Natural Numbers

The first numbers developed were the Natural Numbers, also called the Counting Numbers.

1, 2, 3, 4, 5, ...

The three dots, (…), means that these numbers continue forever: there is no largest counting number.

Think of counting objects as you put them in a container those are the counting numbers.

Natural Numbers

Natural numbers were used before there was history. All people use them.

This "counting stick" was made more than 35,000 years ago and was found in Lebombo, Swaziland.

The cuts in this bone record the number "29."

http://www.tanetc.org/math.html
**Natural Numbers and Addition**

They were, and are, used to count objects
> goats,
> bales,
> bottles,
> etc.

Drop a stone in a jar, or cut a line in a stick, every time a goat walks past.

That jar or stick is a record of the number.

---

**Numbers versus Numerals**

Numbers exist even without a numeral, such as the number indicated by the cuts on the Lebombo Bone.

A numeral is the name we give a number in our culture.

---

**Whole Numbers**

Adding zero to the Counting Numbers gives us the **Whole Numbers**.

0, 1, 2, 3, 4, ...

Counting numbers were developed more than 35,000 years ago.

It took 34,000 more years to invent zero.

This the oldest known use of zero (the dot), about 1500 years ago.

It was found in Cambodia and the dot is for the zero in the year 605.

---

**Why zero took so long to Invent**

Horses versus houses.

<table>
<thead>
<tr>
<th>Horses</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Houses</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

zero horses = zero houses

---

**Why zero took so long**

Would I tell someone I have a herd of zero goats?

Or a garage with zero cars?

Or that my zero cars have zero tires?

Zero just isn't a natural number, but it is a whole number.
Whole Numbers
Each time a marble is dropped in a jar we are doing addition.

A number line allowed us to think of addition in a new way.

Integers
The below number line shows only the integers.

Review: Fractions
Remember that division led to a new set of numbers: fractions.
Fractions are the result when you ask questions like:
\[ \frac{1}{2} = ? \quad \frac{1}{3} = ? \quad \frac{2}{3} = ? \quad 1 + \frac{1,000,000}{1} = ? \]
\[ \frac{1}{2} = ? \text{ asks the question:} \]
If I divide 1 into 2 equal pieces, what will be the size of each?
The answer to this question cannot be found in the integers.
New numbers were needed: fractions.

Fractions
There are an infinite number of fractions between each pair of integers.

Using a magnifying glass to look closely between 0 and 1 on this number line, we can locate a few of these fractions.

Notice that it's easier to find their location when they are in decimal form since it's clear which integers they're between...and closest to.

Rational Numbers
Rational Numbers are numbers that can be expressed as a ratio of two integers.

This includes all the fractions, as well as all the integers, since any integer can be written as a ratio of itself and 1.

Fractions can be written in "fraction" form or decimal form.
When written in decimal form, rational numbers are either:

Terminating, such as 
\[ \frac{1}{2} = 0.500000000000 = 0.5 \]

Repeating, such as 
\[ \frac{1}{7} = 0.142857142857142857... = 0.142857 \]
\[ \frac{1}{3} = 0.33333333333333333... = 0.33 \]
Powers of Integers

Just as multiplication is repeated addition, exponents are repeated multiplication.

For example, $3^5$ reads as "3 to the fifth power" = $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$

In this case "3" is the base and "5" is the exponent.
The base, 3, is multiplied by itself 5 times.

Powers of Integers

When evaluating exponents of negative numbers, keep in mind the meaning of the exponent and the rules of multiplication.

For example, 

$(-3)^2 = (-3)(-3) = 9,$

is the same as 

$(3)^2 = (3)(3) = 9.$

However, 

$(-3)^2 = (-3)(-3) = 9$

is NOT the same as 

$-3^2 = -(3)(3) = -9,$

Similarly, 

$(3)^3 = (3)(3)(3) = 27$

is NOT the same as 

$(-3)^3 = (-3)(-3)(-3) = -27,$

Special Term: Squares and Cubes

A number raised to the second power can be said to be "squared."

That's because the area of a square of length $x$ is $x^2$: "$x$ squared."

A number raised to the third power can be said to be "cubed."

That's because the volume of a cube of length $x$ is $x^3$: "$x$ cubed."

The Root as an Inverse Operation

Performing an operation and then the inverse of that operation returns us to where we started.

We already saw that if we add 5 to a number and then subtract 5, we get back to the original number.

Or, if we multiply a number by 7 and then divide by 7, we get back to the original number.

The Root as an Inverse Operation

Inverses of exponents are a little more complicated for two reasons.

First, there are two possible inverse operations.

The equation $16 = 4^2$ provides the answer 16 to the question: what is $4$ raised to the power of 2?

One inverse operation is shown by: 

$4 = \sqrt{16}$

This provides the answer 4 to the question: What number raised to the power of 2 yields 16?

This shows that the square root of 16 is 4.

It's true since $(4)(4) = 16$
The other inverse operation will not be addressed until Algebra II. Just for completeness, that inverse operation is \( 2 = \log_{10} 100 \).
It provides the answer 2 to the question:
To what power must 10 be raised to get 100.
You'll learn more about that in Algebra II, but you should realize it's the other possible inverse operation.

1. What is \( \sqrt{1} \)?

2. What is \( \sqrt{10} \)?

3. What is the square of 15?

4. What is \( \sqrt{256} \)?
<table>
<thead>
<tr>
<th>6</th>
<th>What is ( \sqrt{16} )?</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>What is the square of 18?</td>
</tr>
<tr>
<td>8</td>
<td>What is 11 squared?</td>
</tr>
<tr>
<td>9</td>
<td>( \sqrt{36} = ? )</td>
</tr>
<tr>
<td></td>
<td>A  6</td>
</tr>
<tr>
<td></td>
<td>B -6</td>
</tr>
<tr>
<td></td>
<td>C is not real</td>
</tr>
<tr>
<td>10</td>
<td>( \sqrt{-81} = ? )</td>
</tr>
<tr>
<td></td>
<td>A  9</td>
</tr>
<tr>
<td></td>
<td>B -9</td>
</tr>
<tr>
<td></td>
<td>C is not real</td>
</tr>
</tbody>
</table>

Review of the Square Root of a Number

The following formative assessment questions are review from 8th grade. If further instruction is needed, see the presentation at:

http://njctl.org/courses/math/8th-grade/numbers-and-operations-8th-grade/
11. \( \sqrt{400} = ? \)
   - A. 20
   - B. -20
   - C. Is not real

12. Ashley and Brandon have different methods for finding square roots.

   **Ashley’s Method**
   To find the square root of a number, find a number so that the product of the number and itself is \( \sqrt{x} \). For example, \( 2 \times 2 = 4 \), so the square root of 4 is 2.

   **Brandon’s Method**
   To find the square root of a number, multiply \( x \) by \( \frac{1}{2} \). For example, \( 4 \times \frac{1}{2} = 2 \), so the square root of 4 is 2.

Which student's method is not correct?
- A. Ashley’s Method
- B. Brandon’s Method

On your paper, explain why the method you selected is not correct.

---

13. \( \sqrt{5^2} = ? \)

14. \( \sqrt{11^2} = ? \)

15. \( (\sqrt{5})^2 = ? \)
   - A. 25
   - B. 5
   - C. \( \sqrt{5} \)
   - D. \( \sqrt{25} \)

16. \( \sqrt{-9} = ? \)
   - A. 3
   - B. -3
   - C. No real roots
17. The expression equal to $\sqrt{54-b}$ is equivalent to a positive integer when $b$ is

- A -10
- B 64
- C 16
- D 4

Review of the Square Root of Fractions

The following formative assessment questions are review from 8th grade. If further instruction is needed, see the presentation at:

http://njctl.org/courses/math/8th-grade/numbers-and-operations-8th-grade/

18. $\sqrt{\frac{49}{100}} =$

- A $\frac{7}{10}$
- B $\frac{7}{50}$
- C $\frac{7}{5}$
- D no real solution

19. $\sqrt{\frac{16}{100}} =$

- A $\frac{2}{5}$
- B $\frac{8}{10}$
- C $\frac{4}{10}$
- D no real solution

21. $\sqrt{-\frac{16}{25}} =$

- A $\frac{4}{10}$
- B $\frac{8}{10}$
- C $\frac{4}{5}$
- D no real solution
### Review of the Square Root of Decimals

The following formative assessment questions are review from 8th grade. If further instruction is need, see the presentation at:

http://njctl.org/courses/math/8th-grade/numbers-and-operations-8th-grade/

<table>
<thead>
<tr>
<th>Problem</th>
<th>Solution</th>
<th>Option A</th>
<th>Option B</th>
<th>Option C</th>
<th>Option D</th>
</tr>
</thead>
<tbody>
<tr>
<td>22 ( \sqrt{\frac{36}{121}} = )</td>
<td>( \frac{6}{12} )</td>
<td>( \frac{6}{13} )</td>
<td>( \frac{6}{11} )</td>
<td>no real solution</td>
<td></td>
</tr>
<tr>
<td>23 Evaluate ( \sqrt{49} )</td>
<td>( \frac{7}{100} )</td>
<td>( \frac{7}{10} )</td>
<td>( \frac{7}{25} )</td>
<td>no real solution</td>
<td></td>
</tr>
<tr>
<td>24 Evaluate ( \sqrt{36} )</td>
<td>( 0.06 )</td>
<td>( 0.08 )</td>
<td>( 6 )</td>
<td>no real solution</td>
<td></td>
</tr>
<tr>
<td>25 Evaluate ( \sqrt{0.0121} )</td>
<td>( 0.11 )</td>
<td>( 1.1 )</td>
<td>( 0.8 )</td>
<td>no real solution</td>
<td></td>
</tr>
<tr>
<td>26 Evaluate ( \sqrt{0.0064} )</td>
<td>( \frac{4}{5} )</td>
<td>no real solution</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Review of Approximating Non-Perfect Squares

The following formative assessment questions are review from 8th grade. If further instruction is needed, see the presentation at:

http://njctl.org/courses/math/8th-grade/numbers-and-operations-8th-grade/

28 The square root of 40 falls between which two integers?
   A 3 and 4
   B 5 and 6
   C 6 and 7
   D 7 and 8

29 Which integer is $\sqrt{40}$ closest to?

$\ldots < \sqrt{ \ldots } < \sqrt{ \ldots }$ Identify perfect squares closest to 40

$\ldots < \sqrt{ \ldots } < \sqrt{ \ldots }$ Take square root

Identify nearest integer

30 The square root of 110 falls between which two perfect squares?
   A 36 and 49
   B 49 and 64
   C 64 and 84
   D 100 and 121

31 Estimate to the nearest integer.

$\sqrt{110}$
32. Estimate to the nearest integer.
\[ \sqrt{219} \]

33. Estimate to the nearest integer.
\[ \sqrt{90} \]

34. Approximate \( \sqrt{29} \) to the nearest integer.

35. Approximate \( \sqrt{96} \) to the nearest integer.

36. Approximate \( \sqrt{167} \) to the nearest integer.

37. Approximate \( \sqrt{140} \) to the nearest integer.
38 Approximate $\sqrt{40}$ to the nearest integer.

39 The expression $\sqrt{93}$ is a number between:
- A 3 and 9
- B 8 and 9
- C 9 and 10
- D 46 and 47

40 For what integer $x$ is $\sqrt{x}$ closest to 6.25?

41 For what integer $y$ is $\sqrt{y}$ closest to 4.5?

42 Between which two positive integers does $\sqrt{56}$ lie?
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10

43 Between which two positive integers does $\sqrt{80}$ lie?
- 1
- 2
- 3
- 4
- 5
- 6
- 7
- 8
- 9
- 10
44 Between which two labeled points on the number line would \( \sqrt{10} \) be located?

\[ 3.0 \quad 3.1 \quad 3.2 \quad 3.3 \quad 3.4 \quad 3.5 \quad 3.6 \quad 3.7 \quad 3.8 \quad 3.9 \quad 4.0 \]

\[ \text{A} \quad \text{B} \quad \text{C} \quad \text{D} \quad \text{E} \quad \text{F} \quad \text{G} \quad \text{H} \quad \text{I} \quad \text{J} \]

---

**Review of Irrational Numbers & Real Numbers**

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---

**Irrational Numbers**

Just as subtraction led us to zero and negative numbers.

And division led us to fractions.

Finding the root leads us to irrational numbers.

Irrational numbers complete the set of Real Numbers.

Real numbers are the numbers that exist on the number line.

---

**Irrational Numbers**

Irrational Numbers are real numbers that cannot be expressed as a ratio of two integers.

In decimal form, they extend forever and never repeat.

There are an infinite number of irrational numbers between any two integers (think of all the square roots, cube roots, etc. that don't come out evenly).

Then realize you can multiply them by an other number or add any number to them and the result will still be irrational.

---

**Rational & Irrational Numbers**

\( \sqrt{9} \) is rational.

This is because the radicand (number under the radical) is a perfect square. \((3^2 = 9)\)

If a radicand is not a perfect square, the root is said to be irrational.

Ex: \( \sqrt{12} \)

---

**Irrational Numbers**

Irrational numbers were first discovered by Hippasus about the about 2500 years ago.

He was a Pythagorean, and the discovery was considered a problem since Pythagoreans believed that "All was Number," by which they meant the natural numbers \((1, 2, 3...,)\) and their ratios.

Hippasus was trying to find the length of the diagonal of a square with sides of length 1. Instead, he proved, that the answer was not a natural or rational number.

For a while this discovery was suppressed since it violated the beliefs of the Pythagorean school.

Hippasus had proved that \( \sqrt{2} \) is an irrational number.
Irrational Numbers

An infinity of irrational numbers emerge from trying to find the root of a rational number.

Hippasus proved that √2 is irrational goes on forever without repeating.

Some of its digits are shown on the next slide.

Square Root of 2

Here are the first 1000 digits, but you can find the first 10 million digits on the Internet. The numbers go on forever, and never repeat in a pattern:

1.414213562373095048801688724209698078569671875376948073
17667793790732478462107039885038753432764157275013984623
0912290702494836056550737212641412497099958314132226659
2750559275755995056151275820657147019559571605097274534
59688204172851741864088919860955232923048430871432145083
9762603627952514079896872533965463318082696406206152583
553295054750287759961729835575220337531857011354374003
40849847160386899076900481503054402779031645424782306
849293619862158057846311596668713013015618568972372352
8850926486124949771542183342042656860614684274077743585
4874156570696776537202264854470158588016207584740228572
2600208554665214583988939437092659180031398824648157
0826301005948587040031864803241948972782906410450726388
13137398552561137220240250912277002269411275736272284957
38108967504018369668368450725993647290687052896943180475
654823728971803268024744206296912485905218010044598415
059112024944134172853147810583630314770773919828931417101
71111683916581726889419758761582512122951848847208969...

Roots of Numbers are Often Irrational

Soon, thereafter, it was proved that many numbers have irrational roots.

We now know that the roots of most numbers to most powers are irrational.

These are called algebraic irrational numbers.

In fact, there are many more irrational numbers that rational numbers.

Principal Roots

Since you can't write out all the digits of √2, or use a bar to indicate a pattern, the simplest way to write that number is √2.

But when solving for the square root of 2, there are two answers: +√2 or -√2.

These are in two different places on the number line.

To avoid confusion, it was agreed that the positive value would be called the principal root and would be written $\sqrt{2}$.

The negative value would be written as $-\sqrt{2}$.

Algebraic Irrational Numbers

There are an infinite number of irrational numbers.

Here are just a few that fall between -10 and +10.

Transcendental Numbers

The other set of irrational numbers are the transcendental numbers.

These are also irrational. No matter how many decimals you look at, they never repeat.

But these are not the result of solving a polynomial equation with rational coefficients, so they are not due to an inverse operation.

Some of these numbers are real, and some are complex.

But, this year, we will only be working with the real transcendental numbers.
We have learned about Pi in Geometry. It is the ratio of a circle’s circumference to its diameter. It is represented by the symbol π.

discuss why this is an approximation at your table.

Is this number rational or irrational?

# is a Transcendental Number

The most famous of the transcendental numbers is #.

# is the ratio of the circumference to the diameter of a circle.

People tried to find the value of that ratio for millennia.

Only in the mid 1800's was it proven that there was no rational solution.

Since # is irrational (it's decimals never repeat) and it is not a solution to an equation...it is transcendental.

Some of its digits are on the next page.
46 What type of number is -? Select all that apply.
- Irrational
- Rational
- Integer
- Whole Number
- Natural Number

47 What type of number is √8? Select all that apply.
- Irrational
- Rational
- Integer
- Whole Number
- Natural Number

48 What type of number is -√64? Select all that apply.
- Irrational
- Rational
- Integer
- Whole Number
- Natural Number

49 What type of number is √2.25? Select all that apply.
- Irrational
- Rational
- Integer
- Whole Number
- Natural Number

Properties of Exponents

The properties of exponents follow directly from expanding them to look at the repeated multiplication they represent.

Don't memorize properties, just work to understand the process by which we find these properties and if you can't recall what to do, just repeat these steps to confirm the property.

We'll use 3 as the base in our examples, but the properties hold for any base. We show that with base a and powers b and c.

We'll use the facts that:

\[
(3^2) = (3 \cdot 3)
\]

\[
(3^3) = (3 \cdot 3 \cdot 3)
\]

\[
(3^4) = (3 \cdot 3 \cdot 3 \cdot 3)
\]
Properties of Exponents

We need to develop all the properties of exponents so we can discover one of the inverse operations of raising a number to a power...finding the root of a number to that power.

That will emerge from the final property we'll explore.

But, getting to that property requires understanding the others first.

Multiplying with Exponents

When multiplying numbers with the same base, add the exponents.

\[(a^b)(a^c) = a^{b+c}\]

\[(3^3)(3^2) = 3^{3+2}\]

\[(3 \cdot 3 \cdot 3)(3 \cdot 3 \cdot 3) = (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)\]

\[(3^3)(3^3) = 3^{3+3}\]

Dividing with Exponents

When dividing numbers with the same base, subtract the exponent of the denominator from that of the numerator.

\[\frac{(a^b)}{(a^c)} = a^{b-c}\]

\[\frac{(3^3)}{(3^2)} = 3^{3-2}\]

\[\frac{(3 \cdot 3 \cdot 3)}{(3 \cdot 3)} = 3\]

\[(3^3) + (3^2) = 3^{3+2}\]

\[(3^3) + (3^2) = 3^1\]
53
- $8^2$
- $8^3$
- $8^9$
- $8^{18}$

54
- $5^2$
- $5^3$
- $5^6$
- $5^8$

55 Simplify: $4^3 (4^7)$
- A $4^{15}$
- B $4^8$
- C $4^2$
- D $4^7$

56 Simplify: $5^7 + 5^3$
- A $5^7$
- B $5^{10}$
- C $5^{21}$
- D $5^4$

An Exponent of Zero
Any base raised to the power of zero is equal to 1.

$a^{0} = 1$

Based on the multiplication rule:

$(3^0)(3^3) = (3^{3+0})$
$(3^0)(3^3) = (3^3)$

Any number times 1 is equal to itself

$(1)(3^0) = (3^0)$
$(3^0)(3^3) = (3^3)$

Comparing these two equations, we see that

$(3^0)= 1$

This is not just true for base 3, it's true for all bases.

57 Simplify: $5^0$ =
58 Simplify: $8^0 + 1 = \text{?}

59 Simplify: $(7)(3^0) = \text{?}

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**Negative Exponents**

A negative exponent moves the number from the numerator to denominator and vice versa.

$\left(\frac{a}{b}\right)^{-n} = \frac{1}{a^n}$

By definition:

$x^{-1} = \frac{1}{x}, \ x \neq 0$

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Based on the multiplication rule and zero exponent rules:

$(3^{-1})(3^{11}) = (3^{11+(-1)})$

$(3^{-1})(3^{11}) = (3^0)$

$(3^{-1})(3^{11}) = 1$

But, any number multiplied by its inverse is 1, so

$\frac{1}{3^1} (3^{11}) = 1$

$\frac{1}{3^1} (3^{11}) = 1$

Comparing these two equations, we see that

$(3^{-1}) = \frac{1}{3^1}$

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60

- $4^2$
- $\frac{1}{4^2}$

61

- $4^2$
- $\frac{1}{4^2}$
62

- B
- C \(\frac{x}{x^4}\)
- D \(\frac{1}{x^4}\)

63

- B \(\frac{b}{a}\)
- C \(\frac{a}{b}\)
- D \(\frac{1}{a}\)

64 Which expression is equivalent to \(x^{-4}\)?

- A \(\frac{1}{x^4}\)
- B \(x^4\)
- C \(-4x\)
- D 0

65 What is the value of \(2^{-3}\)?

- A \(\frac{1}{6}\)
- B \(\frac{1}{8}\)
- C \(-6\)
- D \(-8\)

66 Which expression is equivalent to \(x^{-1}\cdot y^2\)?

- A \(xy^2\)
- B \(\frac{x}{y^2}\)
- D \(xy^{-2}\)

67 a) Write an exponential expression for the area of a rectangle with a length of \(7^{-2}\) meters and a width of \(7^5\) meters.

b) Evaluate the expression to find the area of the rectangle.

When you finish answering both parts, enter your answer to Part b) in your responder.
### 68 Which expressions are equivalent to \( \frac{3^{-8}}{3^{-4}} \)?

- [ ] \(3^{-12}\)
- [ ] \(3^{-4}\)
- [ ] \(3^{2}\)
- [ ] \(\frac{1}{3^2}\)
- [ ] \(\frac{1}{3}\)
- [ ] \(\frac{1}{3^{10}}\)

From PARCC EOY sample test non-calculator #13

### 69 Which expressions are equivalent to \(3^2 \cdot 3^{-5}\)?

- [ ] \(3^3\)
- [ ] \(3^{-3}\)
- [ ] \(3^{-10}\)
- [ ] \(\frac{1}{27}\)
- [ ] \(\frac{1}{3}\)
- [ ] \(\frac{1}{3^{10}}\)

### Raising Exponents to Higher Powers

When raising a number with an exponent to a power, multiply the exponents.

\[(a^b)^c = a^{bc}\]

\[(3^2)^3 = 3^6\]
\[(3^3)(3^2)(3^2) = 3^{2+2+2}\]
\[(3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3) = (3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3)\]
\[(3^3)^3 = 3^6\]

### 70

- [ ] \(2^{1.75}\)
- [ ] \(2^3\)
- [ ] \(2^{11}\)
- [ ] \(2^{28}\)

### 71

- [ ] \(g^{27}\)
- [ ] \(g^{12}\)
- [ ] \(g^6\)
- [ ] \(g^3\)
74 The expression \((x^2z^2)(xy^2z)\) is equivalent to:

- A \(x^2y^2z^3\)
- B \(x^3y^3z^3\)
- C \(x^3y^2z^2\)
- D \(x^4y^2z^2\)

75 The expression \(\frac{10w^3}{5w}\) is equivalent to:

- A \(2w^2\)
- B \(2w^5\)
- C \(20w^8\)
- D \(20w^5\)

76 When \(-9x^6\) is divided by \(-3x^3\), \(x \neq 0\), the quotient is:

- A \(3x^2\)
- B \(-3x^3\)
- C \(-27x^3\)
- D \(27x^8\)

77 Lenny the Lizard's tank has the dimensions \(b^5\) by \(3c^2\) by \(2c^3\). What is the volume of Larry's tank?

- A \(6b^5c^3\)
- B \(6b^5c^6\)
- C \(5b^5c^5\)
- D \(5b^5c^8\)

78 A food company which sells beverages likes to use exponents to show the sales of the beverage in \(a^2\) days. If the daily sales of the beverage is \(5a^6\), what is the total sales in \(a^2\) days?

- A \(a^6\)
- B \(5a^8\)
- C \(5a^6\)
- D \(5a^3\)
A rectangular backyard is 5² inches long and 5³ inches wide. Write an expression for the area of the backyard as a power of 5.

- A 5¹⁵ in²
- B 8⁴ in²
- C 25⁵ in²
- D 5⁶ in²

Express the volume of a cube with a length of 4³ units as a power of 4.

- A 4⁹ units³
- B 4⁶ units³
- C 12⁵ units³
- D 12⁶ units³

Future Topics for Algebra II

Imaginary Numbers

The operation of taking the root combined with negative number allows us to ask for the square root of a negative number.

The letter i represents $\sqrt{-1}$.

An infinite set of square roots of negative numbers emerge from the use of i.

These numbers are no more or less real than any other numbers, and they have very practical uses.

Understanding their meaning and how to use them is best after studying both Algebra I and Geometry.

Complex Numbers

Combining a real number and an imaginary number results in a Complex Number.

Complex numbers are written as a + bi, where a is the real part and bi is the imaginary part.

All numbers are included in the complex numbers since setting b to zero results in a real number and setting a to zero results in a pure imaginary number.

Like Terms
**Like Terms**

**Like Terms:** Terms in an expression that have the same variable(s) raised to the same power

<table>
<thead>
<tr>
<th>Like Terms</th>
<th>NOT Like Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>6x and 2x</td>
<td>6x and x²</td>
</tr>
<tr>
<td>5y and 8y</td>
<td>5y and 8</td>
</tr>
<tr>
<td>4x² and 7x²</td>
<td>4x² and x⁴</td>
</tr>
</tbody>
</table>

**81 Identify all of the terms like 14x².**

- A 5x
- B 2x²
- C 3y²
- D 2x
- E -10x²

**82 Identify all of the terms like 0.75w⁵.**

- A 75w
- B 75w⁵
- C 3v²
- D 2w
- E -10w⁵

**83 Identify all of the terms like \( \frac{1}{4}u^2 \).**

- A 5u
- B 2u
- C 3u²
- D 2u²
- E -10u

**Combining Like Terms**

**Combine like terms**

\[
6x + 3x = (6 + 3)x = 9x
\]

*Notice when combining like terms you add/subtract the coefficients but the variable remains the same.*

**84 4x + 4x is equivalent to 8x².**

- True
- False
85 ) $3z^2 + 7z + 5(z + 3) + z^2$ is equivalent to $3z^2 + 12z + 15.$
   - True
   - False

86 ) $9r^3 + 2r^2 + 3(r^2 + r) + 5r$ is equivalent to $9r^3 + 5r^2 + 6r.$
   - True
   - False

87
   - True
   - False

88
   - True
   - False

89
   - True
   - False

90
   - True
   - False
### Evaluating Expressions

When evaluating algebraic expressions, the process is fairly straightforward:

1. Write the expression.
2. Substitute in the value of the variable (in parentheses).
3. Simplify/Evaluate the expression.

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<thead>
<tr>
<th>Slide 153 / 178</th>
<th>Slide 154 / 178</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>91</strong> Evaluate $3x + 17$ when $x = -13$</td>
<td><strong>92</strong> Evaluate $4u^2 - 11u + 5$ when $u = -5$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slide 155 / 178</th>
<th>Slide 156 / 178</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>93</strong> Evaluate $8v^2 + 9vw - 6w^2$ when $v = 4$ and $w = -3$</td>
<td><strong>94</strong> Evaluate $-10p^2 + 16pq - 64q^2$ when $p = -2$ and $q = \frac{1}{4}$</td>
</tr>
</tbody>
</table>
95 Evaluate $x^3 - 2x^2y + 64xy^2 + 16y^3$ when $x = -3$ and $y = \frac{3}{2}$

**Ordering Terms**

**Putting Terms in Order**

A mathematical expression will often contain a number of terms. As you'll recall, the terms of an expression are separated by either addition or subtraction.

While the value of an expression is independent of the order of the terms, it'll help a lot to put them in a specific order.

**Degree of a Variable**

It's good practice to order the terms by the degree of one of the variables.

The degree of a variable is the value of the exponent in a term.

$$5x^3 - x^4 + 2 - 3x$$

The degree is: $3 \quad 4 \quad 1$

constant; degree = 0

**Order Terms by Degree of a Variable**

For instance, the expression below is not ordered:

$$5x^3 - x^4 + 2 - 3x$$

This is mathematically correct, but will lead to some mistakes later in this course if you're not careful.

**Order Terms by Degree of a Variable**

It's better to write the final expression with the highest degree terms on the left and with each term to the right being next highest.

In this case, the above becomes:

$$-x^4 + 5x^3 - 3x + 2$$

This will make it easier to keep track of these terms when we do polynomials and algebraic fractions.
Order Terms by Degree of a Variable

If there are two or more variables, we have to choose one of them for this purpose.

\[-9xy^2 + x^3 + 7x^2y - y^3\]

We can choose to put the terms in order using either x or y. They would both work. Since x comes before y in the alphabet, I’ll order the terms using x.

\[x^3 + 7x^2y - 9xy^2 - y^3\]

Notice that the degrees of the variable y ascend from left to right. This won’t always happen, but sometimes it will.

Order Terms by Degree of a Variable

Let’s rearrange the terms in descending order using the powers of y.

\[-y^3 - 9xy^2 + 7x^2y + x^3\]

After this rearrangement, the degrees of the variable x ascend from left to right, too.

Both these answers work and are correct.

Order Terms by Degree of a Variable

Let’s try one more example.

\[3x^3 - x^4y^2 + 9 - 3xy^2\]

Since there are more terms in x than y, I’ll order by using x.

\[-x^4y^2 + 3x^3 - 3xy^2 + 9\]

This will make it easier to keep track of these terms when we do polynomials and algebraic fractions.

Order the terms by the degree of the variable in the expression 7x^2 + 23 - 8x + 4x^3.

A 7x^2 - 8x + 23 + 4x^3

B 7x^2 + 4x^3 - 8x + 23 + 23

C 7x^2 + 4x^3 - 8x + 23 + 23

D 4x^3 + 7x^2 - 8x + 23

Order the terms by the degree of the variable in the expression 12 - x^3 + 7x^5 - 8x^2 + 10x.

A 12 + 10x + 7x^5 - x^3 - 8x^2

B 7x^5 - x^3 - 8x^2 + 10x - 12

C 7x^5 - x^3 - 8x^2 + 10x + 12

D - x^3 + 7x^5 - 12 - 8x^2 + 10x

Order the terms by the degree of the variable in the expression 41 + x^3 + 10xy + y^2.

A x^2 + 10xy + y^2 + 41

B 41 + 10xy + x^2 + y^2

C 41 + 10xy + y^2 + 41 + x^2

D x^2 + y^2 + 10y + 41
99 Order the terms by the degree of the variable in the expression \(2y - 3x^2y^3 + 9x^3y^4 - 3x^2 + 11x\):

- \(2y + 11x + 9x^3y^4 - 3x^2\)
- \(9x^3y^4 - 3x^2y^2 - 3x^2 + 11x + 2y\)
- \(11x + 9x^3y^4 + 2y - 3x^2y^2 - 3x^2\)
- \(-3x^2y^2 - 3x^2 + 2y + 9x^3y^4 + 11x\)

---

Integers
Positive numbers, negative numbers and zero

..., -2, -1, 0, 1, 2, ...

symbol for integers

Irrational Numbers
A number that cannot be expressed as a ratio of integers.

\(\sqrt{2}\) 1.41421...
\(\pi\) 3.14159...
\(e\) 2.71828...

Like Terms
Terms in an expression that have the same variable raised to the same power.

<table>
<thead>
<tr>
<th>3x</th>
<th>x^3</th>
<th>5x^3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1/2x</td>
<td>-2x^3</td>
<td>5 NOT LIKE TERMS!</td>
</tr>
<tr>
<td>5x</td>
<td>-5x^3</td>
<td>5x</td>
</tr>
<tr>
<td>-2.3x</td>
<td>1/4x^3</td>
<td>5x^4</td>
</tr>
</tbody>
</table>

Natural Numbers
Counting numbers

1, 2, 3, 4, ...

symbol for natural numbers
Rational Numbers
A number that can be expressed as a fraction.

\[
\begin{array}{ccc}
\frac{1}{4} & 2 & 5.\overline{3} \\
\end{array}
\]

symbol for rational numbers

Real Numbers
All the numbers that can be found on a number line.

Whole Numbers
Counting numbers including 0

\[
\begin{array}{c}
0, 1, 2, 3, ... \\
\end{array}
\]

symbol for whole numbers

Standards for Mathematical Practice

MP1: Making sense of problems & persevere in solving them.
MP2: Reason abstractly & quantitatively.
MP3: Construct viable arguments and critique the reasoning of others.
MP4: Model with mathematics.
MP5: Use appropriate tools strategically.
MP6: Attend to precision.
MP7: Look for & make use of structure.
MP8: Look for & express regularity in repeated reasoning.

Additional questions are included on the slides using the "Math Practice" Pull-tabs (e.g. a blank one is shown to the right on this slide) with a reference to the standards used.

If questions already exist on a slide, then the specific MPs that the questions address are listed in the Pull-tab.