information you still need to create the equation and choose the method to obtain it.

The slope: Use the equation of the parallel line to determine the slope

$$-3x + 2y = -1$$

____
Click

____
Click

Therefore m = ________
click

Math Practice

Writing Equations of Parallel Lines

Step 3: Create the equation.

$y - y_1 = m(x - x_1)$

Point-Slope Form

____

Click

____

Click

Slope-Intercept Form

____

Click

The correct solution to the original problem is either form of the equation. Point-Slope Form and Slope-Intercept Form are two ways to write the same linear equations.

Writing Equations of Perpendicular Lines

2. Write the equation of the line passing through the point (-2, 5) and perpendicular to the line $y = 1/2\ x + 3$

Step 1: Identify the slope according to the given equation.

Given equation: m = ________
click

Perpendicular Line: m = ________
click

Step 2: Use the given point and the point-slope formula to write the equation of the perpendicular line.

________________ Point-Slope Form
Click

____
Click

________________ Slope-Intercept Form
Click

Math Practice

Writing Equations of Perpendicular Lines

3. Write the equation of the line passing through the point

(4, 7) and perpendicular to the line x - 5y = 50

Answer & Math Practice

47 What is the equation of the line passing through (6, -2) and parallel to the line whose equation is $y = 2x - 3$?

A $y = 2x + 2$

B $y = -2x + 10$

C $y = 1/2\ x - 5$

D $y = 2x - 14$

Answer

48 Which is the equation of a line parallel to the line represented by: $y = -x - 22$?

A $x - y = 22$

B $y - x = 22$

C $y + x = -17$

D $2y + x = -22$

Answer

49 Two lines are represented by the equation:

$-3y = 12x - 14$ and $y = kx + 14$

For which value of k will the lines be parallel?

A 12

B -14

C 3

D -4

Answer

50 Which equation represents a line parallel to the line whose equation is: $3y + 4x = 21$

A $12y + 16x = 12$

B $3y - 4x = 22$

C $3y = 4x + 21$

D $4y + 3x = 21$

Answer

51 What is the equation of a line that passes through (9, 3) and is perpendicular to the line whose equation is $4x - 5y = 20$?

A $y - 3 = -5/4(x - 9)$

B $y - 3 = 4/5(x - 9)$

C $y - 3 = 4(x - 9)$

D $y - 3 = -5(x - 9)$

Answer

52 What is an equation of the line that passes through point (6, -2) and is parallel to the line whose equation is

$$y = -\frac{2}{3}x + 5 \,?$$

A $y = -\frac{3}{2}x + 5$

B $y = -\frac{2}{3}x + 2$

C $y = -\frac{3}{2}x + 2$

D $y = -2x + 2$

E $y = x$

Answer

53 What is an equation of the line that passes through the point (5, -2) and is parallel to the line:

$$9x - 3y = 12$$

A $y = 3x - 17$

B $y = x$

C $y = -x + 17$

D $y = -3x + 15$

Answer

54 What is an equation of the line that contains the point (-4, 1) and is perpendicular to the line whose equation is $y = -2x - 3$?

A $y = 2x + 1$

B $y = \frac{1}{2}x + 3$

C $y = -2x - 1$

D $y = -\frac{1}{2}x + 3$

Answer

55 Two lines are represented by the given equations. What would be the best statement to describe these two lines?

$2x + 5y = 15$ $5(x + 1) = -2y + 20$

A The lines are parallel.

B The lines are the same line.

C The lines are perpendicular.

D The lines intersect at an angle other than 90 .

Answer

Triangle Coordinate Proofs

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Triangle Coordinate Proof

Coordinate Proofs place figures on the Cartesian Plane to make use of the coordinates of key features of the figure, combined with formulae (e.g. distance formula, midpoint formula and slope formula) to help prove something.

The use of the coordinates is an extra first step in conducting the proof.

We'll provide a few examples and then have you do some proofs.

Math Practice

Triangle Coordinate Proof

Given: The coordinates A(0, 4); B(3, 0); C(-3, 0) and Q(0, 0) are the vertices of ΔABC and ΔAQB

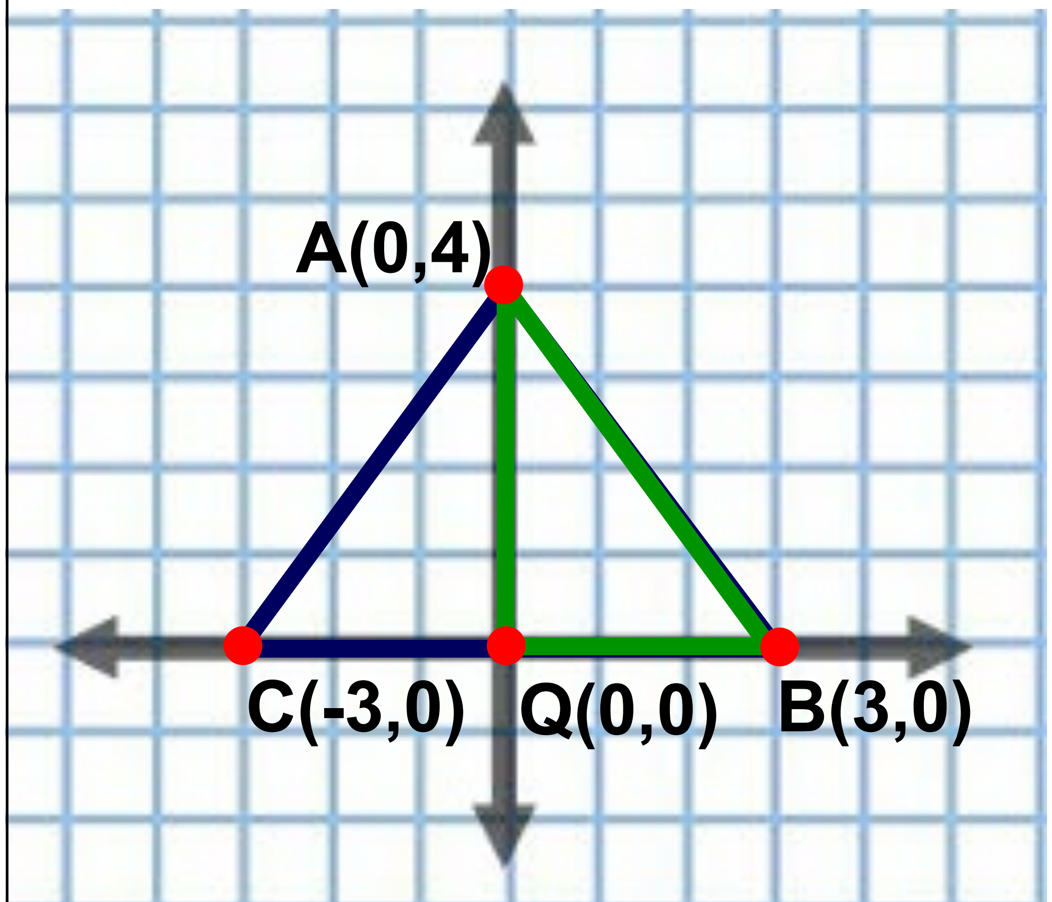
Prove: $\overline{QA}$ bisects ∠CAB

Sketch the triangles on some graph paper and then discuss a strategy to accomplish the proof.

Example

Given: The coordinates A(0, 4); B(3, 0); C(-3, 0) and Q(0, 0) are the vertices of ΔABC and ΔAQB

Prove: $\overline{QA}$ bisects ∠CAB



Does this sketch look like yours?

If not, take a moment to see if this is correct.

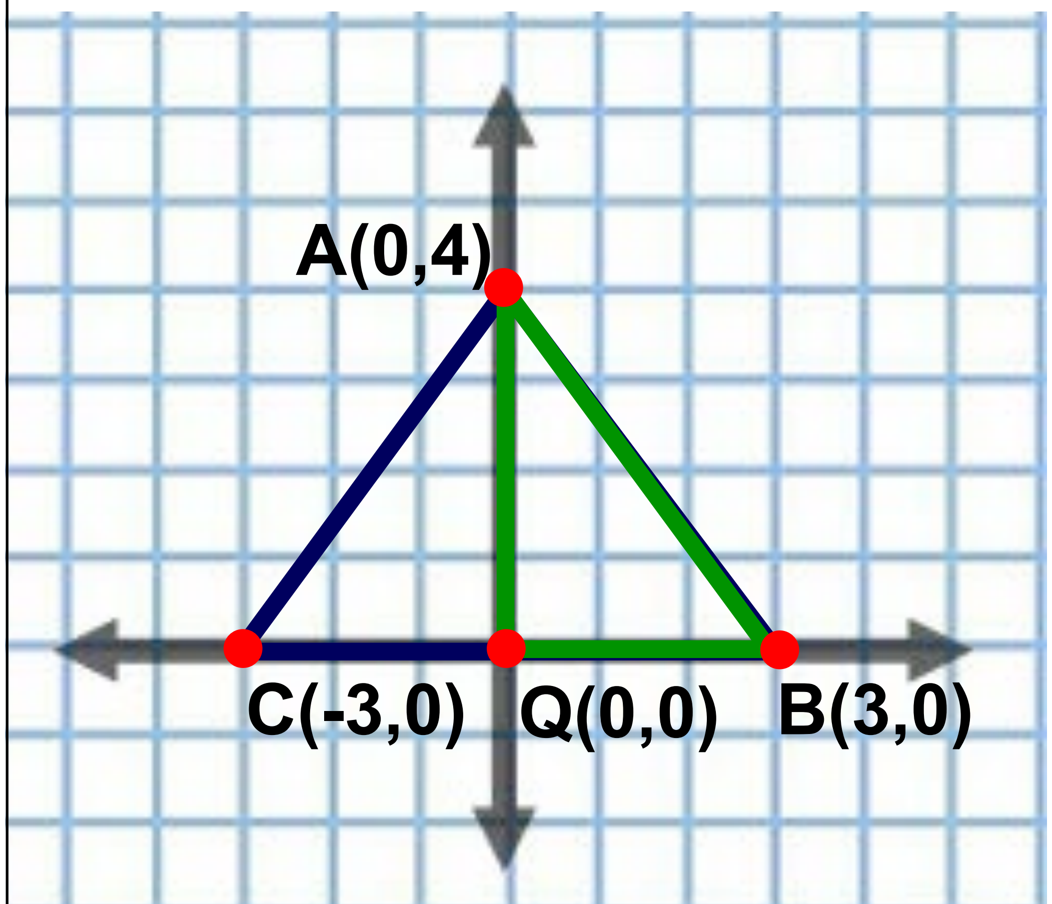
Looking at this, our strategy becomes clear.

If we can prove that ΔAQC ≅ ΔAQB, then we could prove ∠CAQ ≅ ∠BAQ which would mean that segment $\overline{QA}$ bisects ∠CAB: our goal.

If that makes sense, let's get to work.

Example

Given: The coordinates A(0, 4); B(3, 0); C(-3, 0) and Q(0, 0) are the vertices of ΔABC and ΔAQB
Prove: $\overline{QA}$ bisects ∠CAB



We don't need to use the distance formula to find these lengths since they can be read off the graph:

CQ = BQ = 3 & AQ = 4

We can use the distance formula to find these lengths:

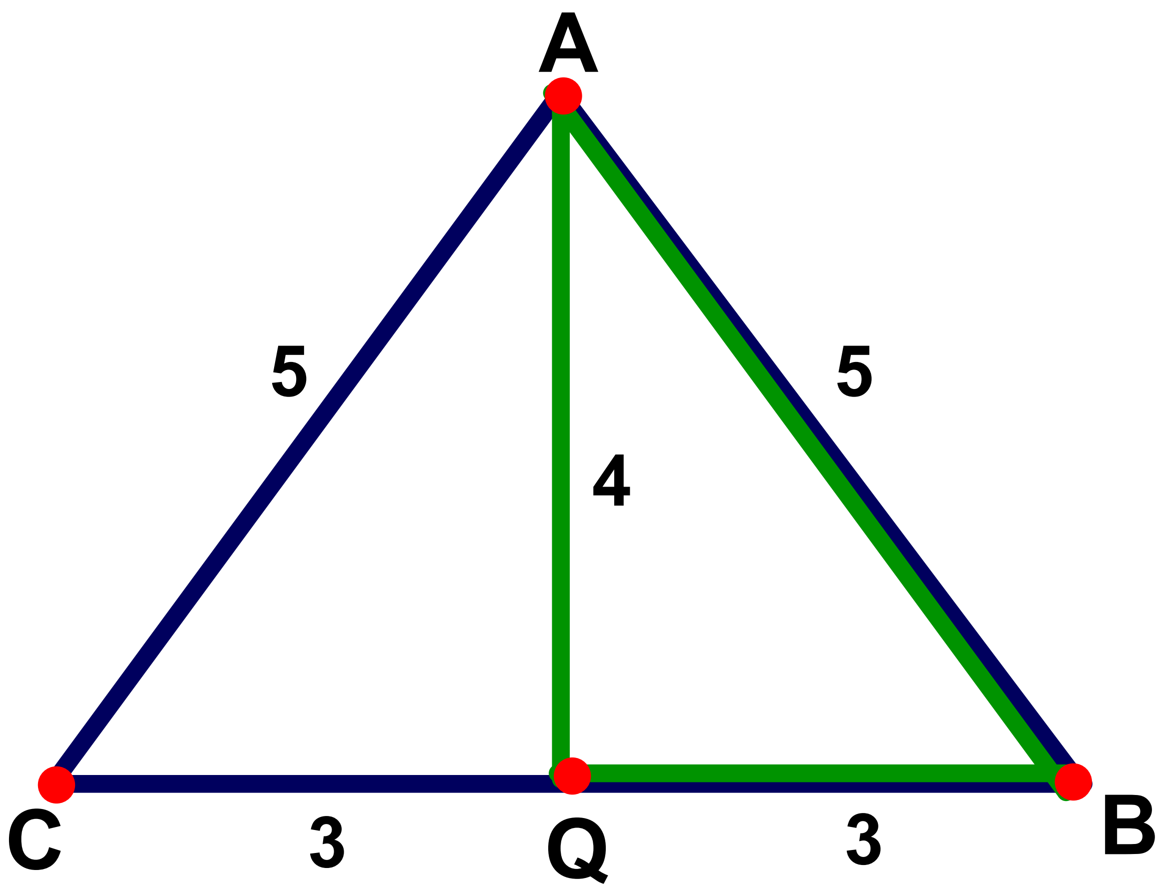
$AB = ((3-0)^2 + (0-4)^2)^{1/2} = (25)^{1/2} = 5$

$AC = ((0-(-3))^2 + (4-0)^2)^{1/2} = (25)^{1/2} = 5$

Example

Given: The coordinates A(0, 4); B(3, 0); C(-3, 0) and Q(0, 0) are the vertices of ΔABC and ΔAQB

Prove: $\overline{QA}$ bisects ∠CAB



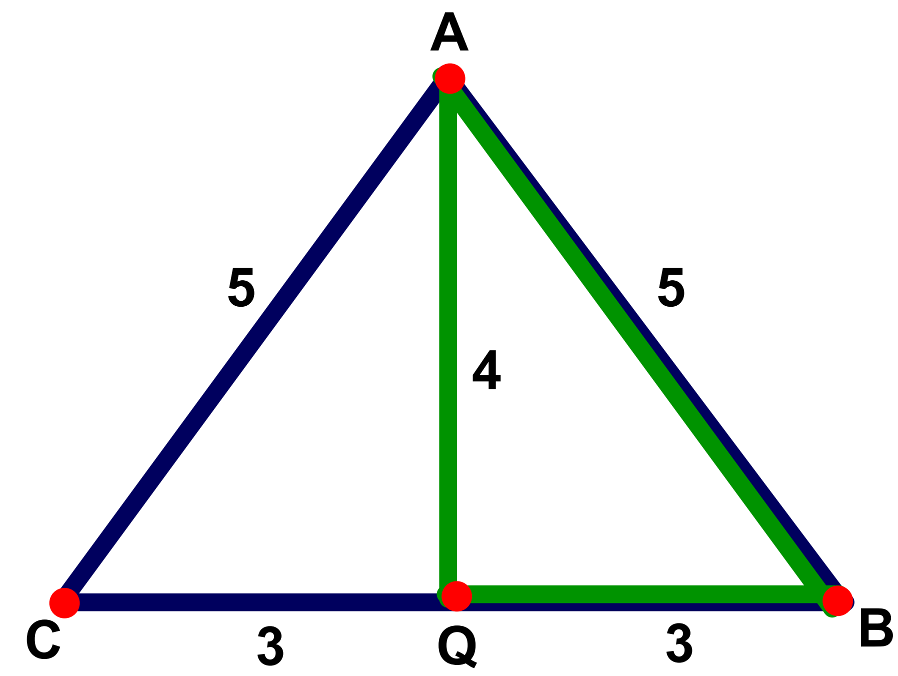
You should be able to see that these are two right triangles with identical length sides, so they must be congruent.

But, let's work out the proof as good practice.

Given: Coordinates of vertices of ΔABC

Coordinates of vertices ΔAQB

Prove: $\overline{QA}$ bisects ∠CAB



Statements	Reasons
1. CQ = 3 and BQ = 3	1. Given in graph
2. AC = 5 and AB = 5	2. Distance Formula
3. $\overline{QC} \cong \overline{QB}$	3. ≅ segments have equal measure
4. $\overline{AQ} \cong \overline{QA}$	4. Reflexive Property of ≅
5. $\overline{AC} \cong \overline{AB}$	5. ≅ segments have equal measure
6. ΔAQC ≅ ΔAQB	6. SSS triangle congruence
7. ∠CAQ ≅ ∠BAQ	7. CPCTC
8. $\overline{QA}$ bisects ∠CAB	8. Definition of angle bisector

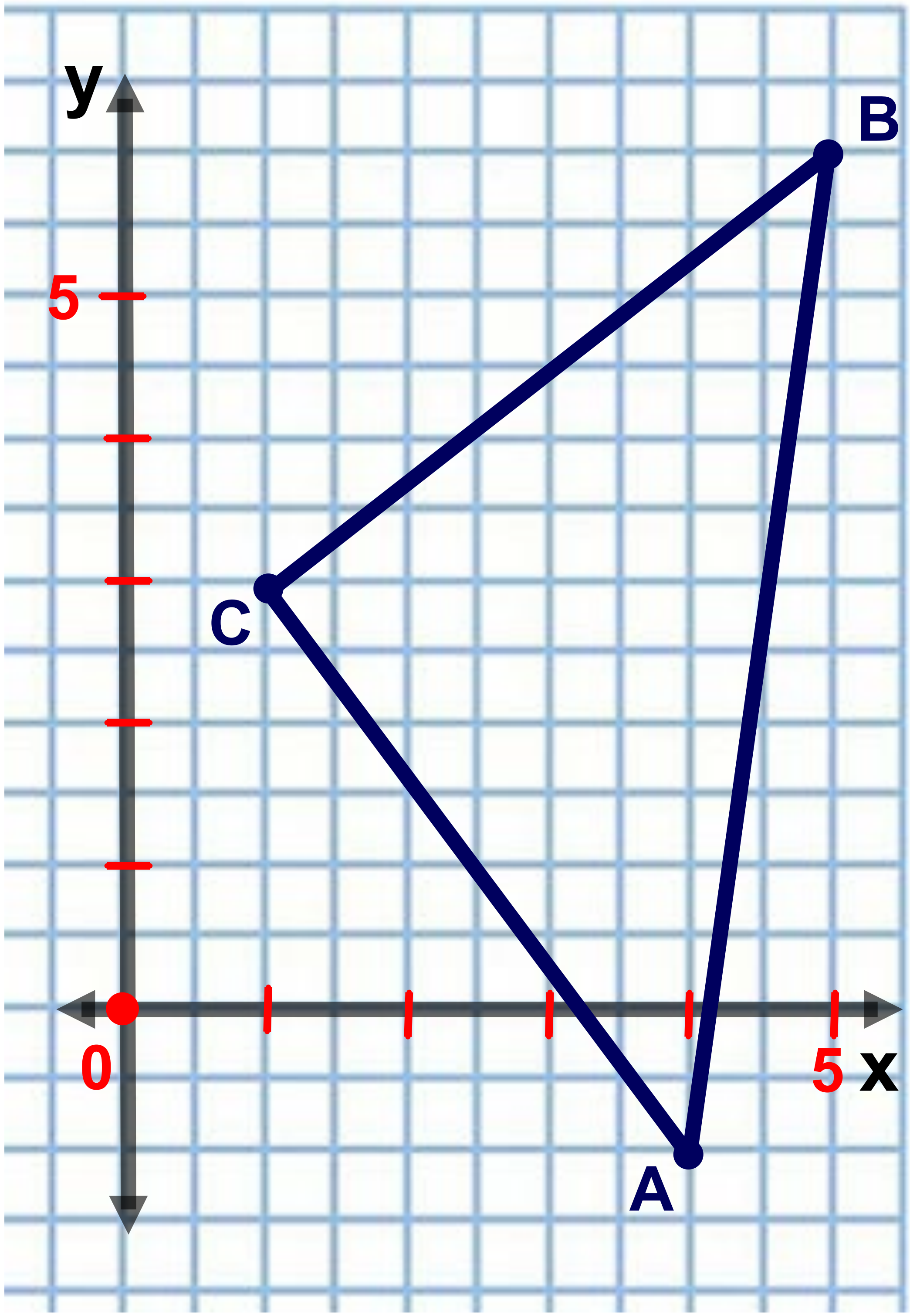
Triangle Coordinate Proof

Given: The points A(4, -1), B(5, 6), and C(1, 3)

Prove: ΔABC is an isosceles right triangle

Make a sketch and think of a strategy for this proof.

Triangle Coordinate Proof



Given: The points A(4, -1), B(5, 6), and C(1, 3)

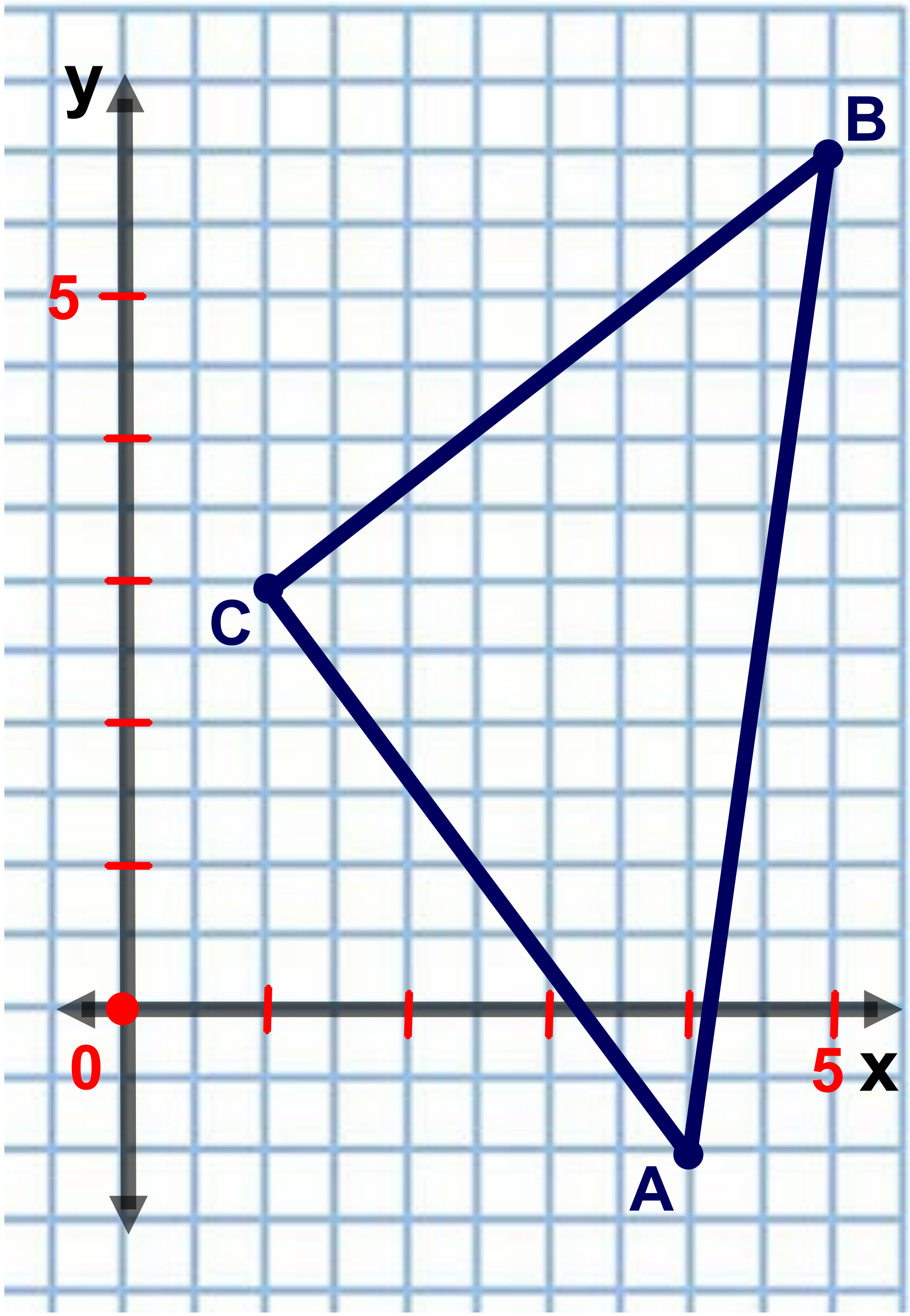
Prove: ΔABC is an isosceles right triangle

If we just had to prove this a right triangle, we could just show that the slope of $\overline{BC}$ and $\overline{AC}$ are negative (or opposite) reciprocals.

But, we also have to show this is an isosceles triangle, so we'd still have to determine the lengths of the sides.

Once we do two sides we may as well do three and then use Pythagorean Theorem to prove it both isosceles and right.

Triangle Coordinate Proof



Let's use the distance formula to find the lengths:

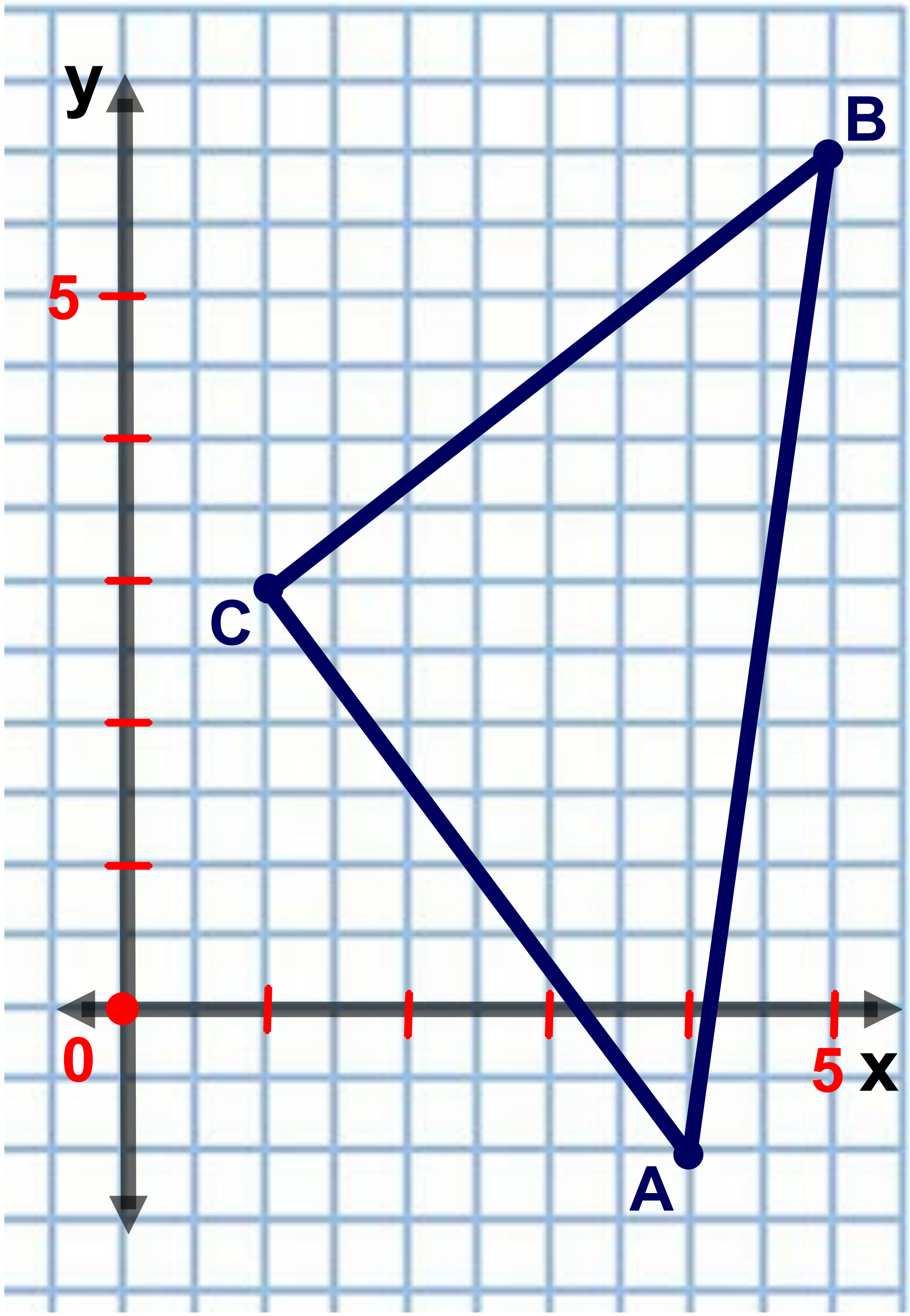
$$d = ((x_2-x_1)^2 + (y_2-y_1)^2)^{1/2}$$

$$AC = ((4-1)^2+(-1-3)^2)^{1/2}$$
$$= (9+16)^{1/2} = (25)^{1/2} = 5$$

$$BC = ((5-1)^2 + (6-3)^2)^{1/2}$$
$$= (16+9)^{1/2} = (25)^{1/2} = 5$$

$$AB = ((4-5)^2 + (-1-6)^2)^{1/2}$$
$$= (1+49)^{1/2} + (50)^{1/2} = 5\sqrt{2}$$

Triangle Coordinate Proof



Given: The points A(4, -1), B(5, 6), and C(1, 3)

Prove: ΔABC is an isosceles right triangle

The lengths are
AC = 5; BC = 5; AB = $5\sqrt{2}$

Since AC = BC this is an isosceles triangle.

Since $(5\sqrt{2})^2 = 5^2 + 5^2 = 50$ this is a right triangle

So, we have proven this to be a right isosceles triangle

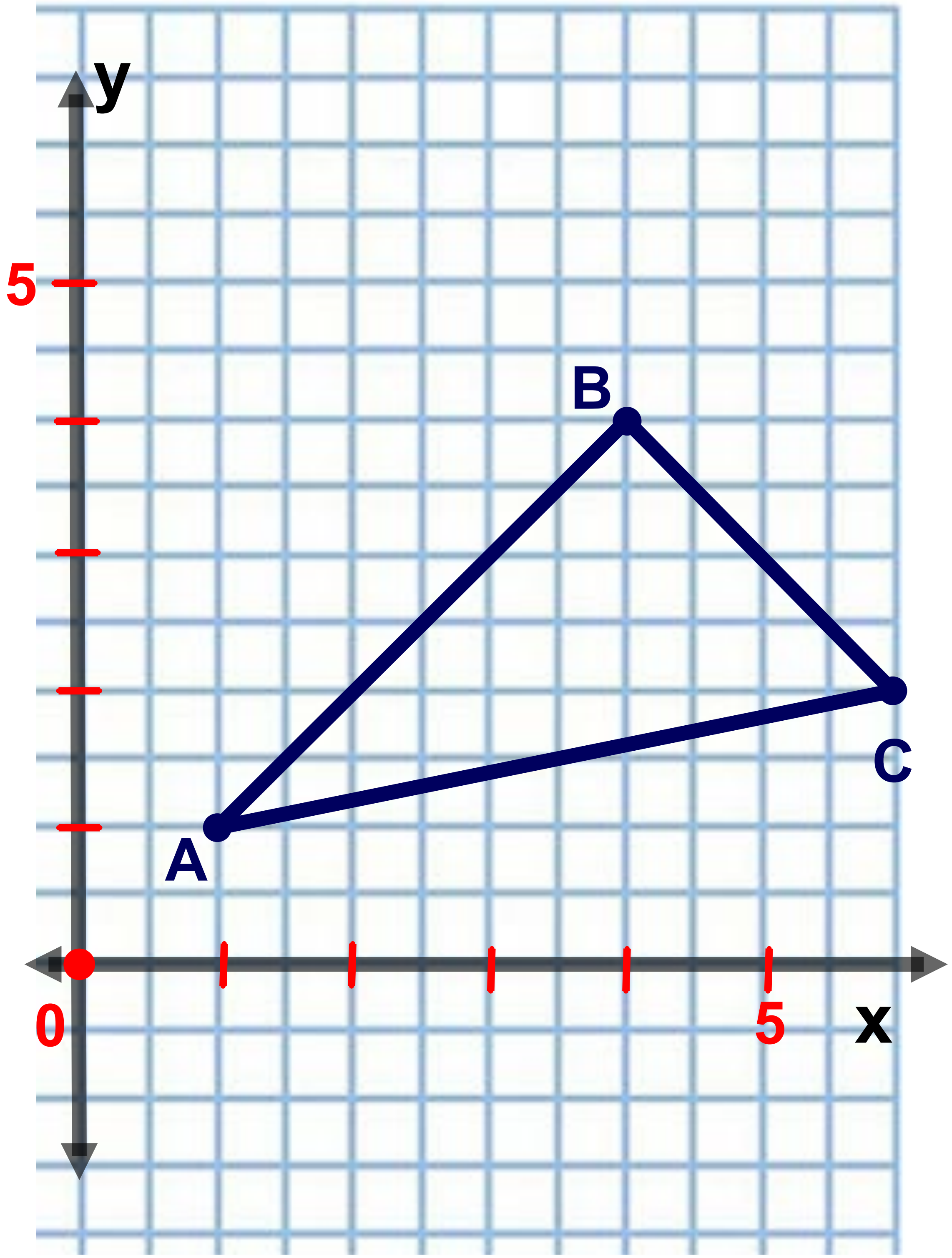
Triangle Coordinate Proof

Given: The points A(1, 1), B(4, 4), and C(6, 2)

Prove: ΔABC is a right triangle

Make a sketch and think of a strategy.

Triangle Coordinate Proof



Given: Points A(1, 1), B(4, 4) & C(6, 2)

Prove: ΔABC is a right triangle

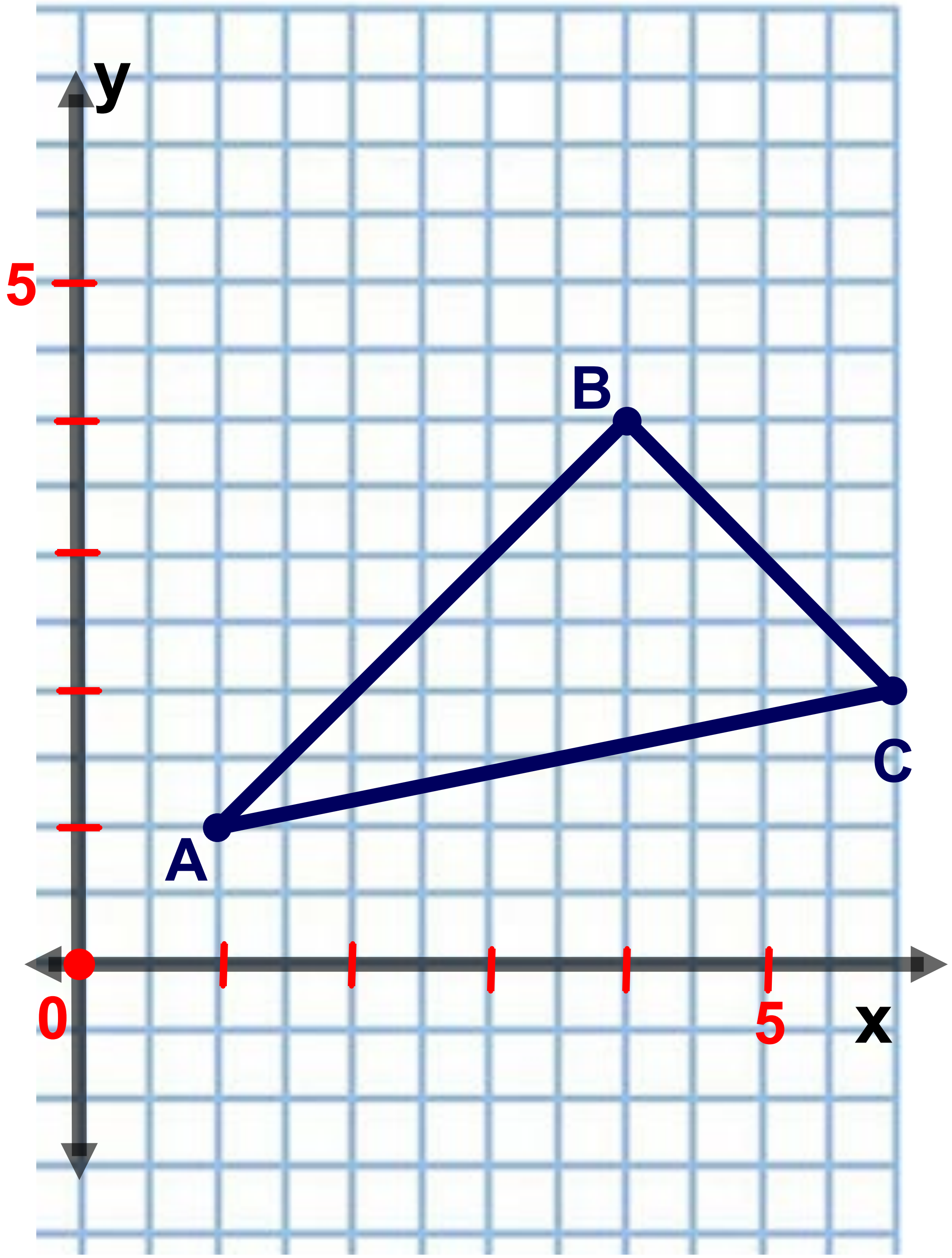
Now we just have to prove that two sides of this triangle are perpendicular.

At a glance, it'd seem like $\overline{AB}$ and $\overline{BC}$ are the likely ones, so let's check them first.

If that failed, we'd have to check the other possibilities, but we let's check the obvious ones first.

We do this by finding if their slopes are negative (or opposite) reciprocals.

Triangle Coordinate Proof



Given: Points A(1, 1), B(4, 4) & C(6, 2)

Prove: ΔABC is a right triangle

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-1}{4-1} = \frac{3}{3} = 1$$

$$m_{BC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-4}{6-4} = \frac{-2}{2} = -1$$

Since -1 is the negative (or opposite) reciprocal of 1, $\overline{AB}$ is perpendicular to $\overline{BC}$ and ΔABC is a right triangle

Equation of a Circle & Completing the Square

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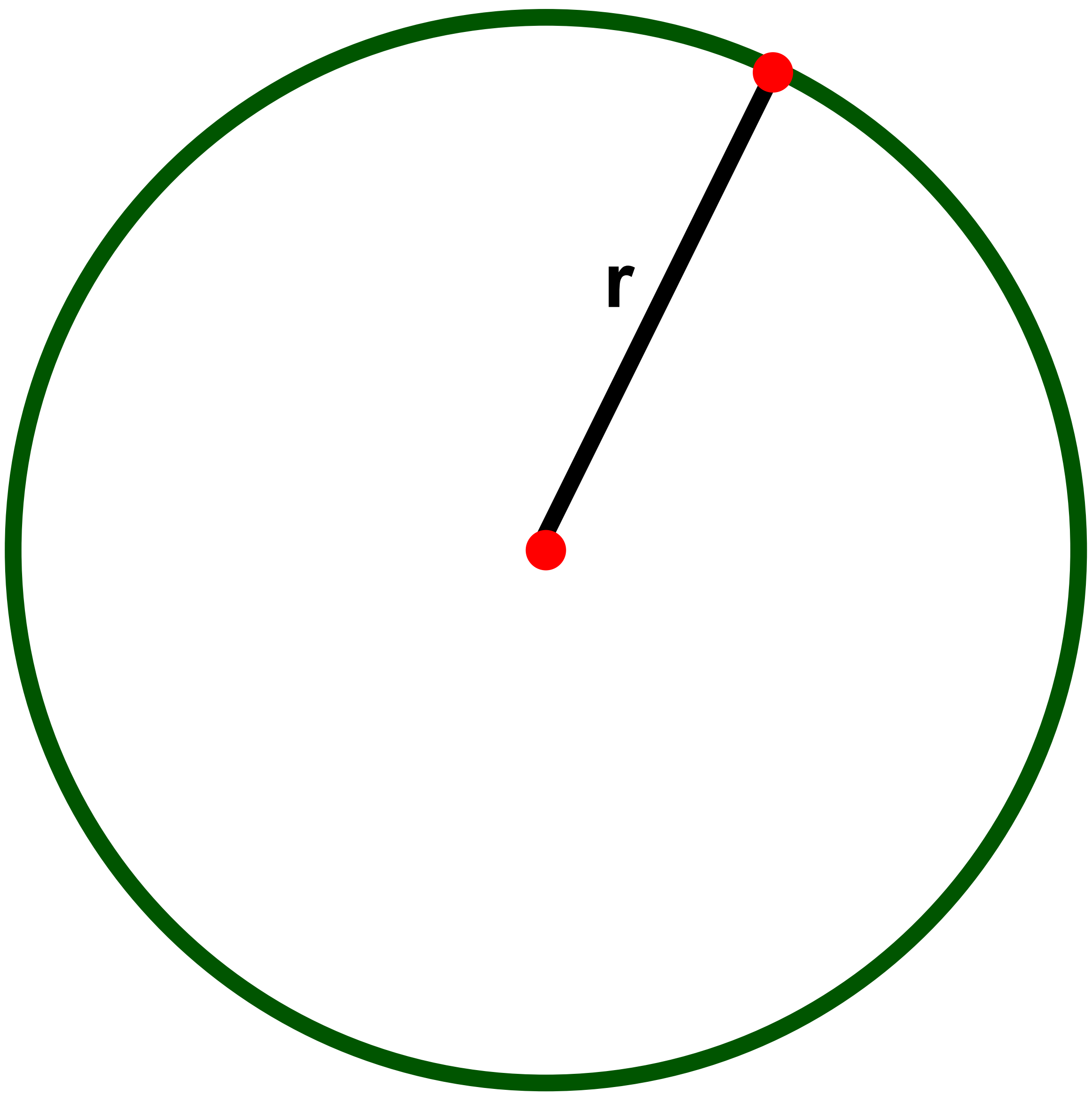
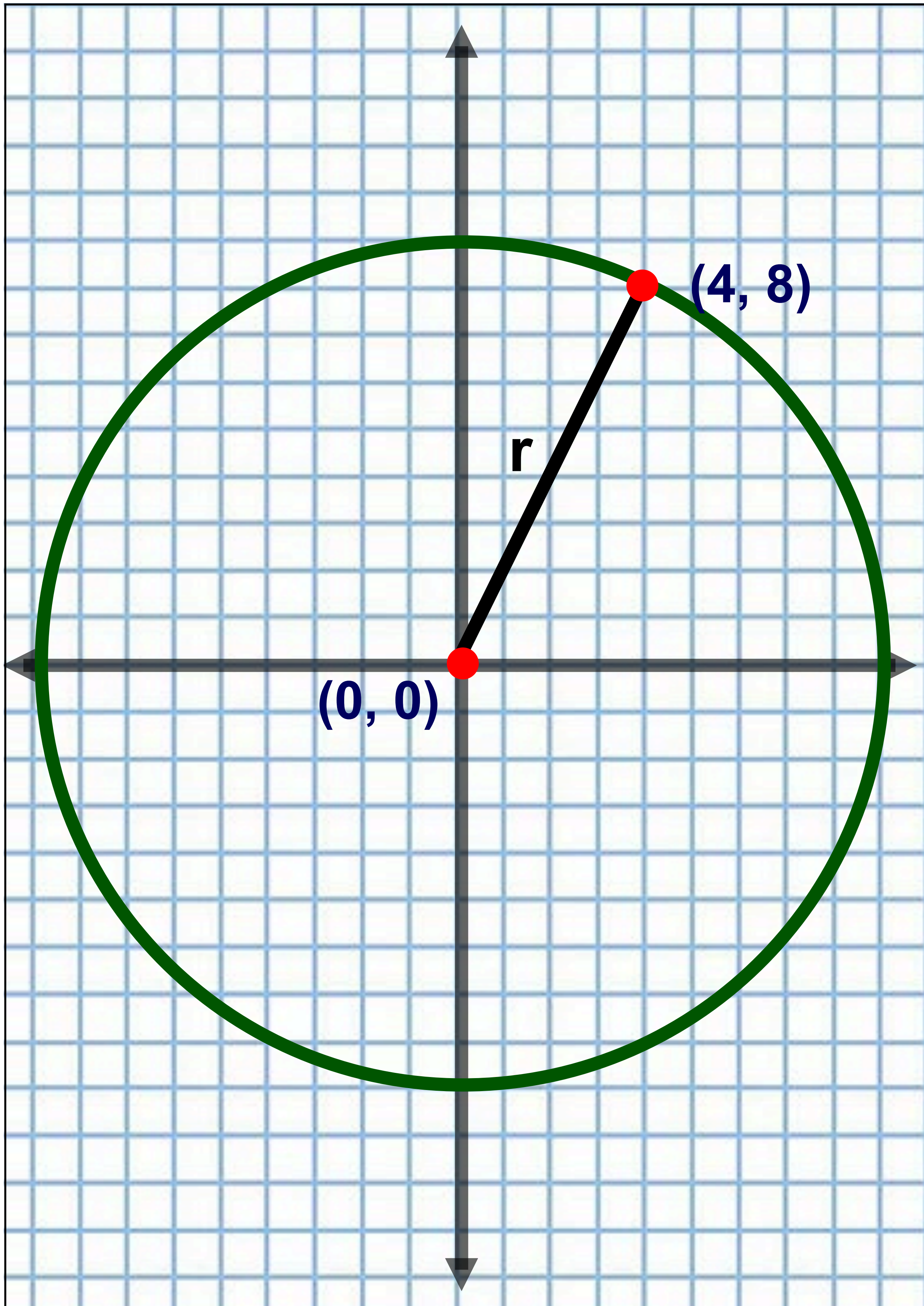
The Circle as a Locus of Points



A Circle is a set of points that are all the same distance from the center of the circle.

The distance from the center to the circumference is the radius, "r."



The Circle as a Locus of Points

Placing the circle on a coordinate plane we can see that if the center of the circle is at (0,0) this particular point is located at about (4,8).

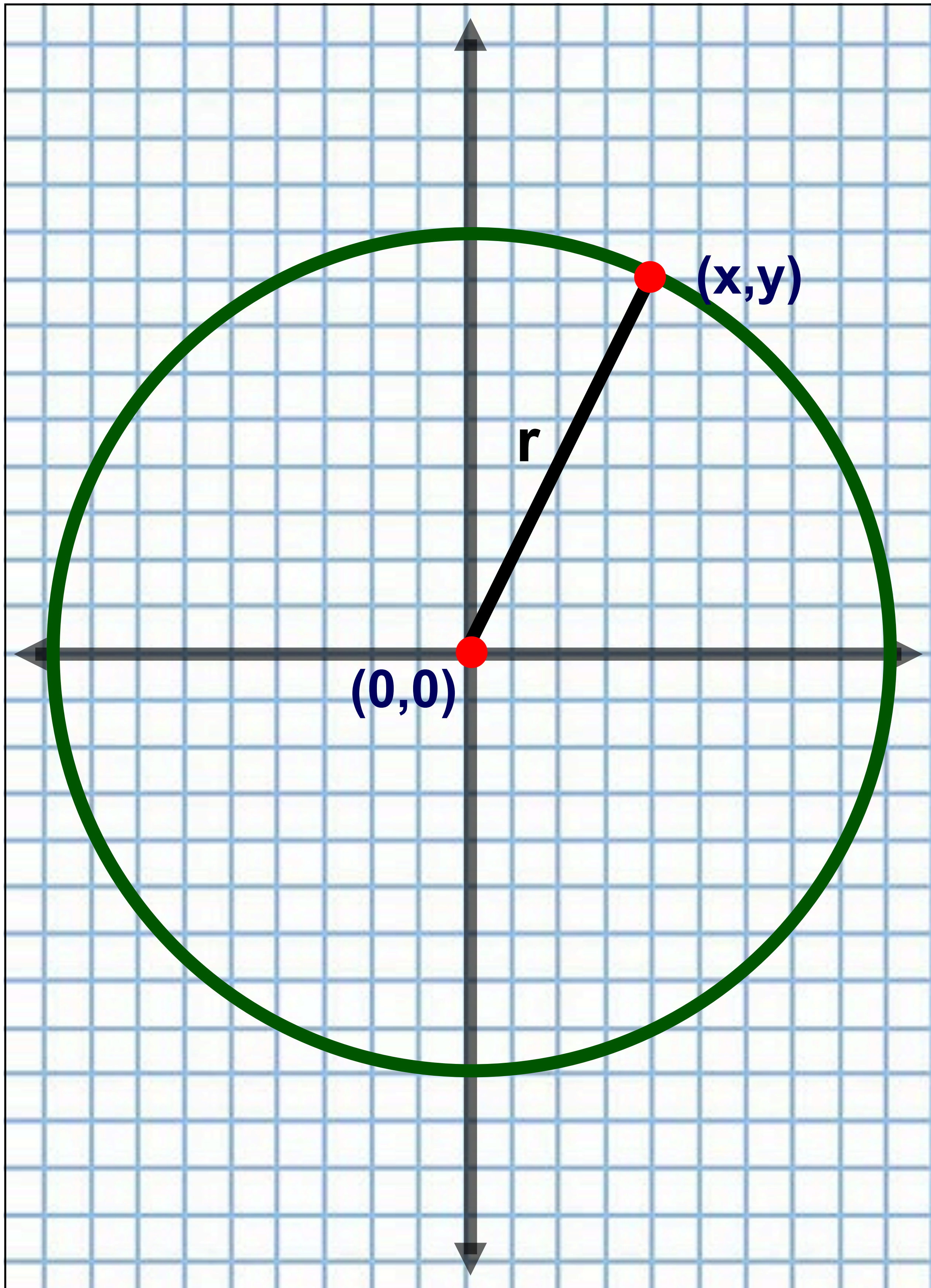
Let's use the distance formula to find the lengths:

$d = ((x_2-x_1)^2 + (y_2-y_1)^2)^{1/2}$

click

click

Math Practice



The Circle as a Locus of Points

We can also solve in general for any point (x,y) which lies on the circumference of the circle whose center is located at the origin (0,0).

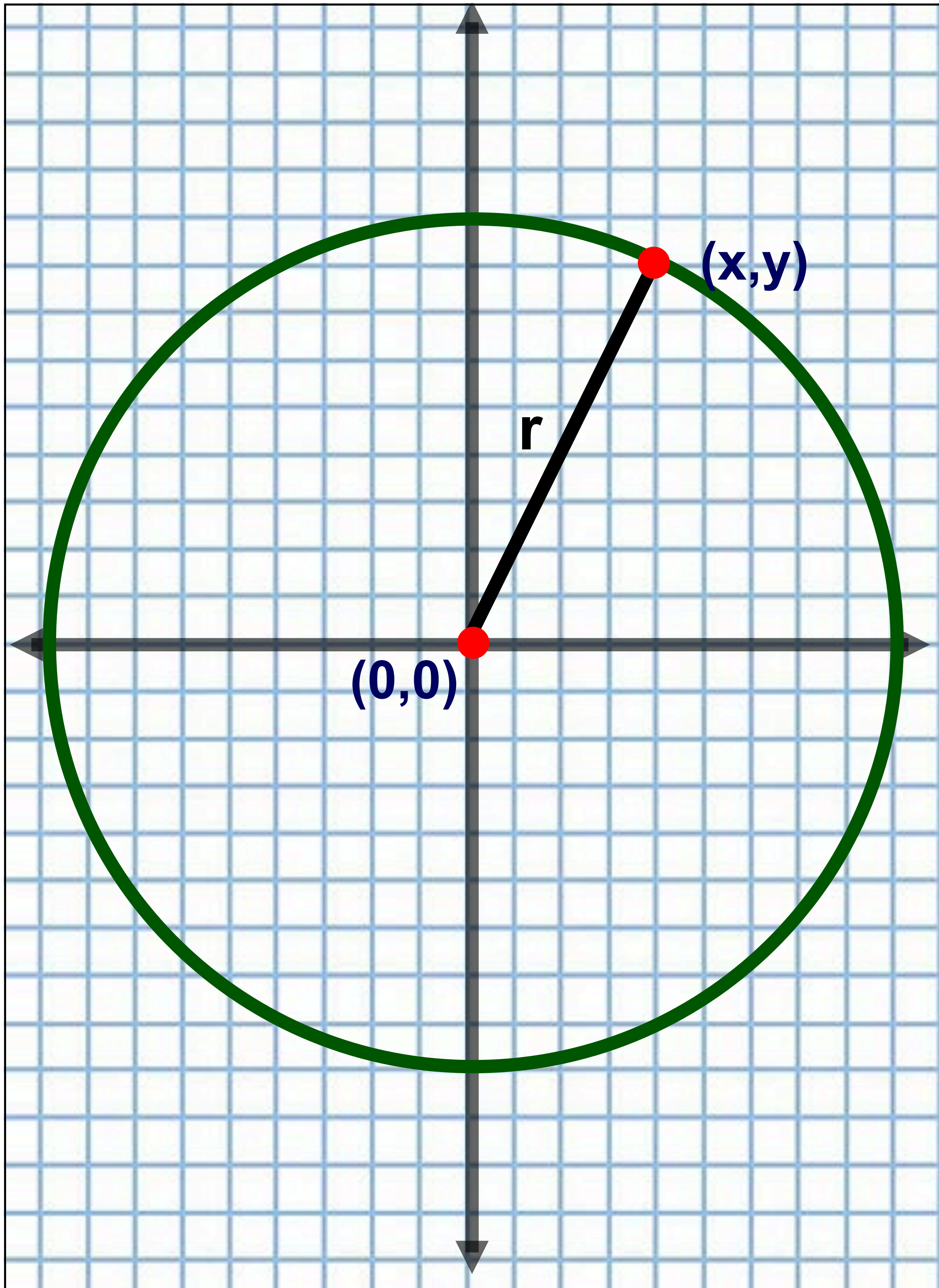
This will give us the equation of a circle with its center at the origin, since every point on the circumference must satisfy this equation.

If we start with the distance formula

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and square both sides, what will be the resulting equation?

click



The Circle as a Locus of Points

$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

Substituting

- r^2 for d^2
- (x, y) for (x_2, y_2)
- $(0, 0)$ for (x_1, y_1)

What will be resulting equation after the substitution?

____

click

The sides are usually swapped to yield:

____

click

The equation of a circle whose center is at the origin.

56 What is the radius of the circle whose equation is $x^2 + y^2 = 25$?

Answer

57 If the y coordinate of a point on the circle $x^2 + y^2 = 25$ is 5, what is the x coordinate?

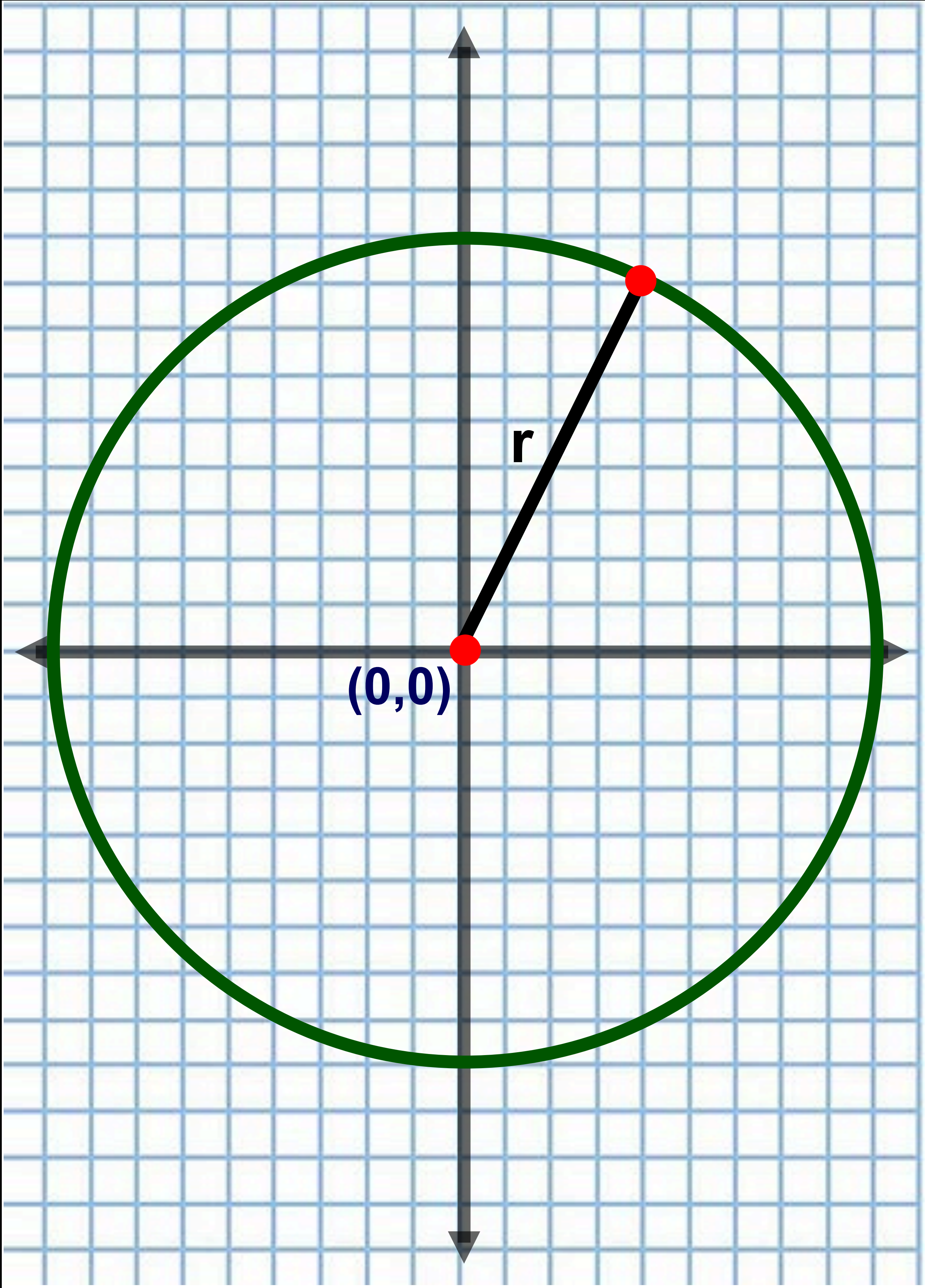
Answer

58 How many points on the circle $x^2 + y^2 = 25$ have an x-coordinate of 3?

Answer

59 How many points on the circle $x^2 + y^2 = 25$ have an y-coordinate of 6?

Answer



The Circle as a Locus of Points

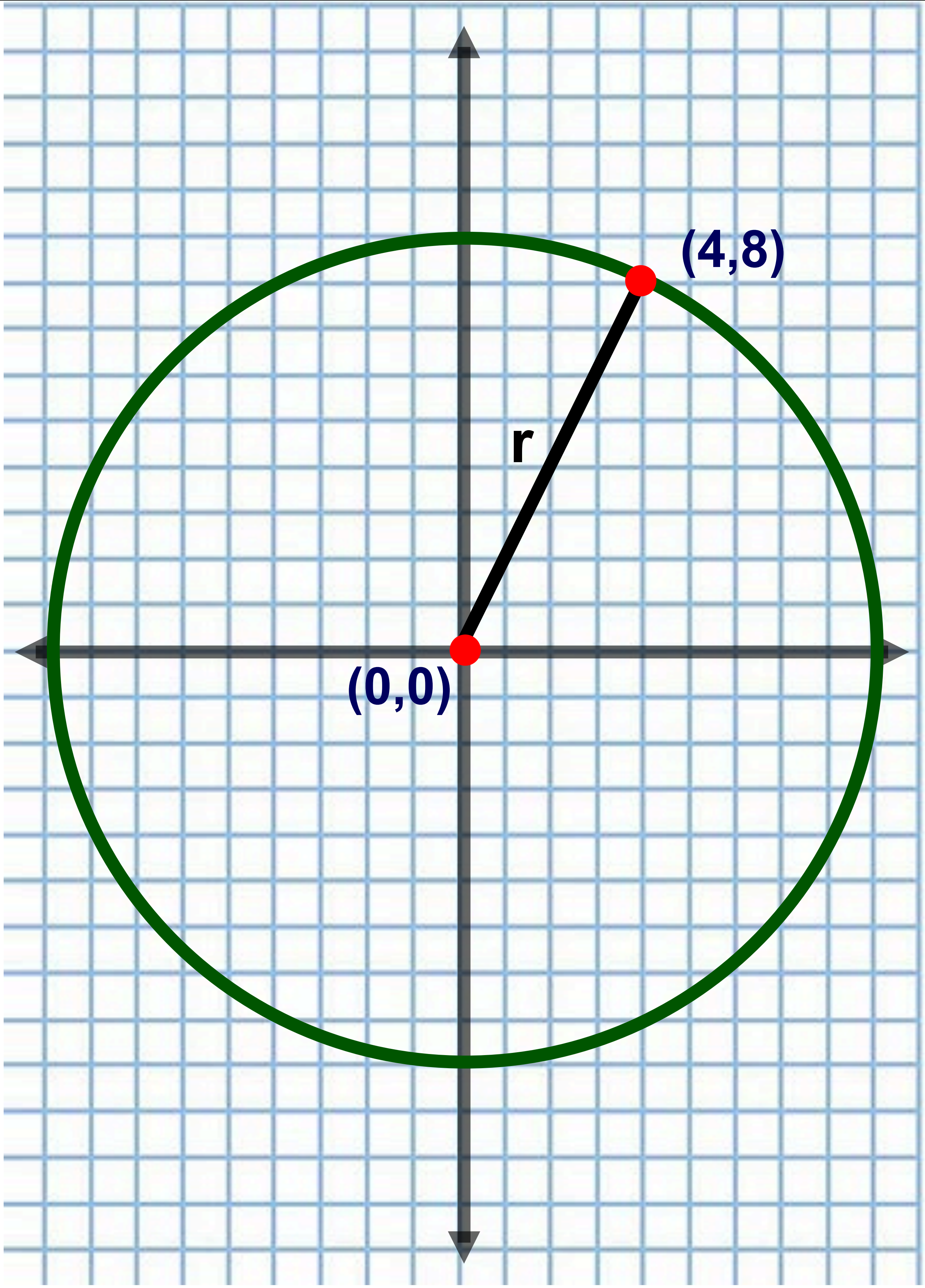
In general, any circle centered on the origin will have an equation

$x^2 + y^2 = r^2$

If a point is on the circle, it must satisfy this equation.

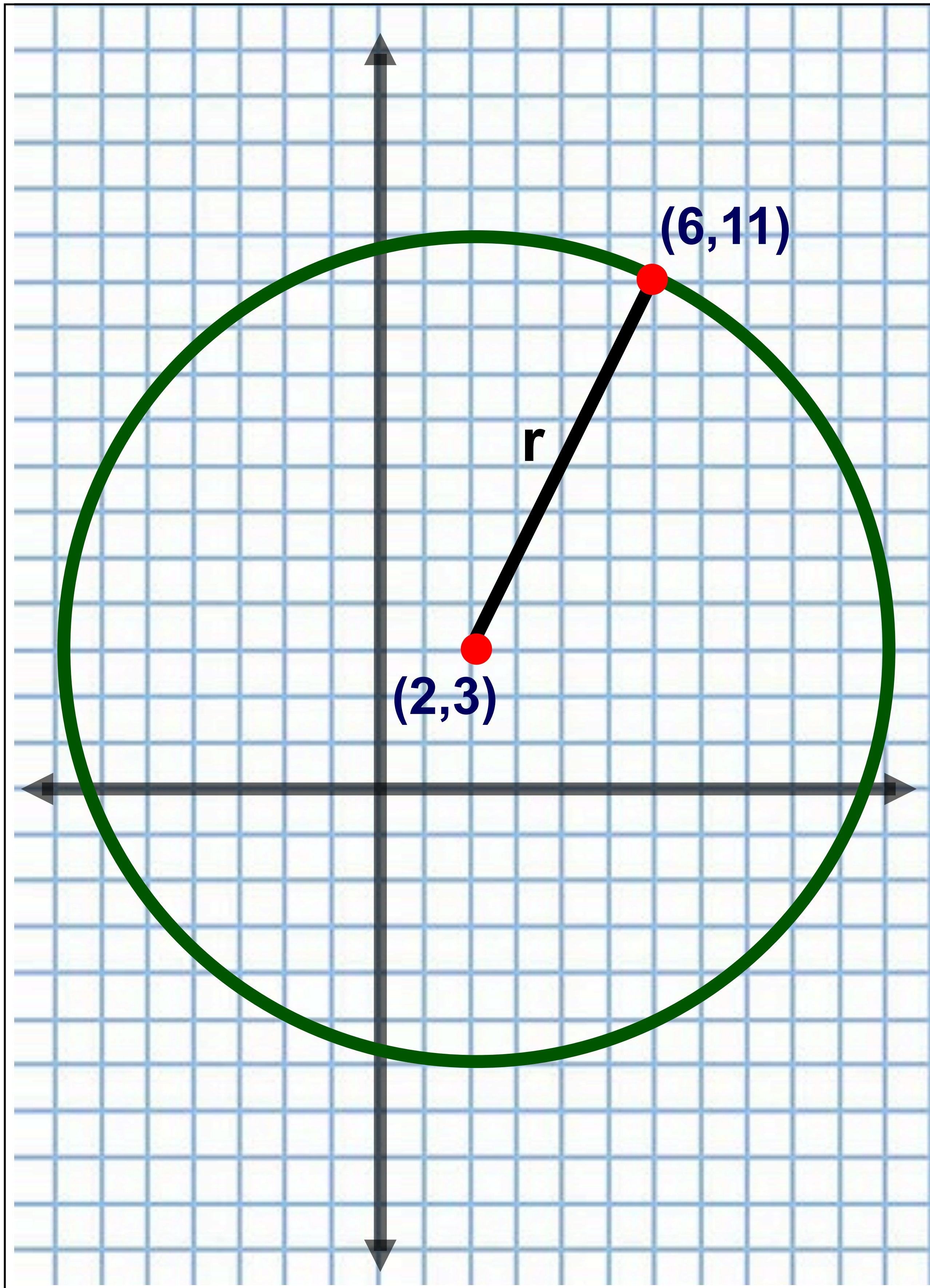
How about circles whose center is not on the origin (0, 0).

The Circle as a Locus of Points



If a circle is not centered on the origin, the equation has to be shifted by the amount it is away from the origin.

For example, let's shift the center of this circle to (2, 3).

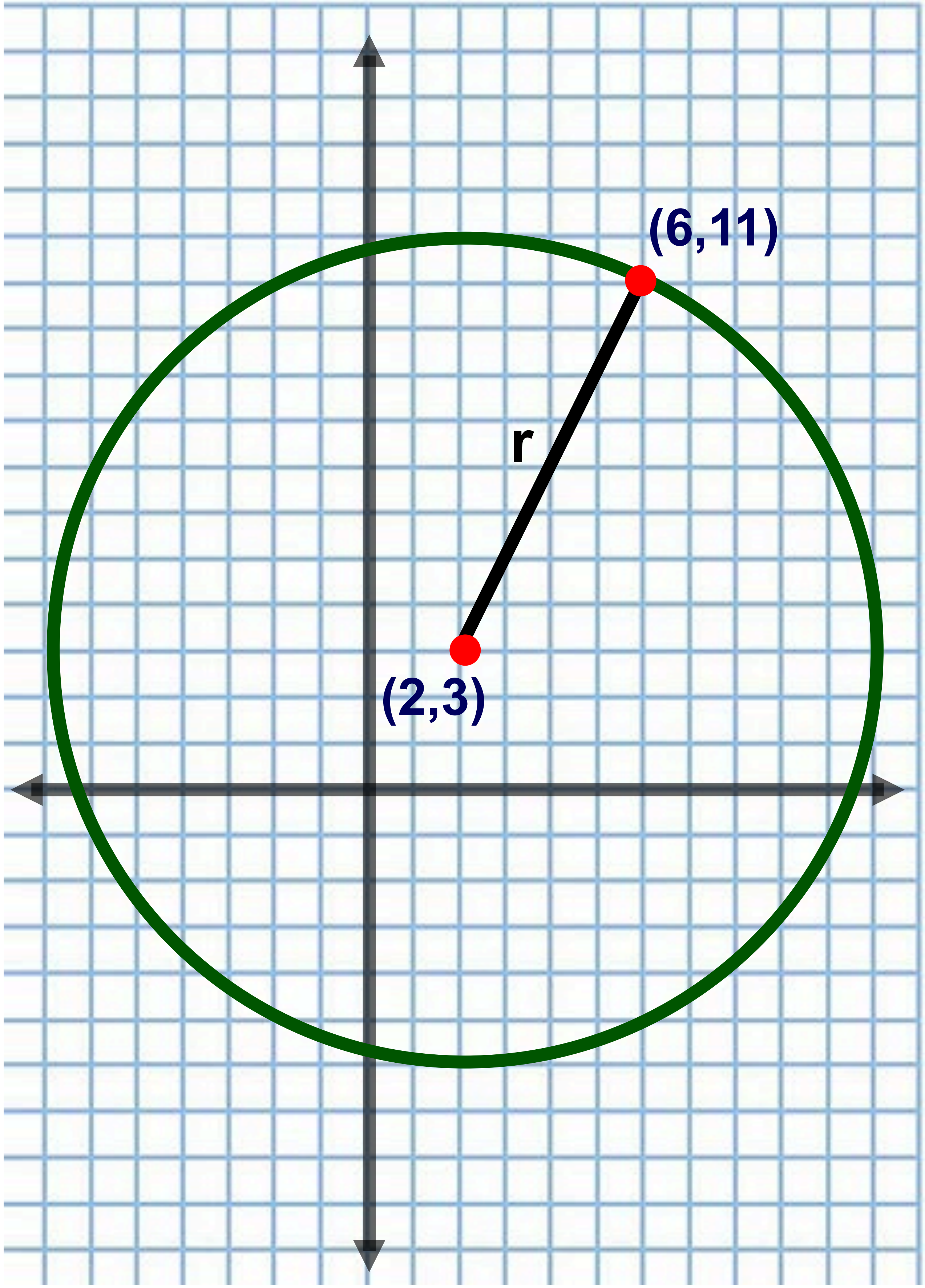


The Circle as a Locus of Points

Shifting the center of this circle from (0, 0) to (2, 3):

You can see that the point on the circle that was at (4, 8) is now at (6, 11)

Moving the center of the circle right 2 and up 3 will add that amount to each x and y coordinate.



The Circle as a Locus of Points

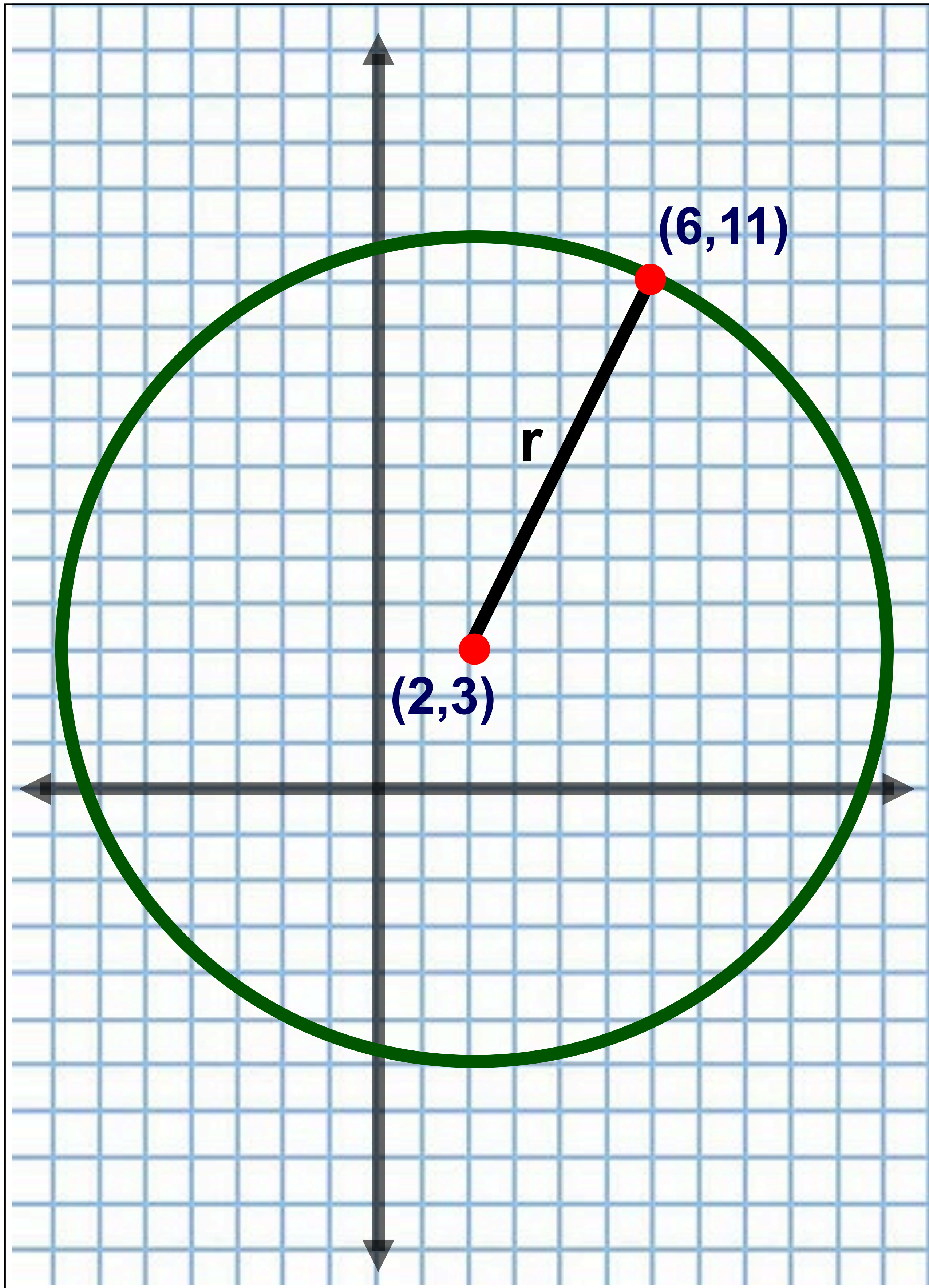
But the distance from the center to each point on the circle has not changed.

So, our equation for this circle has to reflect that.

The new equation will be

$(x - 2)^2 + (y - 3)^2 = r^2$

We can check to see if we still get the same radius.

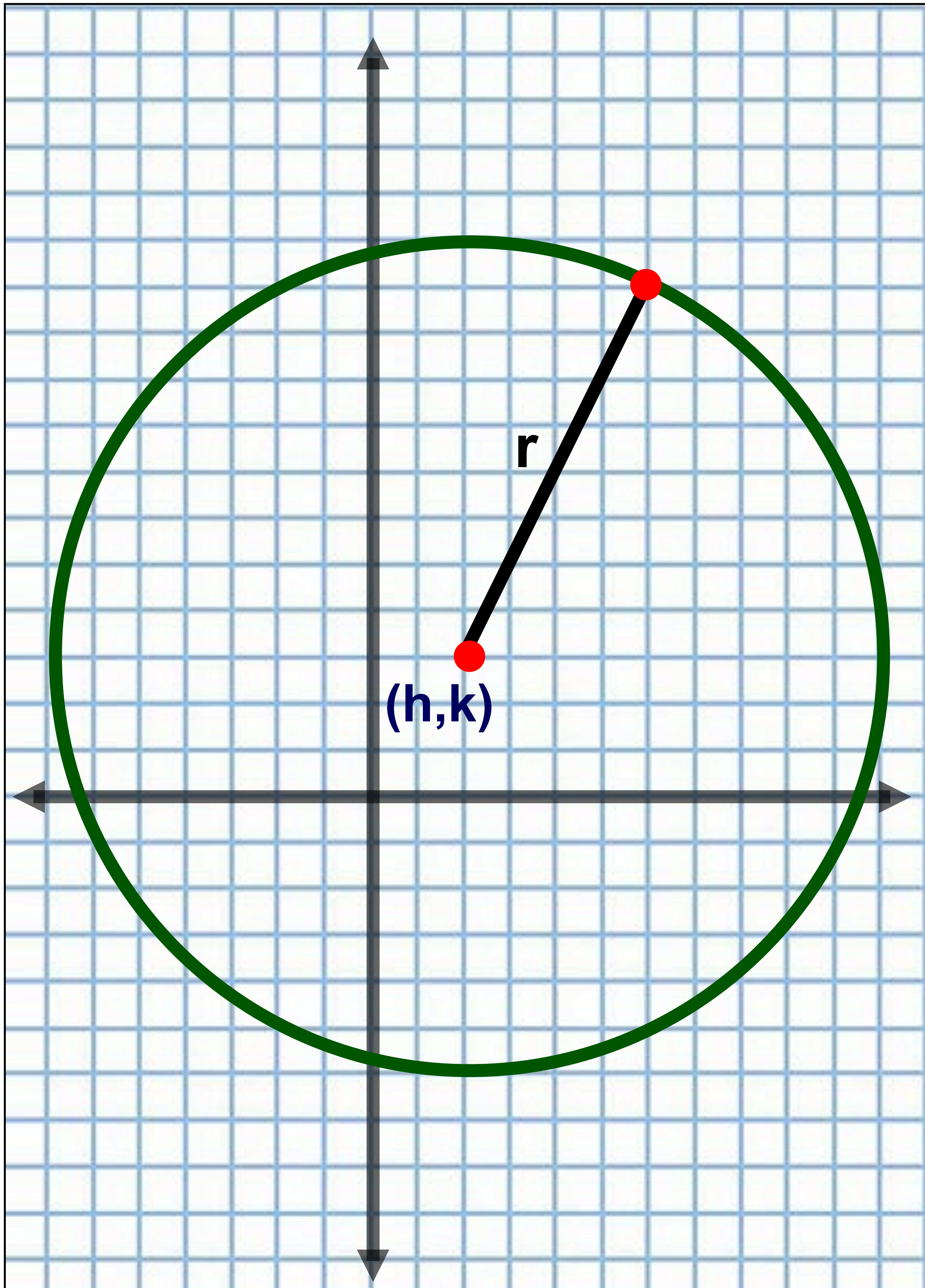


The Circle as a Locus of Points

$r^2 = (6 - 2)^2 + (11 - 3)^2$
$r^2 = (4)^2 + (8)^2$
$r^2 = 16 + 64 = 80$

These are the same values we had before when the circle was centered on (0, 0) and that point was located at (4, 8).

So, this translation of the center did not change the circle or its radius.



The Circle as a Locus of Points

In general, if the center of a circle is located at (h, k) and its radius is r, the equation for the circle is

$$(x - h)^2 + (y - k)^2 = r^2$$

Keep in mind that you are subtracting the x or y coordinate of the center of the circle.

So, if the center is at (3, 5) and the radius is 4, the equation becomes

$(x - 3)^2 + (y - 5)^2 = 16$

Example

Write the equation of a circle with center (-2, 3) & radius 3.8.

Answer
& Math Practice

60 What is the radius of the circle whose equation is

$(x - 5)^2 + (y - 3)^2 = 36$?

Answer

61 What is the radius of the circle whose equation is $(x + 3)^2 + (y - 4)^2 = 67$?

Answer

62 What is the x-coordinate of the center of the circle whose equation is $(x - 5)^2 + (y - 3)^2 = 47$?

Answer

63 What is the center and radius of the circle whose equation is $(x + 3)^2 + (y - 4)^2 = 30$?

Answer

64 What is the center and the radius of the circle whose equation is $(x - 5)^2 + (y - 3)^2 = 57$?

Answer

65 What is the center and the radius of the circle whose equation is $(x + 3)^2 + (y - 4)^2 = 65.36$?

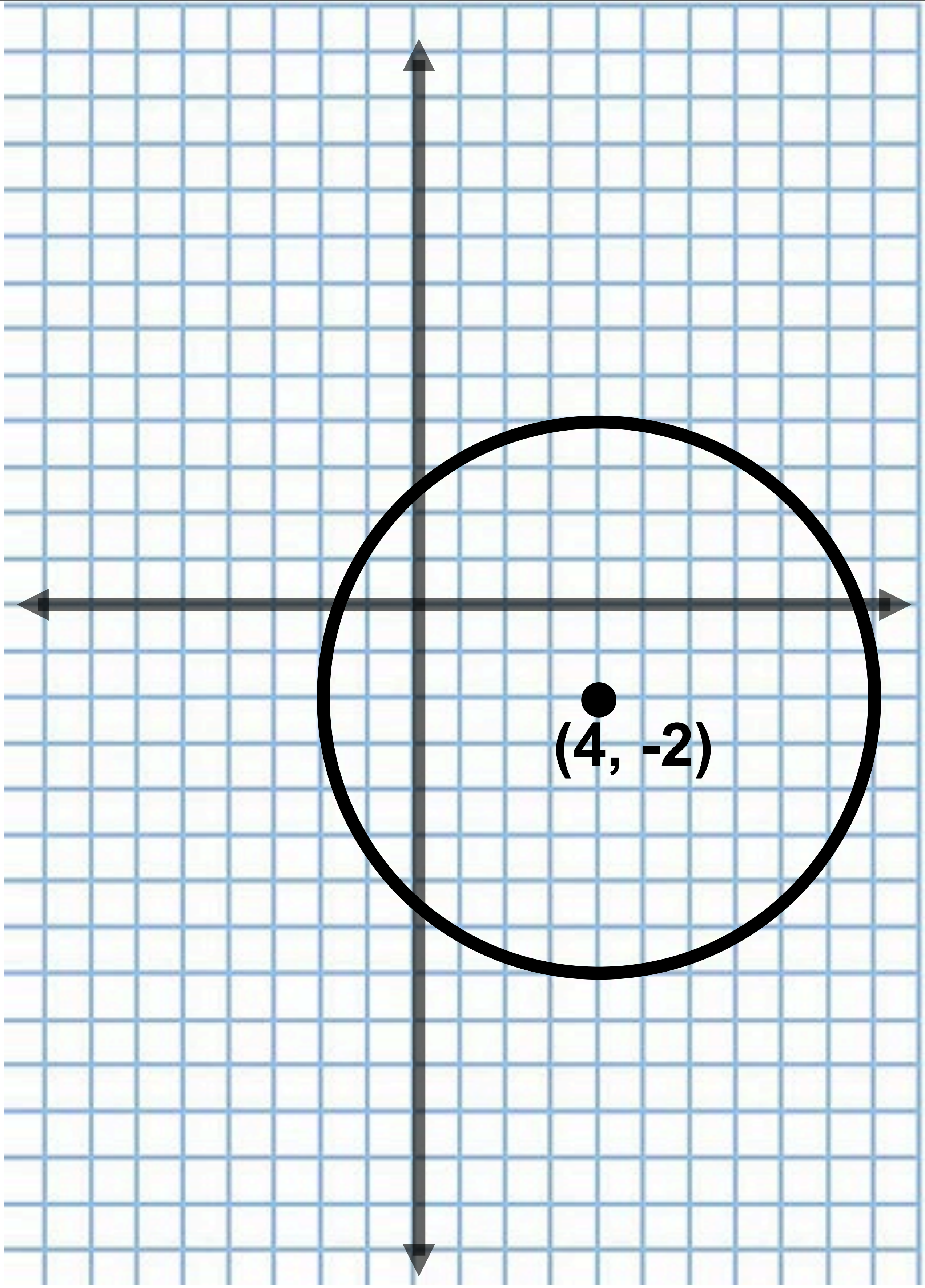
Answer

Example

The point (-5, 6) is on a circle with center (-1, 3).

Write the standard equation of the circle.

Answer



Example

The equation of a circle is
$(x - 4)^2 + (y + 2)^2 = 36$.

Graph the circle.

Click to reveal

Click to reveal

Click on the center of the circle to reveal it in the graph.

Math Practice

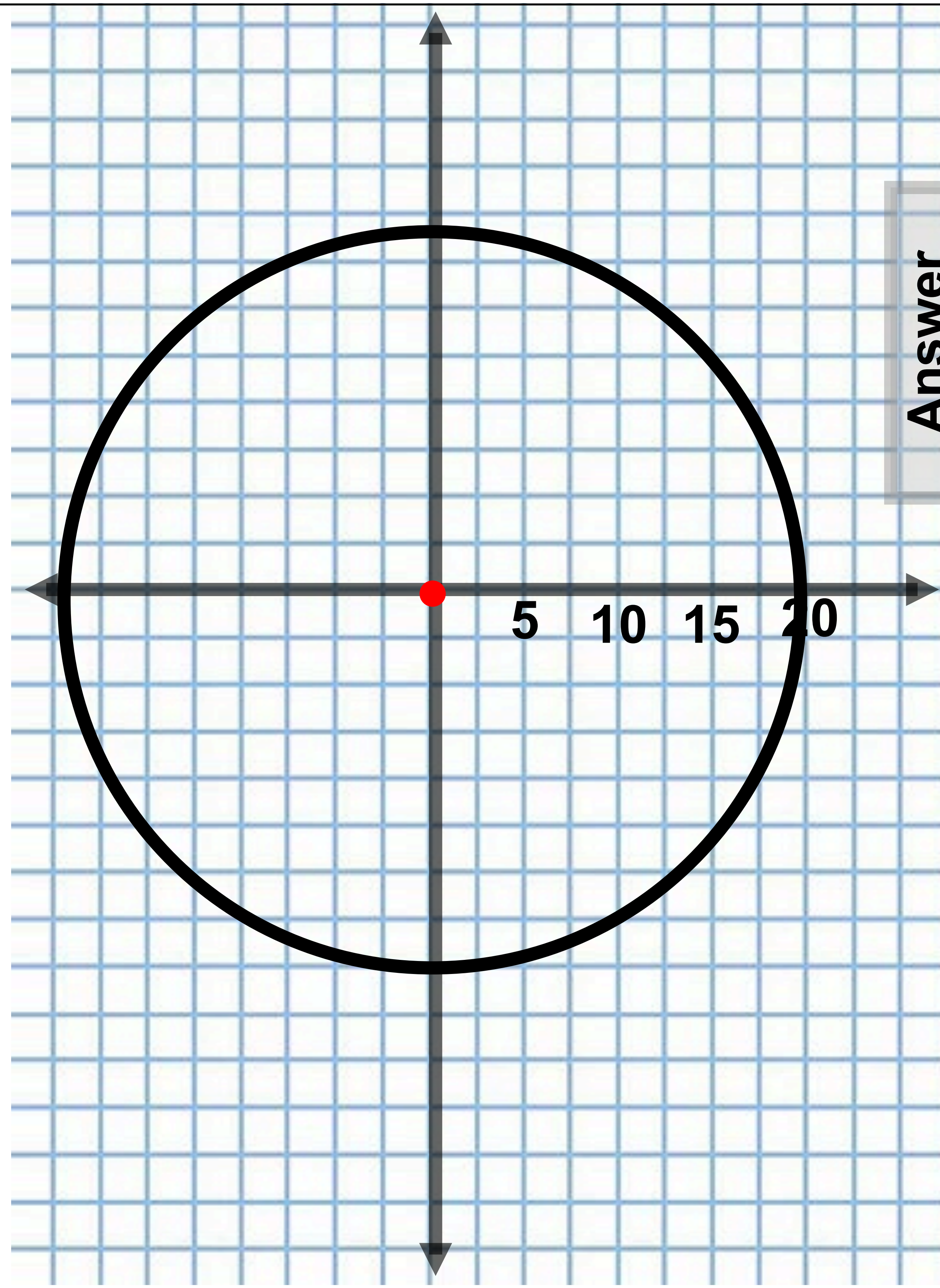
66 Which is the standard equation of the circle below?

A $x^2 + y^2 = 400$

B $(x - 10)^2 + (y - 10)^2 = 400$

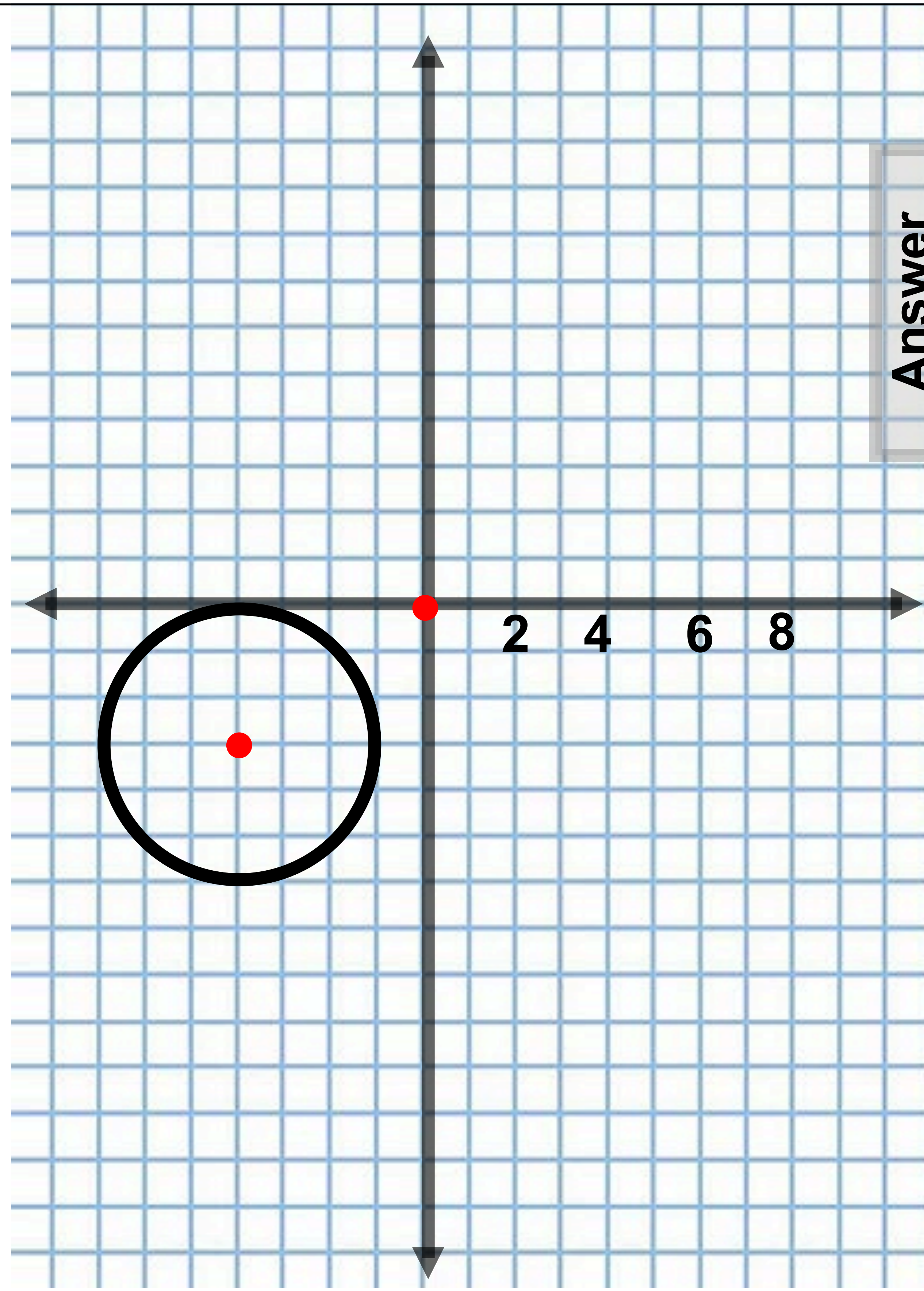
C $(x + 10)^2 + (y - 10)^2 = 400$

D $(x - 10)^2 + (y + 10)^2 = 400$



Answer

67 Which is the standard equation of the circle?



A $(x - 4)^2 + (y - 3)^2 = 81$

B $(x - 4)^2 + (y - 3)^2 = 9$

C $(x + 4)^2 + (y + 3)^2 = 81$

D $(x + 4)^2 + (y + 3)^2 = 9$

Answer

68 What is the center of $(x - 4)^2 + (y - 2)^2 = 64$?

A (0, 0)

B (4, 2)

C (-4, -2)

D (4, -2)

Answer

69 What is the radius of $(x - 4)^2 + (y - 2)^2 = 89$?

Answer

70 What is the diameter of a circle whose equation is

$(x - 2)^2 + (y + 1)^2 = 16$?

A 2

B 4

C 8

D 16

Answer

71 Which point does not lie on the circle described by the equation $(x + 2)^2 + (y - 4)^2 = 25$?

A (-2, -1)

B (1, 8)

C (3, 4)

D (0, 5)

Answer

Completing the Square

You're sometimes going to be given the equation of a circle which is not in standard form.

You need to be able to transform the equation to standard form in order to find the location of the center and the radius.

For instance, it's not clear what the radius and center are of the circle described by this equation.

$x^2 - 4x + y^2 - 12 = 0$

Math Practice

Completing the Square

$$x^2 - 4x + y^2 - 12 = 0$$

To find the radius and the coordinates of the center, we need to transform this into the form

$$(x - h)^2 + (y - k)^2 = r^2$$

The first step is to separate groups of terms which have x, which have y, and are constants.

Just moving those around makes this equation:

$$[x^2 - 4x] + y^2 = 12$$

Take a moment to confirm that this is true.

Completing the Square

$$x^2 - 4x + y^2 - 12 = 0$$

$[x^2 - 4x] + y^2 = 12$

We already see that the y-coordinate of the center is 0 ($k = 0$), since y^2 is by itself.

But what to do with the expression **$(x^2 - 4x)$**?

We have to convert that into the form **$(x - h)^2$** to find the x-coordinate of the center...and then the radius.

Completing the Square

If you recall, when you square a binomial, you get a trinomial.

$(x-h)^2 = x^2 - 2hx + h^2$

Our problem starts with an expression in the form of $x^2 - 2hx$, so let's solve for that so we can see what can replace it:

$x^2 - 2hx = (x-h)^2 - h^2$
So
The coefficient $(-2h)$ of x is $-2h$.

The constant of the trinomial $(-h^2)$ is $-(h)^2$.

So, to get h, divide the coefficient of x by -2

To make the expressions equivalent, subtract h^2 from the binomial

Completing the Square

$$[x^2 - 4x] + y^2 = 12$$

Dividing the coefficient -4 by -2 yields 2, so h = 2

Then $-h^2 = -4$

$[x^2 - 4x] + y^2 = 12$

$[(x - 2)^2 - 4] + y^2 = 12$

$(x - 2)^2 + y^2 = 16$

The center is at (2, 0) and the radius is 4.

The same steps are used to find k, when needed, as in the next example.

Teacher Notes

Example of Completing the Square

Determine the radius and center of this circle.

$$x^2 + y^2 - 2x + 6y + 6 = 0$$

$$[x^2 - 2x] + [y^2 + 6y] = -6$$

$[x^2 - 2x]$

$h = -2/(-2) = 1$

$x^2 - 2x = (x - h)^2 - 1^2$

$x^2 - 2x = (x - 1)^2 - 1$

$[y^2 + 6y]$

$k = +6/(-2) = -3$

$[y^2 + 6y] = (y -(-3))^2 - (3)^2$

$[y^2 + 6y] = (y + 3)^2 - 9$

$$(x - 1)^2 - 1 + (y + 3)^2 - 9 = -6$$

$$(x - 1)^2 + (y + 3)^2 = 4$$

The center is (1, -3) and the radius is 2

Math Practice

72 What is the radius of the circle described by this equation?

$$x^2 + y^2 - 2x + 6y + 6 = 0$$

Answer

73 What is the x-coordinate of the center of the circle described by this equation?

$$x^2 + y^2 - 2x + 6y + 6 = 0$$

Answer

74 What is the x-coordinate of the center of the circle described by this equation?

$$x^2 + y^2 - 8x + 4y - 5 = 0$$

Answer

75 What is the radius of the circle described by this equation?

$$x^2 + y^2 - 8x + 4y - 5 = 0$$

Answer

76 What is the radius of the circle described by this equation?

$$x^2 + y^2 + 16x - 22y + 174 = 0$$

Answer

77 What is the y-coordinate of the center of the circle described by this equation?

$$x^2 + y^2 + 16x - 22y + 174 = 0$$

Answer

78 **Part A**

The equation $x^2 + y^2 - 4x + 2y = b$ describes a circle.

Determine the y-coordinate of the center of the circle.

Answer

PARCC Released Question (EOY) - Non-Calculator Section - Problem 6

79 **Part B**

The equation $x^2 + y^2 - 4x + 2y = b$ describes a circle.

The radius of the circle is 7 units. What is the value of b in the equation?

Answer

PARCC Released Question (EOY) - Non-Calculator Section - Problem 6

80 The equation $x^2 - 8x + y^2 = 9$ defines a circle in the xy-coordinate plane. To find the radius of the circle, the equation can be rewritten as $(\_\_\_\_\_\_\_\_)^2 + y^2 = \_\_\_$.

(Select two answers.)

A $x + 4$

B $x - 4$

C $x + 16$

D $x - 16$

E 25

F 13

G 9

H 5

Answer

PARCC Released Question (EOY) - Calculator Section - Problem 7

PARCC Sample Questions

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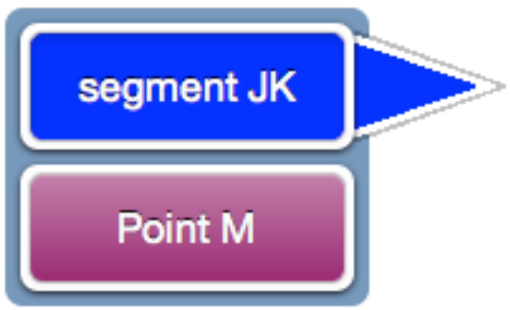
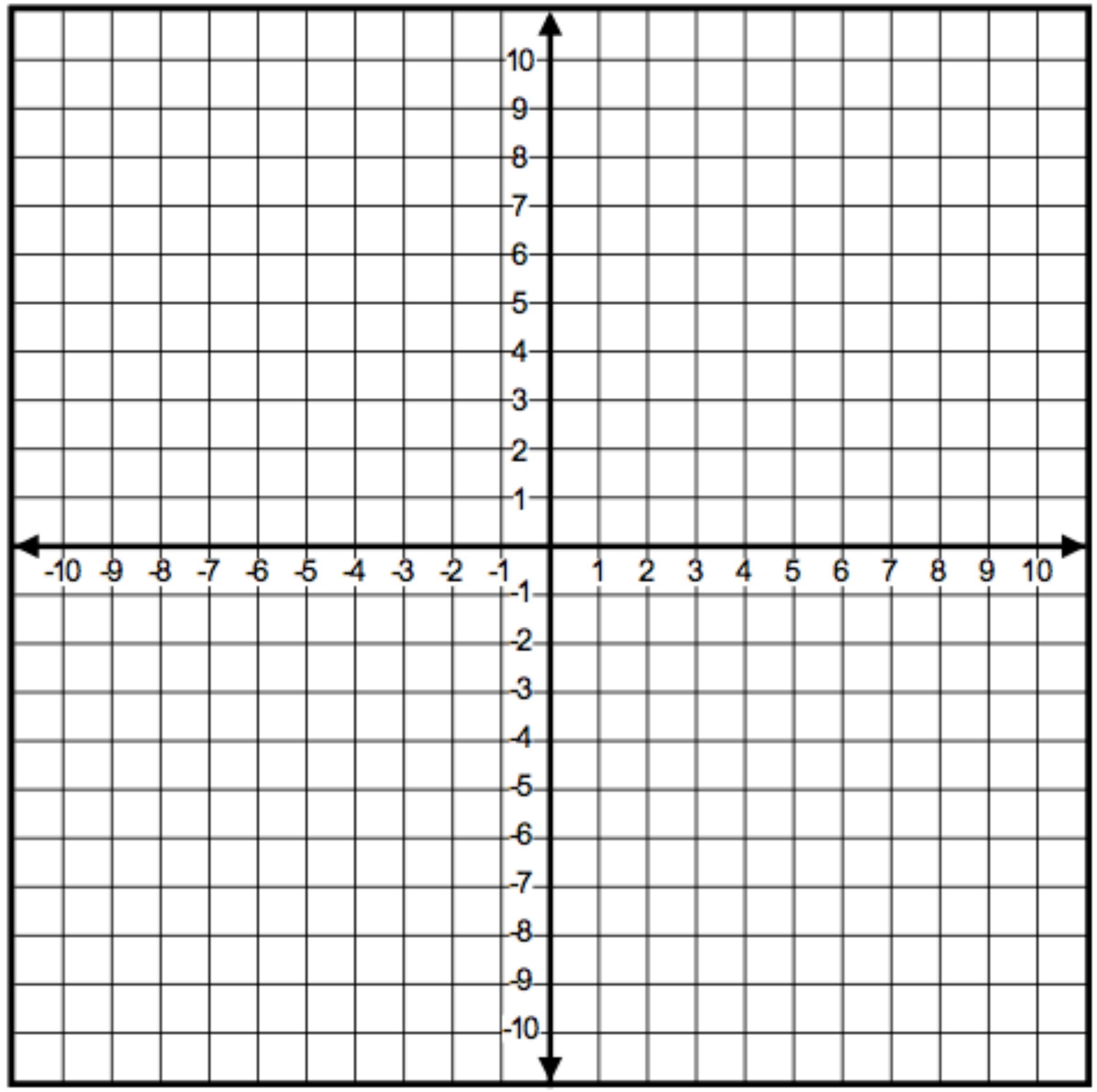
Question 5/7

Topic: Partitions of a Line Segment

Line segment JK in the coordinate plane has endpoints with coordinates $(-4, 11)$ and $(8, -1)$. Graph $\overline{JK}$ and find two possible locations for point M so that M divides $\overline{JK}$ into two parts with lengths in a ratio of 1:3.

To graph a line segment, select segment JK and then plot two points on the coordinate plane. A segment will connect the points. Select Point M and then plot the two points.





Answer

PARCC Released Question (EOY)

Question 6/7

Topic: Equation of a Circle

The equation $x^2 + y^2 - 4x + 2y = b$ describes a circle.

Part A

Determine the *y*-coordinate of the center of the circle.

Enter your answer in the box.

Part B

The radius of the circle is 7 units. What is the value of *b* in the equation?

Enter your answer in the box.

PARCC Released Question (EOY)

Answer

Question 7/25

Topic: Equation of a Circle

The equation $x^2 - 8x + y^2 = 9$ defines a circle in the *xy* -coordinate plane.

Select from the drop-down menus to correctly complete the sentence.

To find the center of the circle and the length of the radius, the equation can be rewritten as

$(\_\_\_\_\_\_)^2 + y^2 = \_\_\_\_$

a. x + 4
b. x - 4
c. x + 16
d. x - 16

e. 25
f. 13
g. 9
h. 5

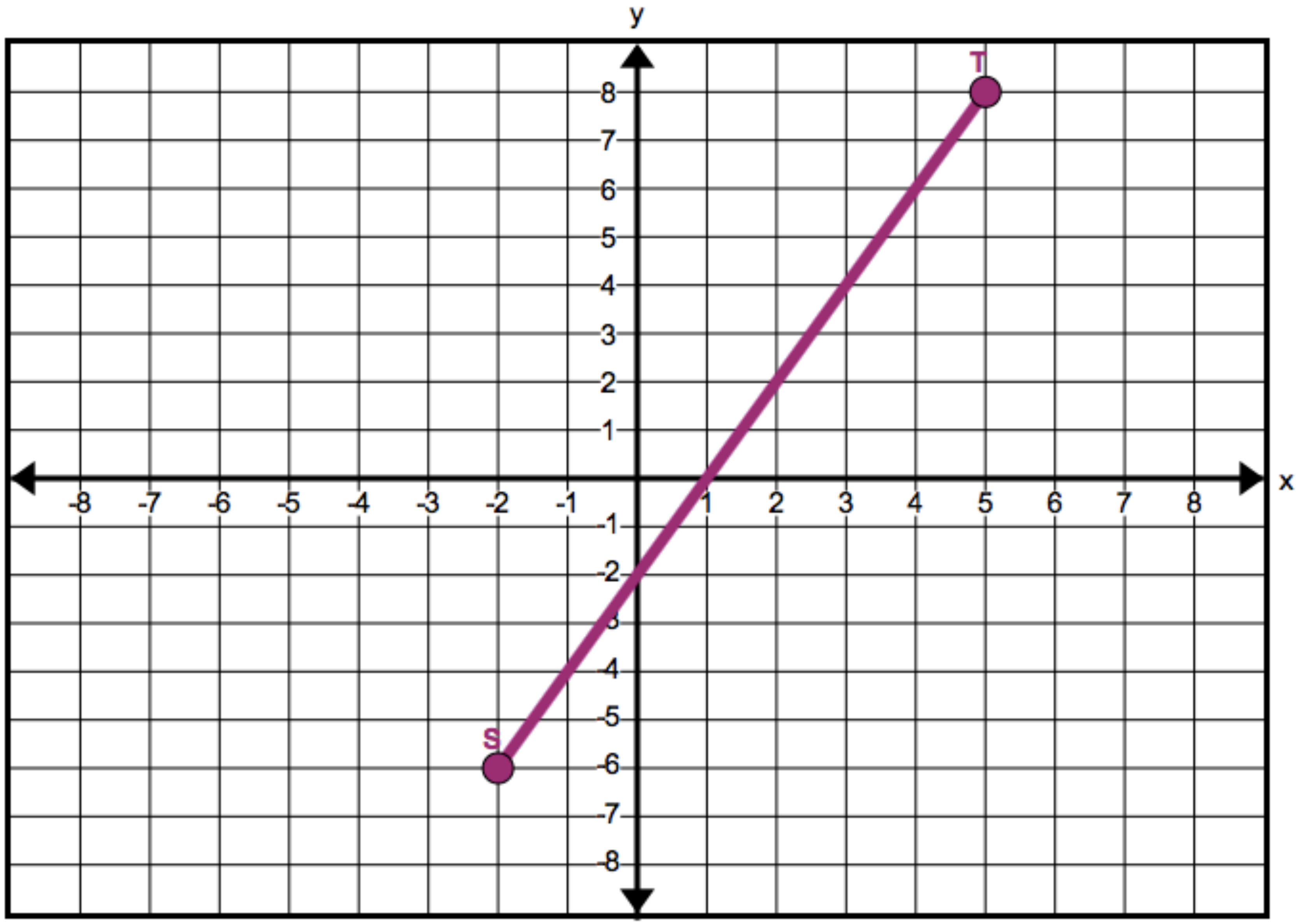
Answer

PARCC Released Question (EOY)

Question 2/7

Topic: Partitions of a Line Segment

Point Q lies on $\overline{ST}$, where point *S* is located at $(-2, -6)$ and point *T* is located at $(5, 8)$. If $SQ : QT = 5 : 2$, where is point Q on $\overline{ST}$? Select a place on the coordinate grid to plot point Q.



Answer

PARCC Released Question (PBA)

General Problems

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of Contents

Write the equation of a line, in slope-intercept form, which has a point of tangency at (3, 6) with a circle whose center is at the origin.

Write the equation of a line, in slope-intercept form, which has a point of tangency at (3, 6) with a circle whose center is at the origin.

Strategy

The slope of the radius of that circle to that point can be determined.

Then the slope of the line tangent at that point will be the negative reciprocal of the slope of the radius since they are perpendicular.

Given a point and the slope, the equation of the line can be written.

Write an equation of a line which has a point of tangency at (3,6) with a circle whose center is at the origin.

Solution

$m_{radius} = (6-0)/(3-0) = 2$
$m_{tangent} = -1/2$

$(y-y_1) = m(x-x_1)$
$(y-6) = (1/2)(x-3)$
$y = 0.5x - 1.5 + 6$
$y = 0.5x + 4.5$

$b = 0.5(0) + 4.5$
$b = 4.5$

81 What is the slope of a line tangent at (7, 2) to a circle whose center is at (2, 3)?

Answer

82 What is the y-intercept of the line in the prior problem which was tangent at (7, 2) to a circle whose center is at (2, 3)?

Answer